

Application of pilot induced oscillations prediction criteria to rotorcraft

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Abstract

This paper investigates the applicability of fixed-wing aircraft pilot induced oscillations criteria to rotorcraft. Three prediction criteria, among the several proposed in literature, were chosen for this study, namely the Bandwidth/Phase Delay method, the Open Loop Onset Point (OLOP) criterion and the Realtime Oscillation VERifier (ROVER). A pilot+rotorcraft mathematical model, developed in MATLAB®, was used to test the criteria. Pilot-in-the-loop simulations were carried out in a fixed-base simulator to verify the criteria's reliability. The global results obtained from the comparison of the PIOs criteria outcomes with the simulation tests data showed an acceptable correlation. The simulator tests provided useful information about the choice of the manoeuvres most suited to detect a rotorcraft's PIOs tendencies.

Introduction

The pilot induced oscillations (PIOs) phenomenon is characterized by large oscillations in pilot control inputs and in aircraft response; it is due to an incorrect and anomalous interaction between the pilot and the vehicle dynamics, so that, when an oscillation is initiated and the pilot tries to damp it out, his/her control inputs act only to sustain or drive the aircraft's oscillatory response to greater amplitude.

From several time traces of past PIOs incidents and flight test programs, some characteristics were detected as recursive and strictly related to the PIOs onset and development [1, 2]:

- the presence of an oscillatory motion;

- the aircraft response is out of phase with the pilot control inputs: a 180 deg and a 90 deg phase difference are measurable between the pilot's inputs and respectively the aircraft attitude and rate;
- the frequency of the oscillation is contained in a specific range: the most common frequencies where PIOs occur are between approximately 1 and 8 rad/s;
- the amplitude of control inputs, aircraft responses, or both, are large enough to have an effect.

As briefly mentioned above, the PIOs phenomenon is the result of an irregular coupling between the pilot and aircraft dynamics, however a third element, commonly called a 'trigger', is necessary for the initiation of the oscillations, such that, if the aircraft is susceptible to PIOs, the trigger event may lead the pilot to make a sudden and abrupt control correction that in turn can activate a PIO [3, 4]. It may come from the

environment, the pilot control action or the vehicle dynamics.

Given the impossibility of avoiding the occurrence of all possible triggers and of predicting exactly the pilot actions, it is necessary to design aircraft such that they do not exhibit tendencies to PIOs, whatever the triggers and the pilot control actions are. In order to expose potential PIOs susceptibility and to implement appropriate modifications to the design, thus preventing risks and avoiding late project changes in advanced design stages, PIOs prediction criteria have been developed to detect aircraft PIOs sensitivity in the early design phases.

The characteristics of the pilot and aircraft dynamics have been adopted as a criterion of the classification of PIOs phenomena [1]:

- Category I: linear pilot-vehicle system oscillations; the effective aircraft dynamics is essentially linear and the pilot behaviour is quasi-linear. These PIOs result from phenomena such as excessive time delay, excessive phase loss due to filters, improper control/response sensitivity, etc. Category I PIOs are the simplest to model, understand, and prevent. They are also the least common in operational flying.
- Category II: quasi-linear events with some non-linear contributions, such as rate or position limiting; these PIOs can be modelled as linear events, with a non-linear contribution that may be treated separately. The most common non-linear contribution is rate limiting of a control effectors actuator; additionally, non-linearities such as stick command shaping or aerodynamic characteristics may also be included.
- Category III: non-linear PIOs with transients; such events are difficult to recognize and rarely occur, but are always severe. These PIOs are mostly associated with non-linear transitions in either the effective controlled vehicle dynamics or in the pilot's behavioural dynamics, that may be related to changes in the flight mission. This mode switching can not be represented by a quasi-linear equivalent model.

The PIOs phenomenon become a research subject specifically for fixed-wing aircraft, most likely because the first major, and in some cases fatal, accidents involved fixed wing vehicles rather than helicopters [5, 6, 7]. Thus, since the end of the 50s, the phenomenon started to be investigated: through the analysis of PIOs incidents, the characteristics and concurring sources of the phenomenon were assessed and a first comprehensive analysis of the phenomenon was proposed in [8]; at the same time, the role of the pilot in the PIOs onset and his dynamics during a fully developed PIOs episode became a research topic [9].

Similarly, PIOs prediction criteria were first developed for fixed wing aircraft [10, 11]. These

methods were assessed and validated against flight tests coming from handling qualities studies and data collected from PIOs episodes during operational flights and flight and ground based test campaigns scheduled during the years.

Compared to the fixed wing world, relatively little has been said about rotorcraft PIOs, although rotorcraft dynamics is not exempt from this phenomenon, as demonstrated by several accidents [3, 12]. Therefore, the research activity dealing with the pilot-rotorcraft coupling phenomenon is not as advanced as in the fixed wing aircraft case and very few published papers analysing this subject can be found in literature [12, 13]. In flight PIOs episodes data are still limited and dedicated flight and ground-based-simulator test campaigns for rotorcraft PIOs are not as numerous.

The PIOs prediction criteria originally used for fixed-wing aircraft, have been applied to rotorcraft, but, as a consequence of the gap explained above, they have not been applied to rotorcraft as extensively as to fixed wing vehicles and, due to the lack of a large experimental database, their validity is not as high.

The present work aims to be an introductory study within a wider context which plans to define a set of guidelines for the evaluation of rotorcraft PIOs susceptibility and to develop a simple mathematical tool for the application of PIOs prediction methods.

The objectives of this work are:

- the definition of a suitable pilot-rotorcraft mathematical model for the application of PIOs prediction criteria;
- the application of the following prediction criteria to this model:
 1. the Bandwidth/Phase Delay criterion;
 2. the Open Loop Onset Point (OLOP) criterion;
 3. the Real-time Oscillations VERifier (ROVER), for the detection of PIOs in real time;
- the identification of the possible changes to be implemented in the application procedure of the criteria, in order to make them suitable for rotorcraft dynamics;
- the verification of the criteria results through pilot-in-the-loop simulations.

