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ON THE PREDICTION OF THE NECESSARY ROTOR DYNAMICS FOR HELICOPTER FLIGHT SIMULATION

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Summary

The paper deals with the question how many degrees of freedom are needed in models for piloted simulation of helicopters.
A brief literature survey is given, from which it appears that there are contradictory opinions on this matter. The paper describes a few theoretical investigations to obtain somewhat more insight. Rigid blade flapping is taken as a first example. In the case of a simple manoeuvre it is heuristically shown when higher-order flap dynamics should be included, as an extension of the conventional 6 degrees of freedom models.
A more general example, based on the known characteristics of the Puma (articulated rotor) and the Bo105 (semi-rigid), is given next. It is found that models of semi-rigid configurations should include the regressing flap mode. A suggestion is done how to include this without a very large demand on computing power.
Finally, some suggestions are made how to proceed to a prediction procedure which may be applied before the - often tedious - task of deriving the complete model is embarked upon.

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Notations

A = matrix of stability derivatives in the 6-dof model
a_0 = coning angle (rad)
a_1 = longitudinal disc tilt w.r.t. no-feathering plane(rad)
B = matrix of control derivatives in the 6-dof model
b_1 = lateral disc tilt w.r.t. plane of no-feathering (rad)
$C_R;C_{NR}$ = flapping damping matrix in the rotating respectively non-rotating system
$F_R;F_{NR}$ = vector of external forces in the flapping equation in rotating respectively non-rotating system
I_x = mass moment of inertia around longitudinal axis(kgm^2)
I_y = mass moment of inertia around lateral axis(kgm^2)
I_z = mass moment of inertia around vertical axis(kgm^2)
I_b = inertia moment of the blade (kgm^2)
K = moment exerted on the body per

radian of disc tilt
K_β = flap hinge spring constant
K_R;K_{NR} = flapping spring matrix in the rotating respectively non-rotating system
m = helicopter mass (kg)
N = number of rotor blades
p = roll velocity of body
q = pitching velocity of body
r = yaw velocity of body
U = vector of controls in the 6 dof model
u = fore/aft translation velocity of body
V = helicopter forward speed (m/s)
v = lateral translation velocity of body
X = vector of state variables in the 6 dof model
w = up/down translation velocity of body
β_k = flapping degree of freedom of the k-th blade
β_{NR} = vector of flapping coordinates in the non-rotating system
γ = Lock-number
θ = perturbation in helicopter attitude angle
θ = blade pitch angle (rad)
$\theta = \theta_0 - \theta_c \cos\psi - \theta_s \sin\psi$
θ_0 = collective pitch (rad) at 0.75R
θ_c = lateral tilt of swashplate (cyclic stick displacement)
θ_s = longitudinal tilt of swashplate
θ_{0tr} = tailrotor collective pitch at 0.75R
θ_f = body attitude (rad)
μ = advance ratio $\mu = V/(\Omega R)$
ν = nondimensional natural frequency of the flap motion

$$\nu^2 = 1 + \frac{K_\beta}{I_b \Omega^2}$$

ρ = air density (kg/m^3)
ϕ = perturbation in helicopter lateral angle
ψ = azimuth angle (rad)
ψ = perturbation in helicopter yaw angle
Ω = angular speed of rotor (rad/sec)

1.Introduction

Piloted simulation of helicopters is relatively new, and not as advanced yet as fixed wing simulation. At the Technical University of Delft a new simulator is being built at present (SIMONA). It is designed as a general purpose research simulator, suitable for different types and categories of fixed wing aircraft as well as helicopters. For this type of simulator, dynamic models are needed based on physical principles, rather than identified models.

One of the questions is, how much detail in such models is required in terms of the number and type of mechanical degrees of freedom and the level of aerodynamic models. Of course, the answer will depend on the particular helicopter configuration, the flight conditions of interest, and the fidelity requirements.
A literature survey, briefly summarized in the next section, revealed that there is some controversy on this subject. Contradictory statements are found when it comes to the question which modes have a substantial effect on the low frequency dynamic characteristics important for flight dynamics.

It was decided to explore this field somewhat more carefully, first by getting some feeling from simple simulation examples in the time-domain, then by relating this to the frequency-domain. The examples concentrated first of all on the flapping-dynamics.

The paper presents a few of the more illuminating results obtained. It ends with a few suggestions on how to proceed, with the final goal in mind to obtain a formal procedure which is able to predict the necessary level of detail in advance, before deriving a complete dynamic model.

2. Brief Literature Survey

In the past few years a number of studies were directed toward solving the problem of proper modelling of helicopters for piloted simulation. Some of the most relevant results, as far as the necessary degrees of freedom is concerned, are summarized.

Curtiss [1] investigates the influence of flap dynamics on the stability and control characteristics of single rotor helicopters in near hovering flight. Using a simple linearized model, he examines the coupling between the body roll mode and lateral flapping motion and compares an articulated rotor and a hingeless one. He concludes that, in the case of articulated rotors, the flapping proves to be rather weakly coupled to the body motion, whereas in the case of hingeless helicopters or rotors with high offset and low Lock number the flapping has significant effect on body motion. The primary parameters influencing the body-flap coupling are the hinge stiffness and the Lock number. Concerning lag dynamics, the lag degree of freedom proves to produce little changes in the basic modes of motion of the articulated and hingeless helicopter. Studying further the influence of feedback of the body motions on the system stability, it is found that both the advancing and regressing flap modes are destabilized by the rate feedback. In ref. [2] Curtiss demonstrates that in the frequency band of 10-20 rad/sec, the regressing lag mode contributes significantly to the helicopter response.

