A NUMERICAL MODEL FOR HAIL IMPACT ANALYSIS

Marco Anghileri, Luigi-M L Castelletti, Fabio Invernizzi, and Marco Mascheroni Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano via La Masa, 34 – 20156 – Milano, ITALIA

Abstract

Hail impact is an actual threat for aircraft structures such as the forward sections of the fuselage, the engine nacelles, and the leading edges of wings and tail planes. When flying through a hailstorms region cannot be avoided, these aircraft parts are called to maintain a certain level of functionality even after being impacted by a number of hailstones. In particular, with regard to nacelle intake, these parts have to be such to avoid hailstone penetrations, which could cause a failure of the control-system of the engine usually placed in the lower part of the intake airframe and then the loss of the engine. For these reasons, and also considering the difficulty in performing specific experimental tests, it is important to develop numerical models, which eventually might represent a tool to develop highefficient and hail-proof structures. In particular, using LSTC LS-Dyna, an explicit Finite Element code, a Smoothed Particle Hydrodynamics (SPH) model of the hailstone has been developed. Initially, this model was validated referring to a laboratory test and, then, referring to the (documented) damages of an airlines aircraft. Subsequently, using the numerical model of an intake in a preliminary design phase, the impact with penetration of a single hailstone and subsequently the simultaneous impact of a number of hailstones were investigated. As a result, the numerical model developed has showed to be feasible and reliable for the analysis of the event investigated.

Keyword

Airworthiness, Hail impact, Explicit Finite Element, Smoothed Particle Hydrodynamics, Nacelle Intake.

Acronyms

FEM	Finite Elements Method
SPH	Smoothed Particle Hydrodynamics
ALE	Arbitrary Lagrangian Eulerian
EOS	Equation of State
CFRP	Carbon Fibre Reinforced Plastic
SAS	Scandinavian Airlines System
TAS	True-Air Speed

Introduction

Aircraft, flying in adverse weather conditions, could pass through stormy regions characterised by hail. Even a short period in these zones is likely to cause serious damages, which go from dents to perforations. As obvious, the worst damages are reported by the frontal sections of the aircraft: the forward sections of the fuselage (nose-cone and pilots' windshields, especially), the intake of the engine nacelles, the leading edges of wings and tail control surfaces, and other accessories such as the radar antenna and the landing lights. Thus, specific requirements (civil and military) have been established for the certification of the mentioned parts. In particular, each part has its own requirement.

It is known (as established by the observation of damaged aircraft and by specific experimental tests) that the extent of the damages depends on the mass, velocity and impact angle of the *hailstone*, and on the design (geometry and material used in manufacturing) of the impacted structures, as well (Ref 1).

Though the analysis of the weather condition (which also makes use of specific radar devices satellite allows avoiding observation) intersection with hailstorms region, it is not always possible to avoid the threat represented by the hail impact with the aircraft structure. Thus, the parts of the aircraft more exposed to this threat are called to maintain a certain level of functionality - even after being impacted by a number of hailstones. In particular, with regard to the nacelle intake, these parts have to be such to avoid hailstone penetrations, which could cause a failure of the control-system of the engine (usually placed in the lower part of the intake airframe) and then the loss of the engine.

As hail impact experimental tests are expensive, time-consuming and particularly difficult to perform, it is straightforward to understand the importance of developing numerical tools to analyze the damages caused by hail impact, which eventually could be used to develop higherficient and hail-proof structures.

Accordingly, in this research, using LSTC LS-Dyna, an explicit Finite Element (FE) code (Ref 2, 3), a Smoothed Particle Hydrodynamics (SPH) model of the hailstone has been developed.

In the first part of the research here described, the numerical model was validated, initially, referring to the results of the experimental tests carried out by the British Royal Aircraft Establishment (RAE) (Ref 5 of 4), and recalled in NASA Technical Notes 4, 5). Subsequently, as a further validation of the model, the impact of the hailstones with the nose-lip of the intake of a turbofan engine was reproduced. In particular, to verify reasonability of the numerical results, considering the numerical model of an intake in initial design phase, the damages caused by two different hail SPH models which differ in the diameter of the hailstone were compared with the photographic documentation of the damages occurred to an airline aircraft (similar in size to that of the intake

model) and caused by hailstone impacts.