Pilot-rotorcraft mathematical model

The PIOs phenomenon is the consequence of an anomalous coupling between the aircraft dynamics and the pilot. Therefore, in order to perform the analysis of the elements involved in the development of the oscillations, to understand the modality of PIOs onset and to investigate the applicability of the prediction criteria, a mathematical model of the rotorcraft-pilot system is needed.

Rotorcraft model

Both the Bandwidth/Phase Delay and the Open Loop Onset Point (OLOP) methods are based on an approach that requires the frequency analysis of the pilot-vehicle system, thus, in order to apply these criteria and to investigate their reliability, a linear model of a rotorcraft was developed.

This model was derived from the comprehensive non-linear model of an example twin-engine medium class helicopter provided by AgustaWestland. Four trim conditions were considered:

- hover;
- forward flight at 40 kts;
- forward flight at 50 kts;
- forward flight at 80 kts.

The hover condition was chosen because the hover is considered, by the literature on the subject, a critical flight regime for PIOs onset and development [1, 14]; the other three flight conditions have also been included in this study in order to investigate potential dependencies of PIOs tendencies on flight speed.

The linear model used included: 6dof rigid body dynamics, main rotor model (with both flap and lag dynamics), three-state Peters-He inflow model, tail rotor dynamics (with cone and inflow dynamics) and flight control system (FCS).

Pilot model

A pure gain pilot was used for the PIOs prediction criteria application. This choice was supported by analytical studies and experimental data published in the literature, according to which the pure gain pilot model is able to provide a good first approximation of the real pilot dynamics during PIOs. In fact, more than one study, including those proposed by Gibson [15], McRuer [3] and Duda [16], state that during a fully developed PIO, especially a large amplitude severe episode, either of Category I or II, the pilot dynamics transits instantly to the synchronous control, which is a particular form of precognitive behaviour wherein the pilot is able to respond to given task requirements with open loop inputs based on an expected response. In the synchronous behaviour the pilot duplicates a sinusoidal input signal with neither time delay nor phase lag.

Fig. 1 shows a scheme of the comprehensive pilot-rotorcraft model used in this investigation.

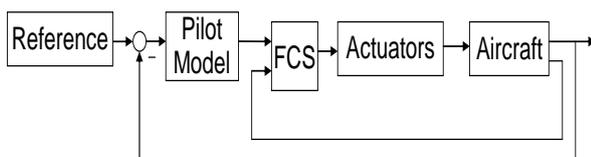


Figure 1 – Scheme of the pilot-rotorcraft model

The pilot is represented as a pure gain on the longitudinal cyclic, lateral cyclic and pedals channels; it processes the error signal resulting from the comparison

of the attitude reference signals and the rotorcraft output data. The FCS elaborates the pilot output through the control laws. Two actuators are placed on the longitudinal and lateral cyclic control channels. The simplified actuator model employed here is identified through two parameters: the bandwidth (BW) and the rate capability (RC).

The comprehensive pilot-rotorcraft model showed good time domain performances, as can be observed by the tracking tasks presented in Fig.2 and Fig. 3.

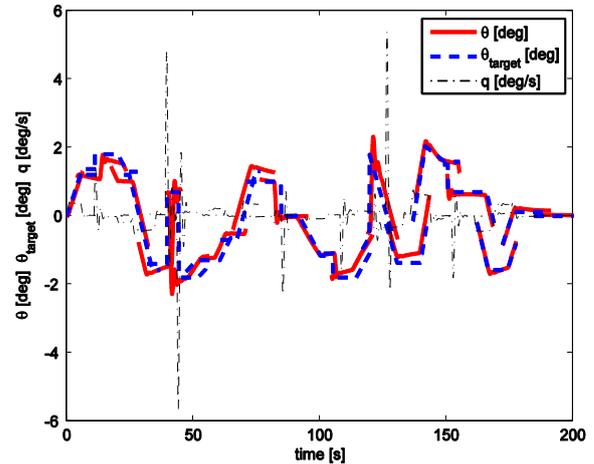


Figure 2 - Pitch attitude tracking time trace

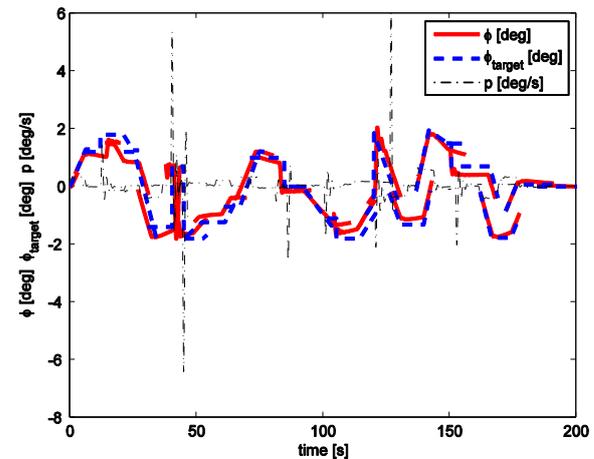


Figure 3 - Roll attitude tracking time trace

Application of PIOs prediction criteria

Several methods for the prediction of aircraft PIOs susceptibility are available in open literature; these criteria focus on PIOs of either Category I or II.

In this analysis, to investigate the applicability of the criteria to rotary-wing aircraft, two PIOs prediction criteria have been selected, namely the Bandwidth/Phase Delay and the Open Loop Onset Point criteria, that respectively address Category I and Category II PIOs phenomena. A third method (ROVER), focusing on the detection of the onset of PIOs in real time, will be presented in a later section of this paper.

The data presented in the following sections have been scaled with respect to a specific factor for data protection.

Bandwidth/Phase Delay criterion

The criterion was developed and proposed by Hoh and Hodgkinson [11] in 1982 as a prediction method for Category I PIOs and validated on the basis of the Neal-Smith and the LAHOS database. It partially derives from the requirements for small amplitude attitude changes in precision tracking tasks, which are defined in the regulations, both MIL-STD-1797A for fixed wing and ADS-33 for rotary wing aircraft.