Hohenemser [5] investigates what is the least sophisticated analytical body-rotor flap model for a hypothetical hingeless helicopter. He develops a linearized flap-body model (9-dof model) and applies two feedback systems to the helicopter: first a rotor tilting feedback system to longitudinal cyclic and second a normal acceleration feedback system to collective pitch. The influence of the feedback systems on the model is determined by eigenvalue comparisons and step control response comparisons. Concerning the first feedback system, the results show that a complete body-flap coupled model is required. For the second feedback system, the 9-dof model seems to be oversophisticated, a model using only the first order rotor dynamics being sufficient. A 6-dof model (zero order rotor dynamics) results always in substantial errors.

Diftler [3] investigates for the UH-60A BLACK HAWK helicopter an unacceptable oscillation noted by the test pilots in hover at approximately 2 Hz when the roll rate feedback gain is increased significantly above the standard values. He looks for a model which predicts this oscillation. Analyzing different approximations, he concludes that a 6-dof model does not predict this oscillation, and in fact does not predict any oscillatory instability as the roll rate gain increases. Adding the flap and lag dynamics, the lag regressing mode proves to be unstable at a roll gain of 2 rad/rad/sec, in agreement with the pilots report. A model including the control system/SAS dynamics shows an instability in flap regressive mode at a frequency of 1.6 Hz which compares reasonably well to the frequency experienced during the test flight. Further, including also higher order dynamics, as the engine/fuel dynamics, the result is not affected. Adding harmonic inflow to the linear model results in a coupled regressive flap/body/inflow mode which is destabilized at approximately the same gain value.

Fu and Kaletka [4] investigate if the higher order models with rotor degrees of freedom can be determined from the Bo105 flight test data by a system identification approach. When the 1st order flapping is added to a 6-dof model, the aperiodic roll mode becomes oscillatory, showing that the roll and flap couple. But the higher frequency modes of the rotor seem to be not adequately described by a first order system. A 2nd order model for the rotor flapping gives good agreement between measurements and model response. Later, Kaletka and Gimonet [8], using identification techniques, compared a conventional 6-dof rigid body model and a 9-dof model for Bo 105 in hover flight. They demonstrate that the response of the extended model shows slightly better agreement with the measured data, the eigenvalues representing the rigid body modes being practically the same for both models.

Houston and Horton [6] verify a theoretical 12-dof model for Puma by deriving correspondent models from flight. They show a deficiency in the theoretical model for Puma flown at 80kn, i.e. the damping ratio is underestimated and the natural frequency is overestimated. Further, the theoretical model fails to predict the trend in damping ratio and natural frequency above 100kn., but accurately predicts the normal acceleration parameters.

3. General Discussion, from an Asymptotic Point of View

As already stated, the paper will discuss primarily the flapping dynamics associated with a rigid blade model. The blade is assumed to be hinged at the hub, including a hinge spring whose stiffness is adjusted to represent different rotorsystems. Zero stiffness corresponds to a teetering rotor, a relatively large stiffness represents the semi-rigid system, whereas the articulated rotor with hinge-offset is an intermediate case.

The flapping motion, as seen from a frame of reference rotating with the blade, can be divided in three distinct time scales:

1. Fast motions, corresponding to transients associated with the eigenfrequency of the blade (angular frequency in the order of the rotor rotational frequency),

2. Intermediate fast motion, corresponding to the steady state response of the blade to control inputs and body rotations,

3. Slow motion, corresponding to the steady state response of the blade to variations of helicopter speed.

In the socalled 6-dof (degree of freedom) models of the helicopter, one concentrates on the intermediate and slow time-scales. This would seem to be obvious at first sight, since these time-scales are clearly relevant for flight dynamics. The fast time-scale is more relevant for vibration, aero-elastic stability, etc. In accordance with this, the fast blade motions are neglected in a 6-dof model, and the blade is assumed to respond instantaneously to control inputs, pitching motion and helicopter velocity. This is in fact an asymptotic approximation to the complete flapping behavior.

However, such an approximation may be misleading. In fact, what is important for flight dynamics is the body motion. The flapping behavior should therefore not be considered in the rotating frame of reference, but rather in the non-rotating frame of reference fixed to the body. It is therefore essential to first transform the blade flapping equations to the body frame. This may be done by applying the socalled Coleman-transformation. The Coleman-transformation is a multiblade transformation which also takes into account the summation or cancelling effects due to different blades of the rotor. When the Coleman-transformation is applied, it appears that in general, the transient blade motion splits into three levels: a relatively low-frequency regressing mode, an intermediate 'coning' mode, and a high-frequency advancing mode.

Under certain circumstances it is therefore conceivable that the regressing mode indeed becomes relevant for flight dynamics, despite the fact that it originates from the fast time-scale motions in the rotating frame. If this happens to be the case, the regressing flapping mode will probably couple to the body motion of the intermediate time scale.