Subsequently, the validated model was used to verify that the structure of the intake was such to avoid hailstone penetration inside the airframe when considering the impact velocity required for certification (such as JAR-E 970 and ACJ E 970). Finally, the simultaneous impact of a number of hailstones was considered.

As a result, the SPH model showed to be efficient and effective for the analysis of impact with penetration and of the simultaneous impact of a number of hailstones.

Hailstone numerical model

As mentioned, the development and the validation of the SPH model was carried out referring to an actual experimental test performed at the British RAE (Ref 4).

Experimental test (Ref 4)

The considered test consisted of the impact of a spherical hailstone, 25.4 mm in diameter, which impacted with a velocity of 192 m/s the centre of an Aluminium Alloy (2014-T4) panel. The panel was square (305 mm in side and 0.91 mm thick) and fixed at the extremities with blind rivets. The free surface, in the centre of the panel, had 200 mm side-edge (Fig 1).

After the impact, the panel had a plastic deformation (showed in Fig 3 referring to the section A-A of Fig 1).

The maximum displacement (measured in the centre of the panel) was 11.20 *mm*.

Numerical model of the panel

The mesh of the panel is made up of 6084 shellelements. The characteristic length of the elements was chosen to be compatible, in the definition of the contact algorithm, with the dimension of the elements of the hail mesh. In particular, the mesh was made finer in the impact region of the panel. Thus, it was possible to achieve a reasonable accuracy, avoiding excessive computational effort.

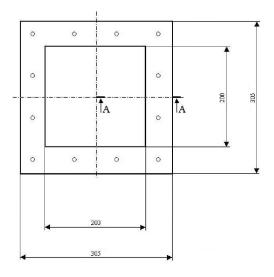


Figure 1 – Experimental test panel.

SPH model of the hailstone

An element of novelty in this research is the model of the hailstone based on the Smoothed Particle Hydrodynamics (SPH) Method.

The SPH Method was initially developed (1997) to analyse astrophysical and cosmological phenomenon. Subsequently, this method was used for problems typical of Computational Fluid Dynamic.

The main difference between SPH and FE method regards the discretisation of the continuum. The SPH is a meshless method: the mesh is replaced with a set of particles endowed with the mass, which interact one with each other through the laws of the dynamics. No direct connectivity exists among the particles. The particles are the basis of an interpolatory scheme based on the *kernel function* that is the core of the method and eventually depends on another very important feature of the method: the *smoothing length*. The value of the smoothing length has to

be chosen carefully and usually results in being a compromise between accuracy in the calculation and CPU time required for the simulation. LS-Dyna implements an algorithm that provides a variable value smoothing length – accordingly with the Conservation of the Mass.

One of the weaknesses of the SPH method is the lack of sharp boundaries, which makes imposing the boundary conditions troublesome. Another weakness is the so-called tension in instability that is a numerical failure of the continuum under tension.

The SPH model of the hailstone was developed starting from both a FE model previously developed (Ref 6, 7) and the real physical and mechanical hail properties (Ref 8).

A feature that the SPH Method shares with most of the other meshless methods is the need of a large number of particles. Consequently, using the SPH discretisation, it is mandatory to define a large number of equally spaced and uniformly distributed particles to guarantee a reasonable level of accuracy. Thus, the number of particles is usually a compromise between accuracy and required CPU-time.

A uniform distribution of particles was obtained using a specific function developed in a MATLAB environment, to guarantee the accuracy in the results.

The distance among the particles was chosen equal to the characteristic length of the element of the hail FE model: 1.2 *mm*. As a result, the model consisted of 4169 particles.

Customarily, when using a FE model of the hailstone (Ref 6, 7), the ice is modelled with the elastic plastic with failure material (*MAT_13 of LSTC LS-Dyna 2, 3)). This model allows for hardening plastic behaviour, that well reproduces the effect of the propagation of the micro-cracks inside the ice, till this, crushed, reaches a fluid-like

state. Table 1 summarizes the principal characteristics of the material model.

Since it was not possible to use this material model because (currently) not implemented by the SPH solver of the code used in the simulations, it was necessary to adopt and to validate a different Constitutive Law (Ref 8).

Several preliminary simulations were carried out considering different material models and for each one of these models different values of the characteristic parameters in order to obtain a convincing behaviour of the hailstone (Fig 2).

Table 1 – Mechanical properties of the hailstone.