Application procedure. The Bandwidth/Phase Delay criterion is based on the analysis of the open-loop transfer function of the aircraft response in pitch(roll) attitude to pilot input, expressed as a stick displacement or force. The calculation of the following parameters is required:

- bandwidth frequency, ω_{BW} ;
- phase delay, τ_p : describing the rate of phase roll-off, i.e. the steepness of the phase curve beyond the neutral-stability frequency, ω_{180} .

It is calculated as:

$$\tau_p = \frac{\pi}{180} \frac{\Delta\Phi_{2\omega_{180}}}{2\omega_{180}} \text{ [s]} \quad (1)$$

- flight path bandwidth, $\omega_{BW\gamma}$ (for longitudinal PIOs): defined as the phase-margin bandwidth frequency of the flight-path-to-stick-force response. A low flight path bandwidth indicates the need for a pilot lead generation to improve the response; in combination with a moderate value of phase delay, PIOs are possible;
- pitch rate overshoot, $\Delta G(q)$ (for longitudinal PIOs): a frequency-domain measure of overshoot that is normally defined in the time domain. High pitch rate overshoot has been known to be undesirable, resulting in bobble tendencies and excessively abrupt short-term response.

Although slight discrepancies [17] concerning the specific values of the criterion boundaries still exist, the metric for the evaluation of the aircraft PIOs susceptibility through the application of the Bandwidth/Phase Delay method was defined as shown in Fig. 4 [3, 13, 18, 19, 20]:

- the aircraft is prone to PIOs if the phase delay parameter is approximately $\tau_p \geq 0.19s$;

- the aircraft may be prone to PIOs if simultaneously $\tau_p \geq 0.14s$ and $\omega_{BW\gamma} < 0.58 \text{ rad/s}$;
- the aircraft may exhibit moderate PIOs if $\Delta G(q) > 12\text{dB}$ and $\omega_{BW} < 1 \text{ rad/s}$, even for $\tau_p < 0.14s$;
- the aircraft can be considered as not susceptible to PIOs if $\omega_{BW} > 1\text{rad/sec}$ and $\tau_p < 0.14s$.

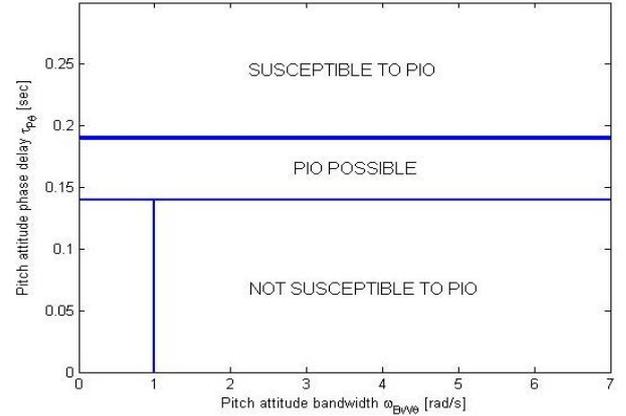


Figure 4 – Bandwidth/Phase Delay criterion chart

The criterion was applied to the pilot-rotorcraft mathematical model described above. As a first attempt, only the longitudinal plane was taken into account and, due to the fact that the phase delay τ_p is the dominant and most critical parameter of the criterion, neither the flight-path transfer function nor the pitch rate overshoot were considered. Thus, only the system bandwidth frequency ω_{BW} and the phase delay τ_p were calculated.

As mentioned above, an actuator model was implemented on the longitudinal and lateral cyclic channels; a second order dynamics actuator model was used:

$$f_{ACT} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (2)$$

where:

- ω_n is the actuator natural frequency, expressed in rad/s;
- the relation between the actuator natural frequency and the bandwidth (BW), both expressed in rad/s, is:

$$\omega_n^2 = BW^2 + 2\zeta BW\omega_n \quad (3)$$

- ζ is the actuator damping.

For the criterion application, the bandwidth of the actuators, BW, starting from the nominal condition, was progressively decreased on both control channels, and hence the natural frequency was decreased as well due to equation (3). In this way, the dynamics of the actuators was slowed down, thus increasing the system phase delay τ_p and the rotorcraft's PIOs susceptibility.

Results. The Bandwidth/Phase delay criterion was applied to several configurations of the pilot-helicopter system, for decreasing values of the actuator's bandwidth. The results did not show strong dependence on flight speed; thus in the following only the hover results are presented.

Table 1 – Hover, Bandwidth method results

<i>Hover condition</i>	
<i>Actuator bandwidth</i> <i>BW [Hz]</i>	<i>Phase delay</i> τ_p [s]
5	0.0946
4	0.1060
3	0.1219
2	0.1506
1.75	0.1627
1.6	0.1724
1.5	0.1778
1.43	0.1912
1	0.2204
0.9	0.2305

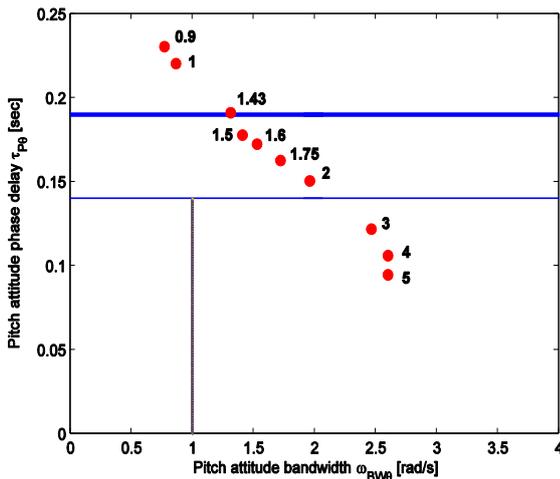


Figure 5 – Bandwidth criterion graph; results for the hover condition

As shown in the (ω_{BW}, τ_p) graph (Fig. 5) and in Table 1, according to the criterion metric explained before, the aircraft is stated as:

- PIOs prone for actuator bandwidth $BW \leq 1.4$ Hz. This value is approximately valid for all the flight regimes considered;
- possibly PIOs prone for actuator bandwidth $BW \leq 2$ Hz. This value is approximately valid for all the flight regimes considered, but, as mentioned above, the phase delay τ_p value is not a sufficient condition, because, at least for the longitudinal PIOs tendencies analysis, the configuration must also meet the condition on the flight path response $\omega_{BW\gamma} < 0.58$ rad/s, which in this analysis was not considered;
- for those configurations in which the phase delay $\tau_p < 0.14$ s, the system bandwidth results $\omega_{BW} > 1$ rad/s, thus no PIOs tendencies are detected.