For certain types of rotorsystem this phenomenon of coupling has indeed been observed. In the above given literature survey it has already been mentioned that a proper identification of Bo105 flight test results could not be achieved with a 6-dof model, but required a 9-dof model.

There are now two questions, to which the remainder of this paper is devoted (in an exploratory way):

1. Under which circumstances will it be essential to derive "coupled" helicopter models?

2. Considering the fact that it is just the regressing mode that is likely to couple with the body motion, is it really necessary to include the complete flapping equations in helicopter models for piloted simulation ?

Especially the latter question is important from the point of view of "real time" computing power required. Recall, that similar couplings might be required in the case of higher flapping modes, lead-lag or torsion modes, all these perhaps even coupled to higher-order body modes.

4. A Simple Manoeuvre, Analyzed in the Time Domain

In order to get some physical feeling for the problem, a very simple manoeuvre is used as an example, i.e. the first few instants during the transition from hover to forward flight, after a step input of longitudinal cyclic pitch. One may assume that just a pitching motion of the helicopter occurs at the very beginning of this manoeuvre, before forward speed builds up and begins to have an influence. For notations, see fig.1.

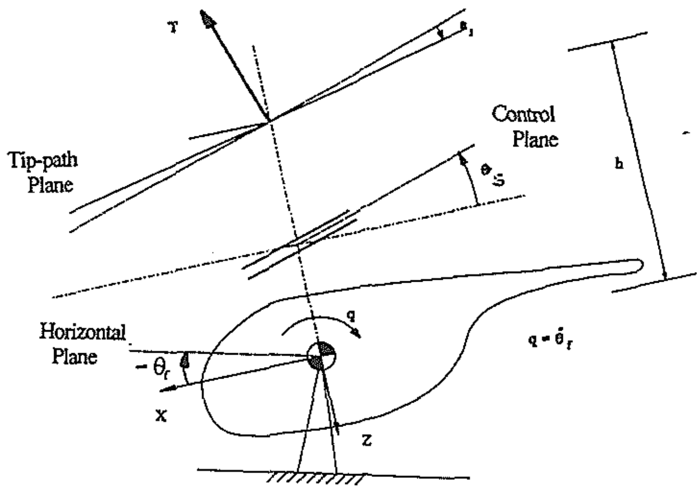

Fig. 1 Helicopter pitch motion after a longitudinal cyclic pitch step

In classical treatments of the subject, the rotordisc tilt is often assumed to respond instantaneously to control inputs, as well as to pitching motion and helicopter velocity. This in fact is equivalent to neglecting the transient flapping motion, which indeed damps out very quickly after a disturbance. Just the quasi-steady response of the rotordisc is taken into account in this classical approach. In the case considered, backward tilt of the rotordisc with respect to the no-feathering plane is given by:

$$a_1 = -\frac{16}{\gamma}\frac{q}{\Omega} \qquad (1)$$

where

a_1 = longitudinal disc tilt w.r.t. plane of no-feathering,
γ = Lock-number,
q = pitching velocity of body,
Ω = angular speed of rotor.

Eq.(1) is combined with the equation describing the pitching of the body:

$$\dot{q} = -\frac{K}{I_y}(\theta_s - a_1) \qquad (2)$$

where

K = moment exerted on the body per radian of disc tilt, due to thrust vector offset w.r. to center of gravity, as well as due to direct spring moments.
I_y = mass moment of inertia around lateral axis
θ_s = longitudinal tilt of swashplate (cyclic stick displacement)

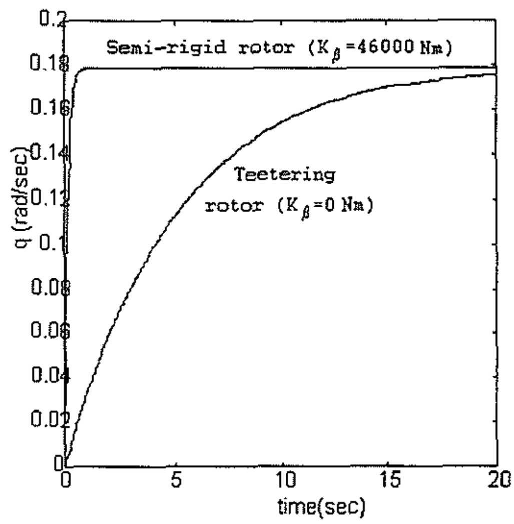

Fig. 2 Pitch response after a θ_s step input for a semirigid and teeter configuration

Fig.2 shows responses for values of K typical for a teeter-rotor and a semi-rigid configuration respectively. The teeter case shows a response characteristic typical for acceleration control, the semi-rigid case is more typical for velocity-control, which requires much less anticipation from the point of view of the pilot.