Properties		j
Density	[kg/m ³]	846
Elastic shear modulus	[GPa]	3.46
Yield strength	[MPa]	10.30
Hardening modulus	[GPa]	6.89
Bulk modulus	[GPa]	8.99
Plastic failure strain		0.35
Tensile failure pressure	[MPa]	-4.00

Initially, the *isotropic elastic plastic material model* characterised by isotropic hardening (*MAT_12 of LSTC LS-Dyna 2, 3)) was considered (Fig 2a). This model appropriately reproduced the behaviour of the ice in the first instants of the impact, when nowhere in the hailstone the failure is reached. Nevertheless, the lack of a specific failure criterion made the model not adequate to represent the behaviour of the hailstone when, cracked after the impact, it behaves like a fluid. As a consequence, the impact is too severe and the panel collapse under such impact loads.

To overcome this problem, the *elastic-fluid* material model (*MAT_1 of LSTC LS-Dyna 2, 3)) was considered (Fig 2b). This model allows reproducing accurately the behaviour of the cracked hailstone, but it is not particularly

appropriate to represent the behaviour of the hailstone in the first instants of the impact, when, before collapsing, is characterised by a relatively high stiffness.

Subsequently, the *elastic piecewise (isotropic)* linear plasticity material model (*MAT_24 of LSTC LS-Dyna 2, 3)) was considered (Fig 2c). The results improved, but were not completely satisfactory. In fact, the failure criterion of this model seemed not appropriate to reproduce the failure mechanisms of the ice.

Eventually, the *elastic plastic hydrodynamic* material model (*MAT_10 of LSTC LS-Dyna 2, 3) was considered (Fig 2d). This model, properly calibrated (the mechanical properties are in Table 2), allowed representing appropriately the behaviour of the hailstone in early instants of the impact, when it is characterised by a high stiffness, and in the subsequent instants of the impact, when, cracked, it behaves like a fluid.

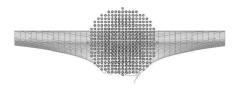
Table 2 – Hailstone mechanical properties for *MAT_10 of LSTC LS-Dyna 2, 3).

Properties		
Density	[kg/m ³]	846
Elastic shear modulus	[GPa]	3.46
Yield strength	[MPa]	10.30
Plastic hardening modulus	[GPa]	6.89
Pressure cut-off	[MPa]	-4.00

However, as a remark, it is worth noticing that the final model had the mass of the particles increased of a 5%, in order to obtain a closer numerical-experimental correlation with regard to the residual plastic strain after the impact (Fig 3). Furthermore, as the material model used requires one, it was chosen to use the Equation of State of the water.

hailstone

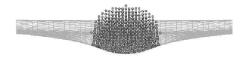
A specific failure criterion based on ultimate tensile stress was defined. This works in such a way that when the ultimate tensile stress is reached, the deviatoric stresses are set to zero and the material can sustain only compressive stresses. The failure criterion based on ultimate plastic strain was not specified because it was unsuitable to correctly reproduce the failure mechanisms of ice. Nevertheless, to bypass this lack, the numerical settling incidence due to instability under tension (typical of the SPH models) was enhanced.



a - Isotropic elastic plastic



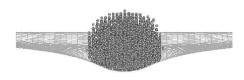
b - Elastic-fluid



c - Elastic piecewise (isotropic) linear plasticity



d - Elastic plastic hydrodynamic



e - Final SPH hailstone model

Figure 2 - Different material models used for the

Final remarks

Considering the graphical output of the simulation (Fig 4), the SPH model gives a reasonable description of the event, though not so like that of a FE model.

Furthermore, from the standpoint of the Constitutive Law, the SPH model provides a somewhat appropriate (with regard to the considered event) description of the behaviour of the ice both in early instants of the impact, when the hailstone is characterised by a high stiffness, and in the following instants, when, cracked because of the impact, it behaves like a fluid.

The comparison between numerical results and experimental data with regard to the plastic strain of the plate measured after the experimental tests shows a good (quantitative) agreement (Fig 4). In particular, in Table 3, the maximum plastic strain (evaluated in the centre of the panel) is directly compared with the displacement measured in the tests: the SPH model gives a relative error within the 1%.

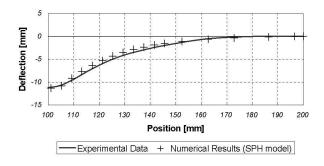


Figure 3 – Numerical-experimental correlation.