Open Loop Onset Point (OLOP) criterion

The OLOP criterion was developed by DLR [16] in the late 90s with the purpose of investigating Category II PIOs, specifically those oscillations caused by actuator rate saturation, which is one of the most common and insidious sources of PIOs phenomena. According to the theoretical basis of the OLOP method [21], it was observed that a high correlation exists between the location of the Open Loop Onset Point in a Nichols chart (a phase-amplitude graph) and the severity of the corresponding jump phenomenon, and associated increase in phase delay, in the closed-loop transfer function of the aircraft-pilot system. A stability boundary in the Nichols chart was proposed to discriminate the cases of rate saturation which could likely incur in instability and PIOs onset from those cases in which the rate limiting does not reduce the pilot-aircraft system stability.

Application procedure. The criterion was applied to the pilot-rotorcraft system, according to the scheme in Fig. 6. Rate Limiter Elements (RLEs) are located both on the longitudinal and lateral cyclic control channels, although in this analysis the criterion was used to detect longitudinal PIOs susceptibility only.

Step 1. A synchronous behaviour pilot model was adopted, due to the fact that, during fully developed PIOs, the pilot acts as a pure gain. The gain K_{Pilot} was adjusted based on the linear aircraft-pilot crossover phase angle Φ_{cr} , that should be varied within the range from $\Phi_{cr} = -120$ deg (low pilot gain) up to $\Phi_{cr} = -160$ deg (high pilot gain). In this investigation it was decided to approach the analysis of the OLOP method considering a medium pilot gain, without investigating any extreme pilot gain values, thus the pilot parameters were set in order to achieve a crossover phase angle Φ_{cr} of about -140 deg.

Step 2. Calculation of the linear closed-loop frequency response from the stick force (or deflection) F_{es} , i.e. the output of the pilot model, to the input of the RLE, which, in this analysis, is the longitudinal cyclic δ_b :

$$F_{fes}^{\delta_b}(j\omega) = \frac{\delta_b}{F_{es}} \quad (4)$$

Step 3. Calculation of the closed loop onset frequency $\hat{\omega}_{onset}$ according to the equation:

$$\hat{F}_{es} \cdot |F_{fes}^{\delta_b}(j\hat{\omega}_{onset})| = \frac{RC}{\hat{\omega}_{onset}} \quad (5)$$

where \hat{F}_{es} is the maximum stick force (or maximum stick deflection) and RC is the rate capability of the RLE.

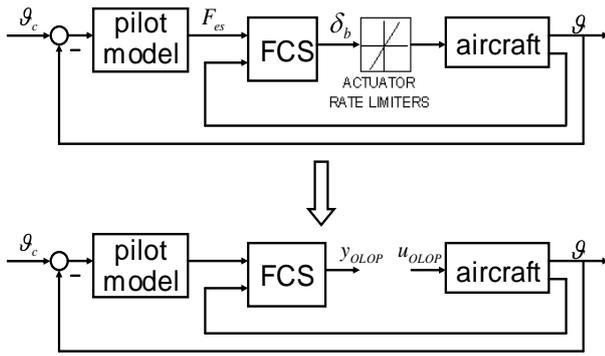


Figure 6 – Closed and Open loop model for frequency response definition

Step 4. Calculation of the required linear open-loop frequency response $F_{OLOP}(j\omega)$. Referring to Fig. 6, $F_{OLOP}(j\omega)$ is determined by cutting the control loop at the rate limiter and analyzing the system with the RLE removed: the input of the open-loop system, u_{OLOP} , corresponds to the output of the rate limiter, while the output of the open-loop system y_{OLOP} is defined as the input of the rate limiter.

Phase and amplitude of the transfer function $F_{OLOP}(j\hat{\omega}_{onset})$ constitute the coordinates of the OLOP in the Nichols chart, $OLOP(\Phi_0, A_0)$.

Results. The method was applied to the four flight conditions examined for decreasing actuator RC. For each flight regime considered the pilot gain was adjusted according to the procedure outlined in Step 1.

As mentioned above, according to the criterion theory, the maximum stick deflection should be used.

This value, for the rotorcraft considered, is equal to 11 deg, i.e. half the maximum deflection of the longitudinal cyclic stick.

In this case, the method appeared to be strongly conservative. In fact it detected PIOs susceptibility for RC = 45 deg/s, which appeared to be too high and not a reliable value. In Table 2 and Fig. 7, the results obtained for hover are presented.

Table 2– OLOP results for $\hat{F}_{es} = 11$ deg, hover

Hover			
Rate capability RC [deg/s]	Onset frequency ω_{onset} [rad/s]	OLOP phase Φ_0 [deg]	OLOP magnitude A_0 [dB]
75	6.2156	-151.42	-6.6032
60	4.8143	-151.25	-1.8568
55	4.5141	-151.02	-0.6761
45	4.2138	-150.76	0.4816
40	4.1597	-150.52	1.2874
35	3.7432	-150.27	2.2546
30	3.5177	-149.99	2.6504
20	2.8125	-149.02	5.3990

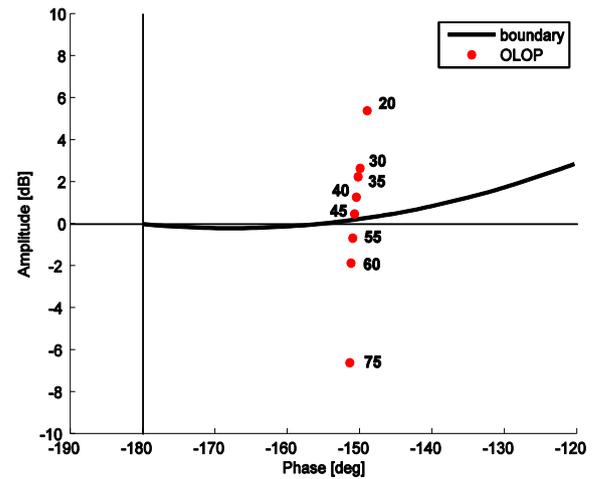


Figure 7 - OLOP criterion graph; results for $\hat{F}_{es} = 11$ deg for the hover condition

In the few papers found in literature, that deal with the OLOP method, no clear information about the choice of the parameter \hat{F}_{es} are given. In [22] the author assumed $\hat{F}_{es} = 1.5$ inches, as determined by a simulation with a pilot model, but [21] noted that this value was far below the maximum stick deflection of the aircraft considered.