Sometimes, a refinement to eq.(1) is introduced, so that some dynamics is included in the tilting of the rotordisc:

$$\tau \dot{a}_1 + a_1 = -\frac{16}{\gamma}\frac{q}{\Omega} \qquad (3)$$

In the appendix an interpretation of this equation is given in terms of the flapping modes observed in the non-rotating frame. According to the theory of the appendix, this kind of extension of the equation for the disc tilt corresponds to taking into account the -low frequency - regressing flapping mode, on top of the steady solution. It is also shown in the appendix that the addition of only da_1/dt still neglects the advancing mode. Fully including all the transient motions of the blade (including the high-frequency advancing mode) would require a further term with d^2a_1/dt^2 in eq. (3). However, the latter is not likely to be necessary, considering the much higher frequency of the advancing mode, which would probably be outside the range relevant for flight dynamics. A more careful examination of this matter is given in the next section.

The addition of the first-order term da_1/dt does influence the response of the semi-rigid system rather profoundly, in such a way that it will probably be noticeable to the pilot (see fig.3).

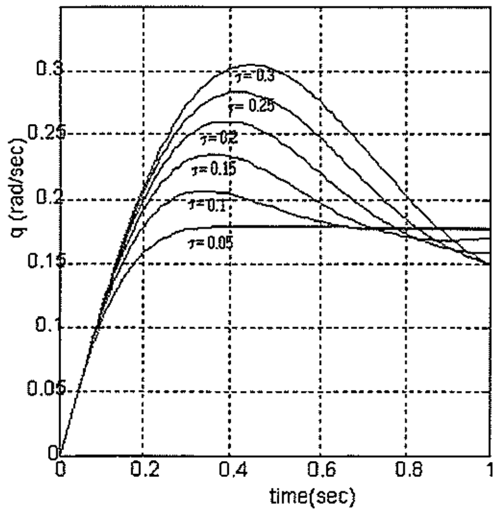

Fig. 3 Influence of da_1/dt on the pitch response of a semi-rigid rotor

On the other hand, in the case of the teetering system, the additional dynamics due to da_1/dt is hardly noticeable (see fig.4).

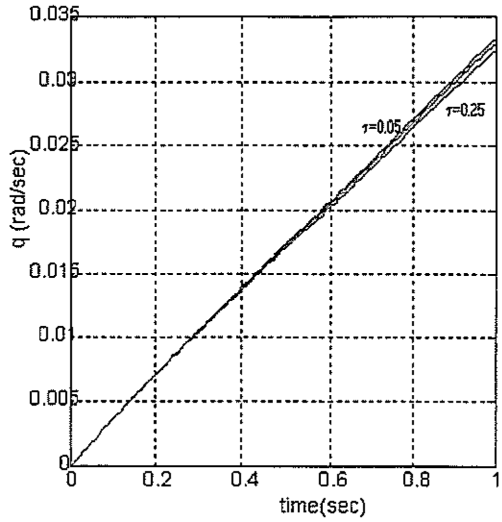

Fig. 4 Influence of da_1/dt on the pitch response of a teetering rotor

5. A Comparison of an Articulated Rotor (Puma) and a Semi-rigid Rotor (Bo 105) in the Frequency Domain

The next section gives a more formal discussion of the above mentioned phenomena. This discussion is on the one hand more general, because it is independent of a particular type of manoeuvre. On the other hand it is slightly restricted in the sense that it considers the frequency-domain, which requires a linearization of the equations of motion. An important simplification in the following is furthermore, that the body motion and the flapping are considered separately. It appears that in this way already a lot of further insight may be obtained.

5.1. Representation of the Body Eigenvalues

For the representation of the body motion, a 6 dof linear model is developed, where only quasi-steady flapping is incorporated. For a complete description of this model the reader is referred to [9,11]. This model is used here to determine the body modes. The model can be applied in the normal speed range of helicopter.

The body motion is described in a body-fixed system of reference by 6 non-linear equations of motion relative to three directions and three rotations. These equations can be linearized about a suitable trim condition. The aerodynamic model is based on linear aerodynamics, so that the model could be derived analytically. The aerodynamic forces and moments are deduced with blade element theory. They can be represented by a matrix of derivatives with respect to three linear and three angular velocity increments. Assuming quasi-steady rotor dynamics, the rotor is considered through its contributions to these derivatives of the body. After calculating the trim parameters, the stability derivatives are deduced. The stability derivatives calculated presume as basic motion a uniform forward flight on a trajectory contained in the longitudinal plane of symmetry. A uniform mean induced velocity derived with the Glauert formula [see ref. 9, pp. 4] is considered along the blade. For the stability derivatives calculations, the longitudinal and lateral motion decouples. The linear equations of motion describing perturbed motion about a general trim condition are in matrix form:

$$\dot{X} = [A]X + [B]U \tag{4}$$

where

A is the matrix of motion derivatives,
B is the matrix of control derivatives,
X the motion states

$$X = [u, v, w, p, q, r, \theta, \phi, \psi]^T$$

U the control states

$$U = [\theta_0, \theta_s, \theta_c, \theta_{0tr}]^T$$

The free motion of the helicopter is described by the homogenous form of eq.(4). Considering fixed stick (U=0) and applying the Laplace transformation, the eigenvalues of the matrix A are defined as solutions of the equation:

$$\det(A - \lambda I) = 0 \tag{5}$$

These eigenvalues characterize the motion of the body and they can be represented as points in the complex plane. They correspond to the natural modes of the motion. The longitudinal motion has 4 roots occurring in two complex conjugate pairs, corresponding to the phugoid and short period modes. The lateral motion results in 5 roots, from which one root is always zero (because of the yaw angle=0 in the basic motion), two other roots are negative real, and a complex conjugate pair corresponds to the dutch roll.