Table 3 – Numerical-experimental correlation with regard to the maximum plastic strain.

	Experimental	Numerical	
	test	model	
Maximum	11.20	11 28	
Displacement [mm]	11.20	11.20	

As a further remark, it is worth noticing that the results obtained with the SPH model were also compared with those obtained with a FE and an ALE model. From the comparison the appropriateness of the SPH model was further verified (Ref 8).

Taking into account the required CPU time, it was observed a drastic drop in the time-step during the impact when considering the FE model of the hailstone - due to the large mesh distortion. On the contrary, considering the other models, the time-step is quite constant during the simulation. In particular, the time-step of the SPH model is higher than those of the other models; accordingly, the SPH model provides a smaller calculation time. Eventually, using a common PC (a Pentium 4 - 1700 MHz CPU, and 256 Mb RAM), the required time for 700 μ s real-time simulation ranged from a minimum of about half an hour (SPH model) to a maximum of about eighteen hours (FE model) - as reported in Table 4.

Table 4 – Time-step and required CPU-time.

Time	_ CPU-Time*	
initial [s]	final [s]	
2.07 ⁻ 10 ⁻⁷	1.85 10 ⁻⁷	1 h, 4 min

^{*} CPU-time referred to a Pentium 4 – 1700 MHz CPU, and 256 MB RAM PC

Hail impact on a turbofan engine intake

The frontal section of an engine intake is one of the parts more seriously damaged when the aircraft cross a hailstorm region (and, this is one of the reasons because of which the nose-lip is made up with metallic material). In particular, the high velocity impact of a hailstone could terminate with the penetration of the hailstone inside the airframe. Usually, part of the electronic equipment that controls the engine is placed in the lower part of the intake. Thus, if the impact is particularly severe, the hailstone could eventually hit the electronic equipment and then cause the loss of the engine. Since hail impact tests are both particularly expensive and difficult to perform, it is straightforward to understand the importance of numerical models to evaluate the severity of an hail impact and hence to develop high-efficiency and hail-proof structure.

The SPH hail model previously developed was used to evaluate the consequence of a hail impact with an intake – the model of which is in a preliminary design phase.

Initially, the feasibility of the numerical model was evaluated considering documented hail impact accident occurred to aircraft with engines of the same power of the engine for which the intake was developed.

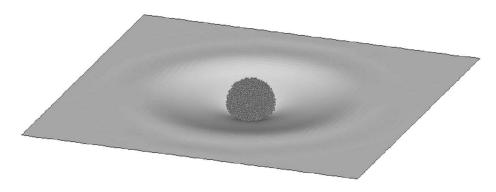


Figure 4 – Hail impact at $t = 0.150 \cdot 10^{-3} s$.

The size and the impact velocity of the hailstone were close to those recorded in the accident and the numerical results were (qualitatively) compared with the photographic documentation of the accident.

Subsequently, using the impact velocity prescribed for hail ingestion (JAR-E 970 and ACJ E 970), it was verified that the structure of the intake was such to safeguard the electronic control system of the engine.

Numerical model of the nacelle intake

The numerical model of the nacelle intake (Fig 5) was characterised by a close reproduction of the geometry of the intake under development – which, as a reference on actual dimensions, has at the leading edge a diameter of 1.60 *m*.

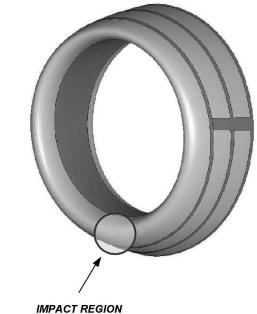
The FE model consisted of 63013 Belytschko-Tsay shell-elements (the default formulation for LSTC LS-Dyna 2, 3)). The mesh, particularly regular, was made finer in the impact region of the nose-lip. Thus, it was possible to achieve a reasonable accuracy, avoiding excessive computational effort.

The nose-lip in Aluminium Alloy 2219-T62 and the airframe in Titanium Alloy Ti-6Al-4V were modelled using the elastic piecewise (isotropic) linear plasticity material model (*MAT_24 of LS-Dyna 2, 3)), using for the mechanical properties of the materials the customary values (also indicated in the MIL handbooks).

The effect on yield stress of the high strain rate was modelled by defining Cowper-Symond coefficient. Furthermore, for the nose-lip, the presence of a residual stress was considered.