In the analysis of [21], the OLOP method was found to be strongly dependent on the value assumed for \hat{F}_{es} , as it also appears in equation (5), and also over conservative when the maximum stick deflection or stick force was used. The reason lies in equation (5): considering the graphical approach for the calculation

of the onset frequency ω_{onset} , an increase of the \hat{F}_{es} parameter induces an upward translation of the curve representing the $\hat{F}_{es} \cdot |F_{fes}^\delta(j\hat{\omega}_{onset})|$ product. Thus the intersection with the function RC/ω occurs at lower frequencies (Fig. 8). Therefore, the onset frequency is lower and it leads to the determination of a higher amplitude A_0 from the y_{OLOP}/u_{OLOP} frequency response (Fig. 9), so that the location of the OLOP above the boundary gets more likely.

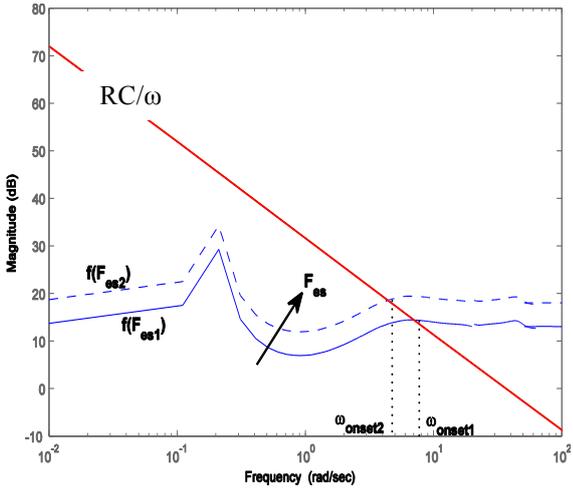


Figure 8 – Definition of the onset frequency for increasing \hat{F}_{es}

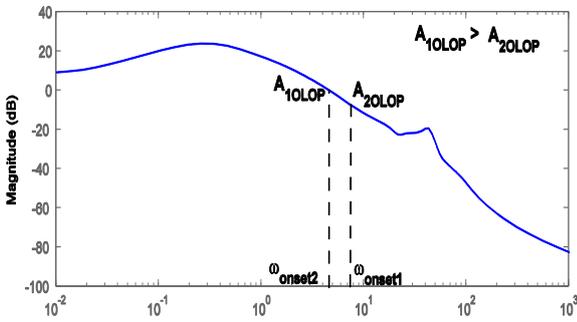


Figure 9 – Calculation of OLOP amplitude for increasing \hat{F}_{es}

Thus the \hat{F}_{es} parameter strongly influences the results provided by the criterion.

For this reason, in this analysis, the OLOP criterion was applied for different values of \hat{F}_{es} :

- $\hat{F}_{es} = 7.5$ deg
- $\hat{F}_{es} = 6$ deg
- $\hat{F}_{es} = 4.5$ deg
- $\hat{F}_{es} = 3$ deg.

For each considered case, appropriate longitudinal cyclic pilot gain was determined in order to achieve a

crossover phase angle included in the range set up by the criterion.

The results obtained for the hover condition are presented in Fig. 10-13, for the four different \hat{F}_{es} values used. It can be observed that when the \hat{F}_{es} parameter is raised, the OLOP method detects PIOs susceptibility for increasing values of the actuator's RC.

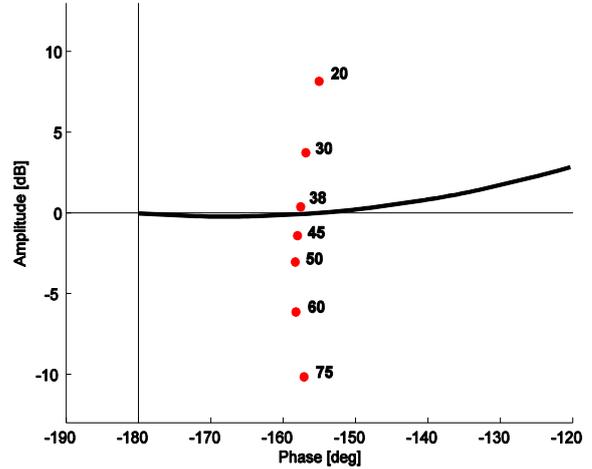


Figure 10 – OLOP criterion chart; results for $\hat{F}_{es} = 7.5$ deg, hover.

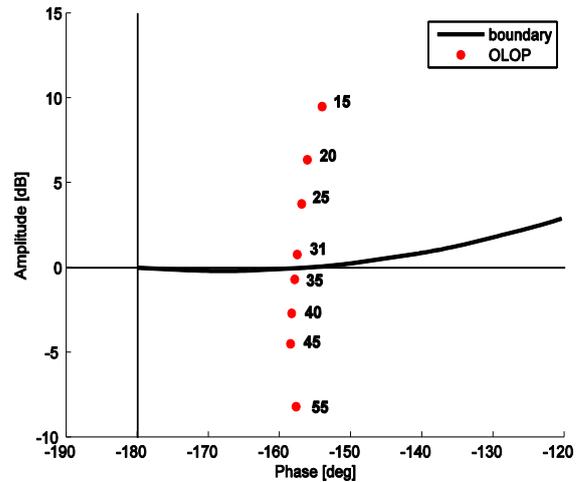


Figure 11 – OLOP criterion chart; results for $\hat{F}_{es} = 6$ deg, hover.