Two configurations are chosen for this study: Puma SA-330 and Bo 105 with the main parameters as defined in Table 1. A range of advance ratios between hover and μ=0.35 in forward flight(144kn for Puma, 148kn for Bo105) was chosen and the variation of the body eigenfrequencies with the advance ratio, step 0.035, was represented in fig. 5a,b. The time is nondimensionalized with the term $m/(\rho A \Omega R)$.

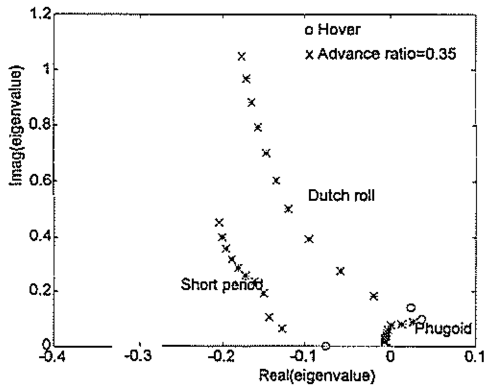

Fig. 5a Loci of Puma Eigenvalues with the Advance Ratio

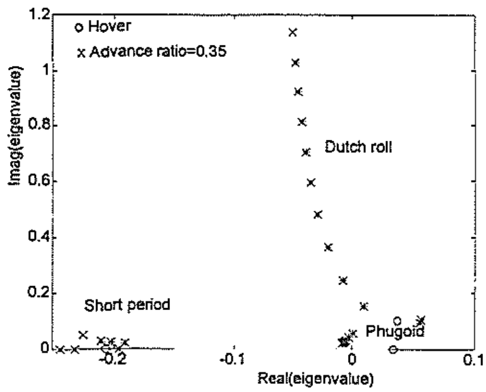

Fig. 5b Loci of Bo105 Eigenvalues with the Advance Ratio

5.2. Representation of the Flap Eigenvalues

For the representation of the flap motion, the flapping equation of motion in the rotating system is derived assuming that the blade is rigid and centrally articulated. Hingeless rotor characteristics are modelled with springs on the flapping hinge[see refs.7,10].

The blade flapping motion can be written like a vibration of a mass-damper-spring system subjected to external forces:

$$\beta_k'' + C_R \beta_k' + K_R \beta_k = F_R \qquad (6)$$

where

β_k represents the flapping degree of freedom of the k-th blade $\beta_R = [\beta_1\ \beta_2\ \beta_3 \ldots \beta_N]^T$

C_R denotes the damping term,

K_R the spring term and

F_R the external forces acting on the blade.

The notation ' stands for the derivative relative to ψ.

The configurations chosen differ in the magnitude of main rotor blade flapping frequency ratio: PUMA SA-330 has a small offset articulated rotor (ν^2= 1.05) and BO 105 has a typical hingeless rotor (ν^2=1.225).

The equations of motion associated with the rotating flapping blade are transformed to the non-rotating system using a Coleman transformation (multiblade coordinate transformation) [see refs.7,10]. This means that the motion of the blades is represented as it is seen by an observer positioned in the fixed frame. This transformation accounts for the cumulative effect of the motion of all rotor blades as seen by the body.

The transformation makes use of the multicyclic symmetry of the rotor to cancel periodic coefficients and leads to simplifications in the flapping equations of motion in the non-rotating frame.

The actual transformation depends on the number of rotor blades. For the two example helicopters in this paper, the number of blades is N=4. The flapping modes of the rotating blade are splitting through this transformation in 4 rotor modes: a 'coning' mode, a high frequency mode called 'progressive' mode, a low frequency mode called 'regressive', and a differential (reactionless) mode -introduced only for a rotor with an even number of blades ≥4. The eigenfrequencies of the flap rotor modes can now be directly compared with the eigefrequencies of the body.

The new coordinates of the flap motion in the non-rotating system correspond to the tip-path-plane coordinates as expressed in a first harmonic Fourier series used in tip-path plane approximation [for a demonstration see ref.10]

The flap equation in the non-rotating system has the form:

$$\beta_{NR}'' + \left[C_{NR}\right]\beta_{NR}' + \left[K_{NR}\right]\beta_{NR} = F_{NR} \qquad (7)$$

where

$\beta_{NR} = \left[a_0\ \ a_1\ b_1\ \beta_{N/2}\right]^T$ is the vector of flapping coordinates,

C_{NR} denotes the damping matrix in the non-rotating system,

K_{NR} the spring matrix in the non-rotating system,and

F_{NR} the external forces in the non-rotating system.

In order to obtain the eigenfrequencies of the disc tilt motion, the periodicity in (7) (i.e. the fast "wobbling motion" of the disc) is neglected. The characteristic equation of system (7) is:

$$\det(\lambda^2 + \lambda C_{NR} + K_{NR}) = 0 \qquad (8)$$

The solutions of eq. (8) representing the flap eigenvalues can be represented as points in the complex plane.