The composite material, a Carbon Fibre Reinforced Plastic (CFRP) woven with resin volume fraction of 42%, which was used in the manufacturing of the inner and outer barrel was modelled referring to a particularly effective

material model (*MAT_54 of LS-Dyna 2, 3)) and using already validated coefficients (Ref 10).



Nose-lip

Tube

Fwd bulkhead

Outer barrel

Figure 5 - FE model of the intake.

As a final remark on the numerical model, it is worth mentioning that the influence of the weather conditions (in particular of the temperature) was not considered.

Documented hail impact accidents

In order to verify the accuracy of the SPH hail model with regard to real hail impact cases, using the numerical model of the intake described, a number of impact simulations were performed. Documented accidents occurred to aircraft of the same class of the one, which the intake described

belong to, were considered. The dents shown in Fig 6a and 6b are from the post-accident photographic documentation of the accidents occurred to a Douglas DC-9-81 (MD 81) of the *Scandinavian Airlines System* (SAS) (this is used also as a quantitative reference (Ref 11) as also the diameter and the depth of the dents are available) and to a GII 183 A-C.

In particular, with regard to the accident occurred to the DC-9-81, the aircraft, flying at a *True-Air Speed* (TAS) of about 140 *m/s* trough a hailstorm region characterised by hailstones up to 50 *mm* in diameter, reported serious dents above all in the leading edge of the nacelle (Fig 6a). In particular, the largest dents were 50-70 *mm* in diameter and 5 *mm* in depth.

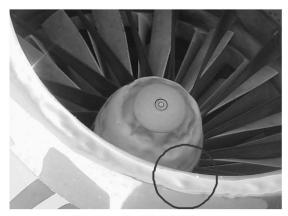
Referring to this accident, the impact of a hailstone at 140 *m*/s was reproduced using the FE numerical model of the intake previously described. This was not developed for the engine of an MD 81, but for an aircraft somewhat similar in dimensions.

The SPH hail models developed were used. As the hailstone are characterised by an extremely various range of shape and dimensions, to reproduce the most common conditions in which the aircraft can incur, were carried out simulations using spherical hailstones characterised by two different radius: 12.70 *mm* (the same previously used) and 21.35 *mm* (the dimension of a golf ball). In particular, the model of the latter was obtained from the former by scaling the dimensions.

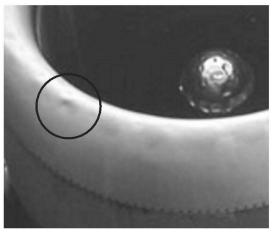
The most critical conditions were considered: the hailstone impacted the most prominent point of the lower part of the nose-lip of the intake, in direction parallel to the engine axis.

As a result of the analyses, it was observed that in none of the considered cases the hailstones were such to cause the failure of the nose-lip and penetrate inside the intake structure (Fig 7). Furthermore, it was noticed that the hail model produced dents analogous to those described in the accident documentation quantitatively (Table 5) and qualitatively (Fig 6).

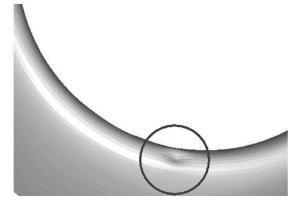
Despite the constructive differences and the lack of further impact data, the damages obtained numerically are close to the measurements relative to the actual dents and visually similar to the photographic documentation images (as evident comparing the dents in Fig 6).



a - Dents on the GII 183 A-C nacelle

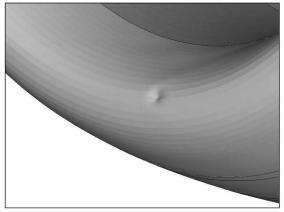


b - Dents on the MD 81 nacelle (Ref 11)

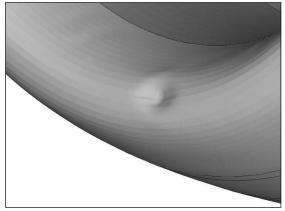


c - Dent due to a 42.7 mm diameter hailstone

Figure 6 – Dents on the nose-lips of a turbofan engine intake.



a - radius 12.70 mm



b - radius 21.35 mm

Figure 7 - Dents due to hailstone.

Table 5 - Dents due to hailstone.