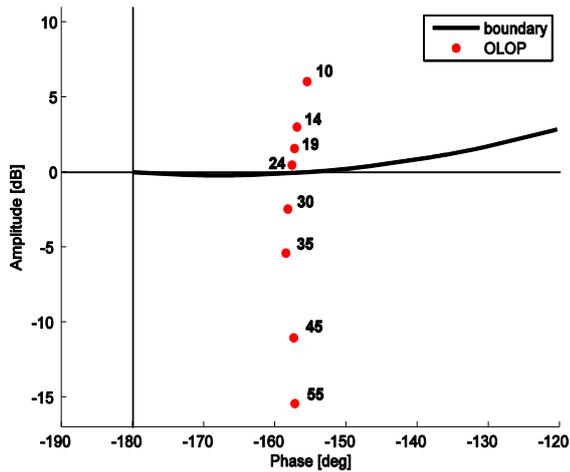


Figure 12 – OLOP criterion chart; results for $\hat{F}_{es} = 4.5$ deg, hover.

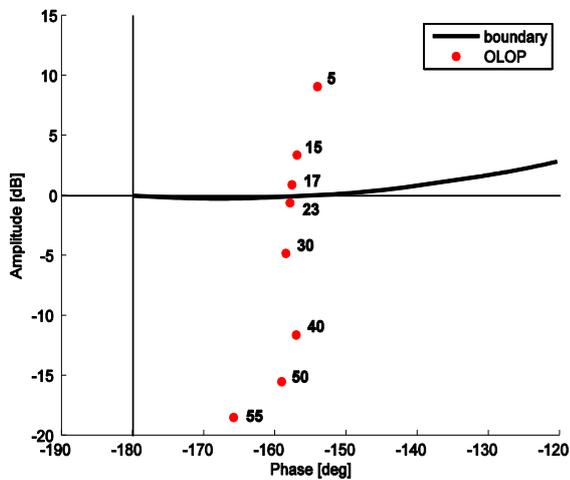


Figure 13 – OLOP criterion chart; results for $\hat{F}_{es} = 3$ deg, hover.

Table 3 shows, for the four flight conditions considered and for varying stick deflection \hat{F}_{es} , the values of the maximum RC that, according to the OLOP method, induce PIOs tendencies. It can be seen how the OLOP results depend strongly on the assumed stick deflection but only mildly on the flight condition analysed.

Table 3 – Maximum rate capability (RC) values for which PIOs susceptibility is detected

Maximum rate capability (RC) values [deg/s]				
Stick deflection [deg]	Hover	40 kts	50 kts	80 kts
$\hat{F}_{es} = 7.5$	38	37	38	39
$\hat{F}_{es} = 6$	31	31	32	32
$\hat{F}_{es} = 4.5$	24	23	24	24
$\hat{F}_{es} = 3$	17	16	16	17

Simulation tests

Pilot-in-the-loop simulations in one of the AgustaWestland simulators were performed with the aim of verifying the reliability of the PIOs prediction criteria that were applied in this investigation. These simulator test activities, due to their limited time duration, can only be considered as a preliminary work. However, this activity provided significant information regarding:

- the validity of the off-line criteria results;
- the type of manoeuvres most appropriate to expose a rotorcraft's PIOs tendencies.

Ground-based simulation facility

AgustaWestland provided the fixed-based simulator used for the execution of the tests [23]. The simulator features a classical pilot+co-pilot lay-out and is driven by the commercial software packages: MATLAB®, VEGA®, VAPS® and Flightlab®.

Simulator tests

It must be considered that the test execution and outcomes were limited by the following:

- simulator facility and test pilot availability: only one-hour test was performed with one pilot;
- uncertainties on the simulator's global time delay;
- well-known limitations (lack of motion cues and global out-of-the-window visual scene) associated with ground-based simulators when used as a tool for PIOs susceptibility investigation [14].

The manoeuvres selected for the simulator tests were:

- *pitch attitude capture task*: starting from a hover stabilized condition, reach 15 deg pitch down in 2 sec, then achieve 40 kts speed and pitch up as required to maintain speed. No constraint is given on total time.
- *acceleration and deceleration manoeuvre*: the task is derived from ADS-33 requirements. Start from a stabilized hover condition, rapidly increase power to approximately maximum, maintain altitude constant with pitch attitude, and hold collective constant during the acceleration to an airspeed of 50 kts. Upon reaching the target airspeed, initiate a deceleration by aggressively reducing the power and holding the altitude constant with pitch attitude. The peak nose-up attitude should occur just before reaching the final stabilized hover. Complete the manoeuvre in a stabilized hover for 5 s over the reference point at the end of the course.

- *low speed forward flight speed capture*: from an out of ground effect hover condition accelerate to 40 kts in 8 s.
- *vertical speed capture*: from a trimmed level flight at 80 kts transition to a climb/descent condition, performing a 300 fpm vertical speed capture in 3 s.

The first two tasks were chosen because they are characterized by high pilot gain. In fact, the onset of PIOs is commonly associated with and favoured by an increase in pilot gain. The latter two tasks, which in literature are not included in the set of manoeuvres considered appropriate for PIOs detection, were performed to verify if the simulator test outcomes would agree with the indications found in literature. The four tasks were performed by independently decreasing the actuator's BW and RC.

The non-linear model running in the simulator was the same used to generate the linearized model analysed with the off-line criteria.

Table 4 describes the tests performed and summarizes the pilot's comments.