5.3. Comparison Between the Flap and Body Eigenvalues

The flap and body motion can now be compared in the complex plane. The root loci of the body and flap eigenvalues when the helicopter velocity varies (μ from 0 to 0.35 in forward flight) for Puma and Bo105 are shown in fig.6 a,b.

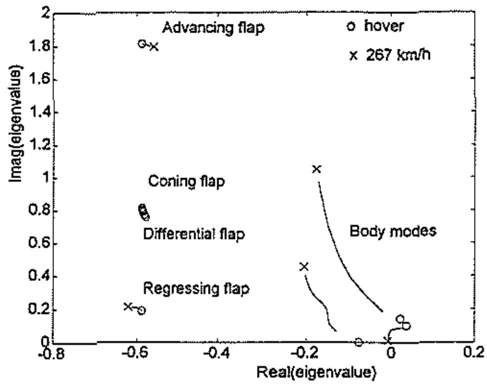

Fig. 6a Puma Body and Flap Eigenvalues as a Function of Forward Speed

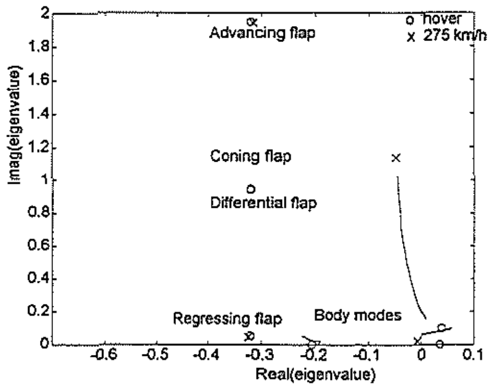

Fig. 6b Bo 105 Body and Flap Eigenvalues as a Function of Forward Speed

From the flap eigenvalues, the flap regressing mode has the closest position to the body modes. Furthermore, it appears that the regressing mode of the Bo 105 hingeless helicopter is much closer to the short period than the one found for the Puma.

This conclusion can be seen more clearly when the regressive mode and the short period mode of both Puma and Bo105 are shown on an expanded scale (see fig.7). Four advance ratios are chosen μ=0; 0.07; 0.107; 0.2. It can be seen that, in comparison with Puma, the Bo 105 flap regressive mode is situated much closer to the body short period mode in damping and frequency ratio.

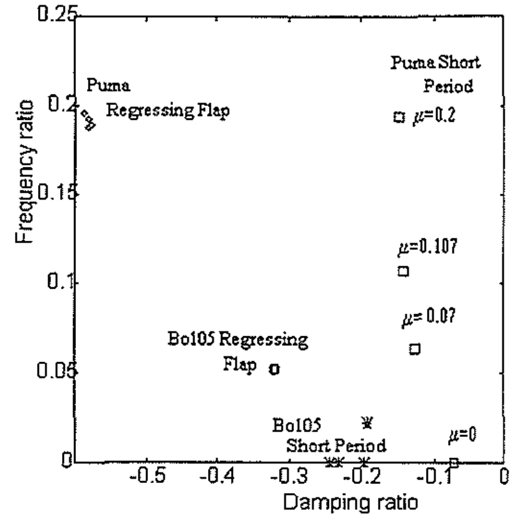

Fig. 7 Short period mode and flapping regressive mode for Puma and Bo105

5.4. Preliminary Conclusions from the Analysis in the Frequency Domain.

The latter observation may be related to the conclusions in ref.6 for Puma and refs. 4 and 8 for Bo105 where flight test results were analyzed. In both cases it was attempted to identify the flight test results using 6 dof models. This attempt was more successful in the case of PUMA than in the case of the Bo 105. In the latter case, a better identification was possible when using a 9 dof model.

The following tentative conclusions might be drawn:

- If the poles in the complex plane associated with the uncoupled body motion and the uncoupled disc motion (in the non-rotating frame) are close together, one may expect that a coupled model will be necessary. This is most likely to occur in the case of semi-rigid rotorsystems.

- A complete 9 dof coupled model might, however, not be required because the coupling effects will primarily be associated with the regressing flap mode. The coning and advancing flapping mode are situated at frequencies far above the short period mode. It is not likely that such high frequencies will play an important role in piloted simulations.

- A pragmatic approach which may save a lot of computing power is proposed in the Appendix. It consists of taking into account the relevant coupling effects by just adding first order dynamics to the conventional disc tilt equations. A simple example of this kind of approach was already shown in section 4.

6. Suggestions for a General Prediction Procedure

The analysis described here can be applied to any other degree of freedom of the rotor dynamics (lag, torsion, bending) and can be used as a criterion to reveal how many degrees of freedom of rotor dynamics are necessary to be included in a helicopter simulation model. It is only after having performed such a relatively simple exploratory analysis, that one has to proceed and write out the fully coupled, non-linear model that has been determined to be relevant in the case considered.

There are some remaining questions however. The above discussion just outlines some general ideas, which will be pursued further during future research. An obvious question that will have to be answered is the following. When comparing uncoupled roots in the complex plane, one has to judge when roots are 'close to each other' or when they are 'sufficiently remote'. The same kind of judgement is needed as far as the distance between poles and excitation sources is concerned. In other words, a quantitative criterion will have to be developed.