SPH hail model	Diameter [mm]	Depth [mm]
r=12.70 mm	40	4
r=21.35 mm	121	10

Actual velocity hail impact

After having further verified the feasibility of the numerical model of the hailstone with regard to documented hail impacts, the consequences of a hail impact with impact velocity of 185 *m/s* was investigated. Such a velocity, for the considered aircraft, is the true-air speed (TAS) indicated by the JAR/ACJ -E 970 requirements for certification of the intake with regard to hail ingestion.

When considering the impact of the 12.70 *mm* radius hailstone, a little collapse of the nose-lip is observed (Fig. 8a).

When considering the impact of the 21.35 *mm* radius hailstone, the results showed that the hailstone, after having broken the nose-lip, penetrated inside the airframe impacting the tube and the bulkhead (Fig. 8b).

As result of the present investigation the design of the intake was modified and the nose-lip made slightly thicker, though not directly necessary for engine safety. In fact, the momentum of the hailstone after penetrating inside the intake structure was not such to break the bulkhead (reinforced on purpose) placed as a shelter in front of the electronic equipment that controls the engine.

Simultaneous impact of a number of hailstones

As a conclusion of the research, the simultaneous impact of a number of hailstones was investigated.

To consider the more adverse and real conditions an aircraft can face, it is necessary (to be able) to reproduce the simultaneous impact of a number of hailstones. In particular was considered: the simultaneous impact of ten hailstones 12.7 mm in diameter placed at the same initial distance from the nose-lip. The number and the average distance between each other (equals to 100 mm) were chosen, made reference to the statistics.

The impact velocity was the same previously considered (185 m/s).

Considering the simultaneous impact of ten hailstones, the region in the nose-lip invested by the hail was wider. Thus, it was necessary to extend the region characterised by a finer mesh. This adjustment brought the number of elements to 89701.

The contact interaction was defined *only* between each one of the hailstones and the nose-lip of the intake. In fact, since during the preliminary simulations it was observed that the hailstones do not come into contact with other parts of the intake, it was deemed not necessary to define a contact interaction among hailstones and the remaining parts of the intake. Furthermore, it was not defined a contact interaction among the hailstones. Automatically a contact interaction starts when a particle belonging to one of the hailstones enters the influence sphere of another hailstone particle 2, 3).

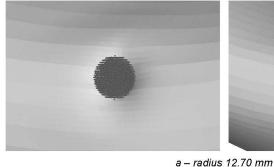
As straightforward to understand, the impact of ten hailstones (Fig 9a) is much more severe than the impact of a single hailstone – and not only for the larger momentum transferred from ten hailstones to the structure. The results obtained evidenced that each one of the hailstones produced a local collapse in the structure of the nose-lip (Fig 9b). Nevertheless, the hailstones, completely cracked, did not penetrate into the intake and, hence, did not cause further damages to the structure.

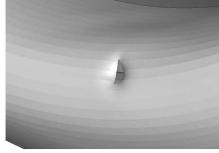
In Fig 9a, a frame from the simulation and the damages produced on the nose-lip by the impact

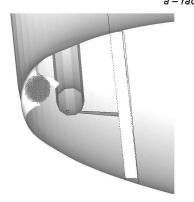
of the hailstones are shown with regard to the same time ($t = 0.30 \cdot 10^{-3} s$).

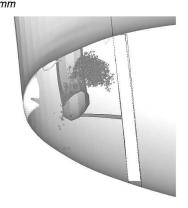
As a concluding remark, it is important to mention that the SPH hailstone model was the only that allowed to reproduce in detail and with acceptable computational efforts the simultaneous impact of a number of hailstones.

Indeed, this event (even more than the impact with penetration) is characterised by large distortions. Thus, a premature termination of the analysis was obtained when the same event was simulated using a FE model for the hailstones. A severe drop in time-step due to the excessive caused mesh distortions that. Simulation performed using an ALE model for the hailstones gave unsatisfactory results. ALE models are basically insensitive to mesh distortions. Nevertheless, the numerical results were not particularly accurate also because of the customary problems arising from the coupling of two solvers deeply different in nature: the Eulerian and Lagrangian solver.