Table 4 – Simulator performed tests

TEST	MANOEUVRE	DATA	REMARKS
Test 0	Low speed forward flight speed capture	BW=1.5 Hz RC=100 deg/s	No PIOs
Test 1	Pitch attitude capture	BW=1.5Hz RC=100 deg/s	Tendency to PIOs
Test 2	Pitch attitude capture	BW=2.5 Hz RC=100 deg/s	Slight tendency to PIOs
Test 3	Pitch attitude capture	BW=5Hz RC=70 deg/s	Handling slightly degraded
Test 4	Pitch attitude capture	BW=5Hz RC=25 deg/s	Slight tendency to PIOs
Test 5	Pitch attitude capture	BW=5Hz RC=15 deg/s	Clear tendency to PIOs
Test 6	Vertical speed capture	BW=2Hz RC=100 deg/s	No tendency to PIOs
Test 7	Vertical speed capture	BW=2Hz RC=100 deg/s	Test 6 re-run
Test 8	Vertical speed capture	BW=1.5Hz RC=100 deg/s	No tendency to PIOs
Test 9	Vertical speed capture	BW=1.0Hz RC=100 deg/s	Task not suitable
Test 10	Acceleration/Deceleration	BW=2Hz RC=100 deg/s	No tendency to PIOs
Test 11	Acceleration/Deceleration	BW=2Hz RC=100 deg/s	Test 10 re-run
Test 12	Acceleration/Deceleration	BW=1.9Hz RC=100 deg/s	Slight tendency to PIOs
Test 13	Acceleration/Deceleration	BW=1.0Hz RC=100 deg/s	Strong tendency to PIOs
Test 14	Acceleration/Deceleration	BW=5Hz RC=70 deg/s	No concern
Test 15	Acceleration/Deceleration	BW=5Hz RC=25 deg/s	No concern
Test 16	Acceleration/Deceleration	BW=5Hz RC=15 deg/s	Tendency to PIOs

Results

Attitude capture task. PIOs tendencies were detected for bandwidth values $BW \leq 2.5$ Hz. Fig. 14 shows the time traces of Test 2: it can be observed that the target pitch down of 15 deg is achieved, but the rotorcraft exhibits an oscillatory response, with a large pitch rate amplitude and moderate longitudinal cyclic input; the 90 deg phase delay between pitch rate (positive pitch up) and longitudinal cyclic (positive forward) can be clearly seen (Fig. 15).

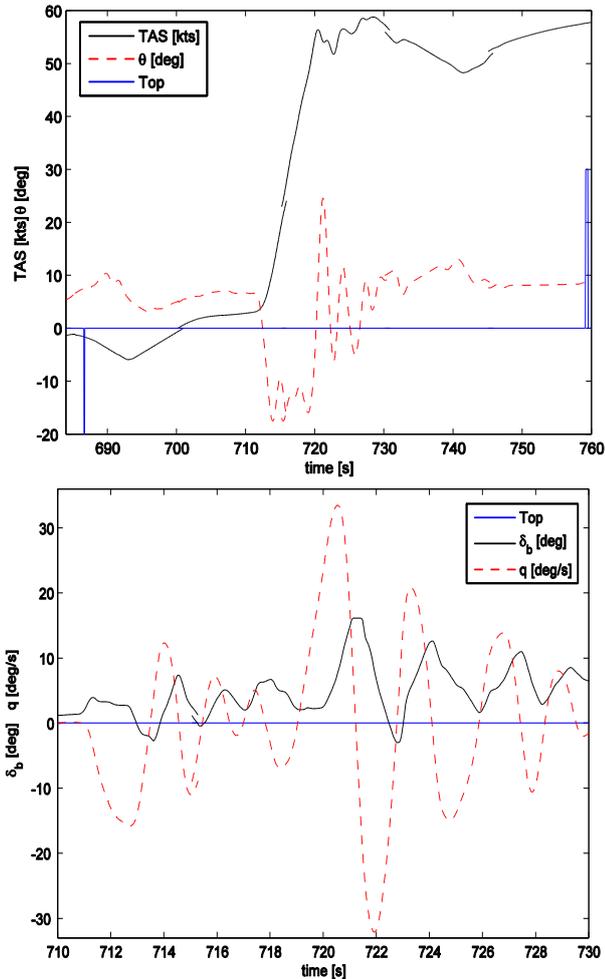


Figure 14 - Attitude capture task, Test 2
(RC=100 deg/s; BW=2.5 Hz)

The rotorcraft's PIOs susceptibility was also exposed for rate capabilities $RC \leq 25$ deg/s. Fig. 15 shows the time traces of Test 5 ($RC=15$ deg/s and $BW=5$ Hz), during which the pilot detected clear tendencies to PIOs. The encountered PIO episode was severe, as the oscillations developed quickly and exhibited large amplitudes, with the pitch rate reaching a maximum peak-to-peak amplitude of about 60 deg/s. The PIO lasted almost 20 s. In the last graphs the typical triangle-like trend of the servo position [24], indicating the occurrence of rate limiting, can be observed.