For this purpose it is intended to consider a number of typical cases (different rotor parameters, aerodynamic models as well as different manoeuvres). Comparisons will be made between predictions obtained by the above sketched simplified procedure and simulation results obtained by fully coupled, non-linear helicopter models.
In order to avoid the tedious work involved in deriving such fully coupled, non-linear models for a large number of cases, the procedure proposed by Van Holten will be used. This procedure, described in refs. 12 and 13 enables one to perform simulations of complex systems in the time-domain, without the need to derive explicitly the equations of motion. It is specifically designed to analyze "one-off" cases, saving the analyst a considerable amount of time, at the cost of additional computing time.

Conclusions

A possible approach has been described how to determine the necessary degrees of freedom of helicopter models for piloted simulation, before such a model is actually derived.

The few examples given for the case of flapping, indicate that semi-rigid rotorsystems do require some form of coupled flap-body motion models. However, it appears that the coupling effects are mainly restricted to the regressing disc tilt mode, coupling with the short period motion. Therefore, it might not be necessary to implement a fully coupled (more than 6 dof) model. From the theory given in the paper, it has become clear that the effects that are most important for piloted simulation may be included in a simpler way. The method proposed would save quite a lot of - real time - computing power, but needs further validation.

The procedure outlined to predict the required level of detail in real time simulation is in principle also applicable to other modes such as higher-order flapping, lag, torsion and possibly higher-order body modes. This will be a subject of future research.

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Table 1 Basic Data of Helicopter Models Used in Chapter 5

	Puma	BO 105
Aircraft mass	5805 kg	2096 kg
Distance between center of gravity and rotor shaft axis	0.1125m	-0.0076m
Distance between CG and helicopter longitudinal symmetry plane	0	0.03m
I_x	9638 kg m^2	1803 kg m^2
I_y	33240 kg m^2	4892 kg m^2
I_z	25889 kg m^2	4428 kg m^2
I_{xz}	2226 kg m^2	0 kg m^2
MAIN ROTOR		
Rotor speed Ω	270 rot/min	424 rot/min
Blade mass	68 kg	27.3 kg
Number of blades N	4	4
Blade lift curve slope	5.73 rad^{-1}	5.73 rad^{-1}
Rotor hub height above CG	1.875m	0.94468m
Blade flap moment of inertia	1280 kg m^2	219.5 kg m^2
Blade radius	7.5 m	4.91 m
Blade chord	0.5401 m	0.27 m
Flap frequency ratio υ^2	1.05	1.225
Lock number	9.374	5.087

Consider a four bladed centrally articulated rotor with spring at the flap hinge. The equations for the tip-path-plane tilt modes a_1 and b_1 in the nonrotating frame, after applying a Coleman transformation are (see ref. 10):

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}'' + \begin{bmatrix} \frac{\gamma}{8} & 2 \\ -2 & \frac{\gamma}{8} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}' +$$

$$\begin{bmatrix} \nu^2-1+\gamma\frac{\mu^2}{16}\sin4\psi & \frac{\gamma}{8}+\gamma\frac{\mu^2}{16}-\gamma\frac{\mu}{16}\cos4\psi \\ -\frac{\gamma}{8}+\gamma\frac{\mu^2}{16}-\gamma\frac{\mu^2}{16}\cos4\psi & \nu^2-1-\gamma\frac{\mu^2}{16}\sin4\psi \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}'' = \tag{A-1}$$

$$=\gamma \begin{bmatrix} 0 \\ \frac{\mu}{3} \end{bmatrix} \theta_0 - \gamma \begin{bmatrix} -\frac{\mu^2}{16}\sin4\psi \\ \frac{1}{8}+\frac{3}{16}\mu^2+\frac{\mu^2}{16}\cos4\psi \end{bmatrix} B_1 - \gamma \begin{bmatrix} \frac{1}{8}+\frac{\mu^2}{16}-\frac{\mu^2}{16}\cos4\psi \\ -\frac{\mu^2}{16}\sin4\psi \end{bmatrix} A_1$$

In order to apply an eigenvalue analysis, control inputs as well as the periodicity is neglected so that the left term of (A-1) becomes:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}'' + \begin{bmatrix} \gamma/8 & 2 \\ -2 & \gamma/8 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}' + \begin{bmatrix} \nu^2-1 & \gamma/8+\gamma\frac{\mu^2}{16} \\ -\gamma/8+\gamma\frac{\mu^2}{16} & \nu^2-1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = 0 \tag{A-2}$$

Let us consider first the hovering flight (μ=0).