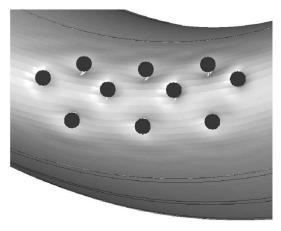




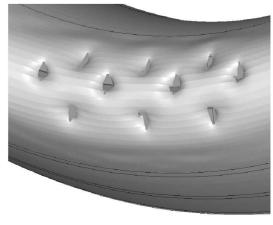
b – radius 21.35 mm

Figure 8 – Actual velocity hail impact

With regard to the required CPU-time, it is worth noticing that the SPH model allows a time-step considerably higher (10⁻⁷ s) than those associated to the other numerical models such as FE or ALE and constant during the analysis. Furthermore, being characterised by a smaller time-per-cycle, the SPH model does not require computational effort.



a – hailstones impact



b - resulting dents

Figure 9 - Ten hailstones simultaneous impact.

Conclusions

Hailstone impacts represent an actual threat for aircraft structures and, therefore, it is important to develop numerical tools to design high-efficiency and hail-proof structures. Thus, in this research, using an explicit FE code (commercially available), LSTC LS-Dyna 960, a SPH model for the hailstone has been developed and validated

with regard to actual hail impact cases. As a result, a close correlation with experimental data was obtained.

In particular, it was necessary to individuate a Constitutive Law different from the customary elastic plastic with failure material. Nevertheless, eventually, the SPH model allowed obtaining results, *qualitatively* and *quantitatively*, close to the experimental data. (*Qualitatively*, referring to the impacted structure deformation, with errors within 1%, and *quantitatively* referring to the panel damages and the hailstone behaviour).

The same model, in a subsequent stage of the used research, was to investigate consequences of the impact of a hailstone against the nose-lip of the intake of a turbofan engine in preliminary design phase. However, before considering the case of interest, to verify the feasibility of the hailstone model, hail impacts were simulated using velocities close to those of a documented case, so that it was possible to compare the numerical results with the measures of the dents and photographic documentation of actual damages. As a result, a close agreement with the real dents (in size and visually) was achieved.

Then, simulations using the impact velocity prescribed for the certification of the intake were carried out and finally also the simultaneous impact of many hailstones were considered.

In all the cases considered, the SPH model of the hailstone has showed to be feasible for the analysis of the event and extremely efficient (able to reproduce the hailstone behaviour even when it crumbles after the impact and able to provide a close numerical-experimental correlation in small calculation times). In particular, since SPH model, being based on the Lagrangian approach and being meshless, seemed particularly feasible and reliable for the analysis of *impact of hailstone with penetration* and of *simultaneous impact of a number of hailstones*.

References

- 1 Souter RK, Emerson JB. Summary of Available Hail Literature and the Effect of Hail on Aircraft in Flight. NACA Technical Note 2734, Washington, 1952, p. 1-33
- 2 Hallquist JO. LS-DYNA Keyword User's Manual, LSTC, Vol.1-2, Mar. 2001
- 3 Hallquist JO. LS-DYNA Theoretical Manual, LSTC, May. 1998
- 4 Thomson RG, Hayduk RJ. An Improved Analytical Treatment of the Denting of Thin Sheets by Hail. NASA Technical Note D-6102, Washington, D.C., Jan. 1971, p.1-36
- 5 Thomson RG, Hayduk RJ. An Analytical Evaluation of the Denting of Airplane Surfaces by Hail. NASA Technical Note D-5363, Washington, D.C., Aug. 1969, p. 1-34
- 6 Kim H, Kedward KT. Experimental and Numerical Analysis Correlation of Hail Ice Impacting Composite Structures. AIAA-99-1366, 1999, p.1416-142
- 7 Kim H, Kedward KT. Modeling Hail Ice Impacts and Predicting Impact Damage Initiation in Composite Structures, American Institute of Aeronautics & Astronautics, Vol. 38, No. 7, Jul. 2000, p. 1278-1288
- 8 Anghileri M, Castelletti L-ML, Invernizzi F, Mascheroni M. A survey of numerical models for hail impact analysis using explicit Finite Element codes, sent to International Journal of Impact Engineering
- 9 Schulson EM. The Structure and Mechanical Behavior of Ice. Army Research Office, JOM 51 (2) 1999, p.21-27
- 10 Anghileri M, Castelletti L-M L, Lanzi L, and Mentuccia F. Composite materials and bird-strike analysis using explicit finite element commercial codes, 8th International Conference on Structures Under Shock and Impact (SUSI) 2004
- 11 Accident Investigation Board Finland, Aircraft
 Damage in Hailstorm West of Helsinki on
 21.7.2001, Investigation report B 5/2001 L, pp.
 1-31