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}'' + \begin{bmatrix} \gamma/8 & 2 \\ -2 & \gamma/8 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}' + \begin{bmatrix} \nu^2-1 & \gamma/8 \\ -\gamma/8 & \nu^2-1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = 0 \tag{A-3}$$

The eigenvalues are found by solving the characteristic equation of this system:

$$\begin{vmatrix} s^2+\frac{\gamma}{8}s+\nu^2-1 & 2s+\frac{\gamma}{8} \\ -2s-\frac{\gamma}{8} & s^2+\frac{\gamma}{8}s+(\nu^2-1) \end{vmatrix} = 0 \quad \Leftrightarrow \tag{A-4}$$

$$(s^2+\frac{\gamma}{8}s+\nu^2-1)^2+\left(2s+\frac{\gamma}{8}\right)^2=0$$

and they correspond to the advancing and regressing mode:

$$\begin{aligned} s_{NR_{adv}} &= -\frac{\gamma}{16} \pm i\left(1+\sqrt{\nu^2-\left(\frac{\gamma}{16}\right)^2}\right) \\ s_{NR_{regr}} &= -\frac{\gamma}{16} \pm i\left(1-\sqrt{\nu^2-\left(\frac{\gamma}{16}\right)^2}\right) \end{aligned} \tag{A-5}$$

In the case of vacuum, γ=0, the solutions (A-5) become:

$$\begin{aligned} s_{NR_{adv}} &= \pm i\ (1-\nu) \\ s_{NR_{regr}} &= \pm i\ (1+\nu) \end{aligned} \tag{A-6}$$

Now we can demonstrate that neglecting the flapping acceleration in the eq. (A-3), the advancing mode is neglected and the flapping equations keep only the regressing mode. Neglecting the flapping accelerations in (A-3) yields:

$$\begin{bmatrix} \gamma/8 & 2 \\ -2 & \gamma/8 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}' + \begin{bmatrix} \nu^2-1 & \gamma/8 \\ -\gamma/8 & \nu^2-1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = 0 \tag{A-7}$$

with the eigenvalues:

$$s_{1,2} = -\frac{\gamma}{16}\frac{\frac{1+\nu^2}{2}}{1+(\gamma/16)^2} \pm i\ \frac{(\gamma/16)^2+\frac{1-\nu^2}{2}}{1+(\gamma/16)^2} \tag{A-8}$$

The following approximation can be made (flap frequency varies between 1...1.15)

$$\frac{1-\nu^2}{2} \approx 1-\nu \tag{A-9}$$

so that:

$$s_{1,2} = -\frac{\gamma}{16}\frac{\frac{1+\nu^2}{2}}{1+(\gamma/16)^2} \pm i\ \frac{(\gamma/16)^2+1-\nu}{1+(\gamma/16)^2} \tag{A-10}$$

In vacuum the solution (A-10) becomes:

$$s_{1,2} = \pm\ i\ (1-\nu) \tag{A-11}$$

which corresponds exactly to the regressing flap mode. With aerodynamics included, let us write the solution (A-10) as:

$$s_{1,2} = -\frac{\gamma}{16}\frac{\frac{1+\nu^2}{2}}{1+(\gamma/16)^2} \pm i\left(1-\frac{\nu}{1+(\gamma/16)^2}\right) \quad \text{(A-12)}$$

Comparing (A-12) with the solution of the regressing mode (A-5), it can be seen that solution (A-12) represents the eigenvalues of the low frequency mode. In frequency, the solution of the first order flapping dynamics is situated close to the exact solution of the regressing mode (A-5) because:

$$\frac{\nu}{1+(\gamma/16)^2} \approx \sqrt{\nu^2-\left(\frac{\gamma}{16}\right)^2} \quad \text{(A-13)}$$

In damping, a term that multiplies the damping ratio appears:

$$\frac{\frac{1+\nu^2}{2}}{1+(\gamma/16)^2} < \approx 1 \quad \text{(A-14)}$$

which for typical rotors just slightly differs from 1.

In forward flight, equations (A-2) have to be considered. Representing the variation of the eigenvalues for Puma and Bo105 for the second order flap dynamics equations and also for the simplified first order flap equations, as shown in fig. A1 a,b, it is visible that the simplified flap corresponds closely to the low frequency regressing mode.

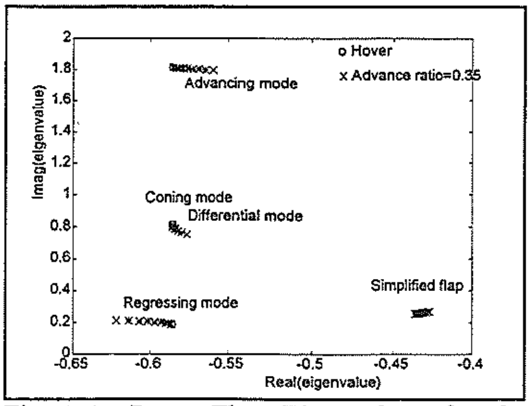

Fig. A-1a Puma Flap Eigenvalues for the Complete and Simplified Flapping Equation

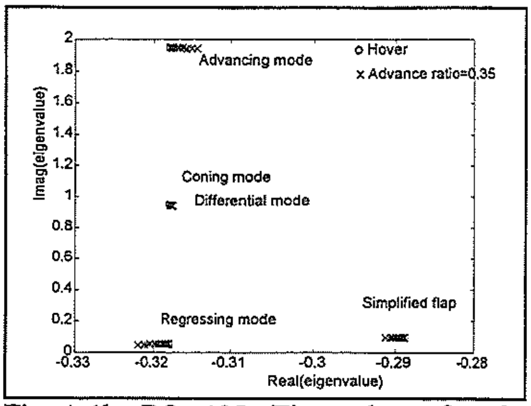

Fig. A-1b BO 105 Eigenvalues for the Complete and Simplified Flapping Equation