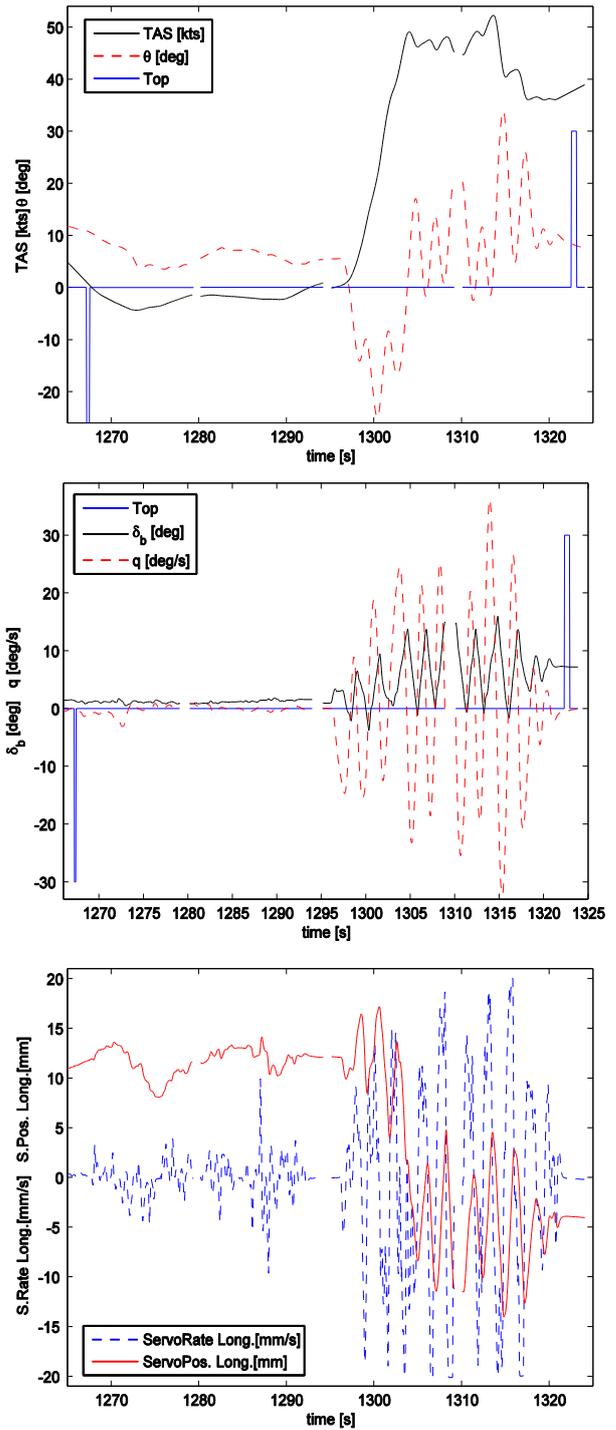


Figure 15 - Attitude capture task, Test 5
(RC=15 deg/s ; BW=5 Hz)

Acceleration-deceleration manoeuvre. PIOs susceptibility was encountered for $BW \leq 1.9$ Hz (Fig. 16). The episode lasted about 10 s, with maximum peak-to-peak oscillation amplitudes of 40 deg/s for the pitch rate and 10 deg for the longitudinal cyclic.

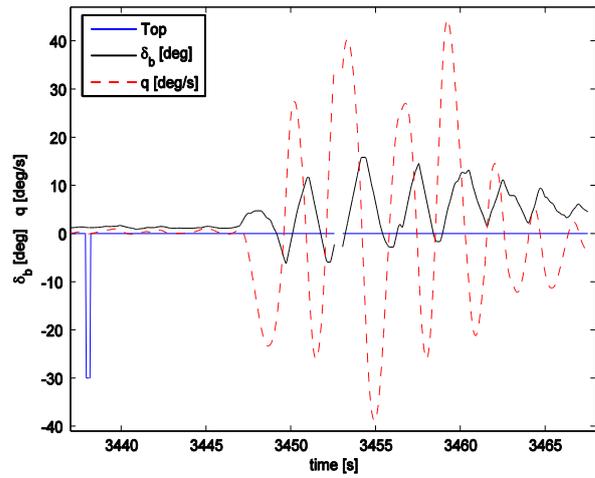
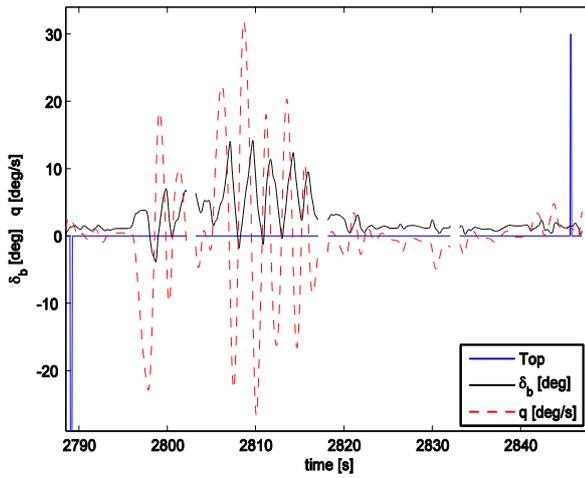
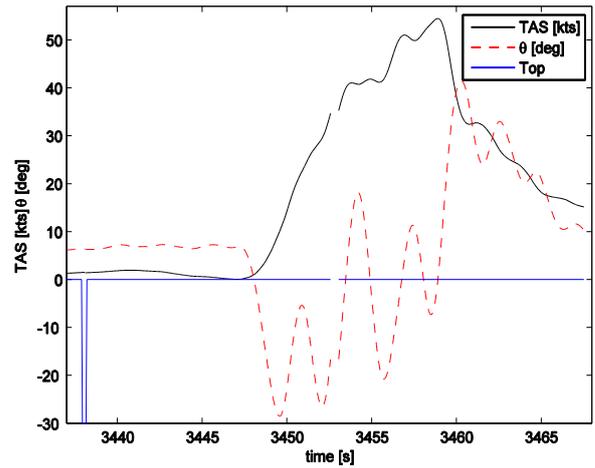
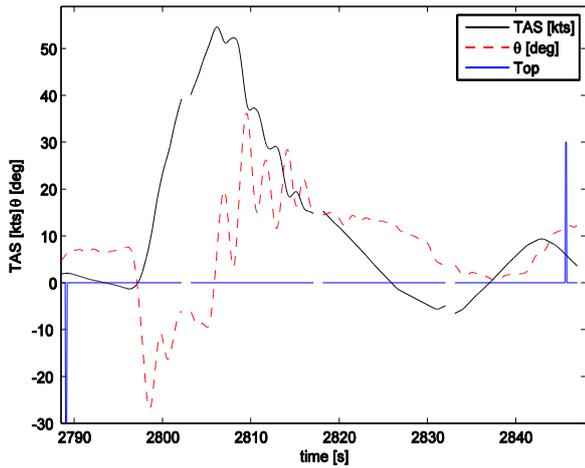


Figure 16 - Acceleration-deceleration task, Test 12 (RC=100 deg/s; BW=1.9 Hz)

The rotorcraft's PIOs susceptibility was also exposed during Test 16 (Fig. 17) for rate capabilities $RC \leq 15 \text{ deg/s}$. The time traces of this test clearly show the triangle-like trend of the servo position due to the rate limiting occurrence. Both the pitch rate and the pilot longitudinal cyclic reached significant amplitudes. The PIOs onset occurred immediately after the pitch down and acceleration phase.

The pilot judged this task a little less aggressive than the attitude capture task, as it required a lower pilot gain. This could explain why in the acceleration-deceleration task PIOs tendencies were encountered for RC and BW values lower than for the attitude capture manoeuvre.

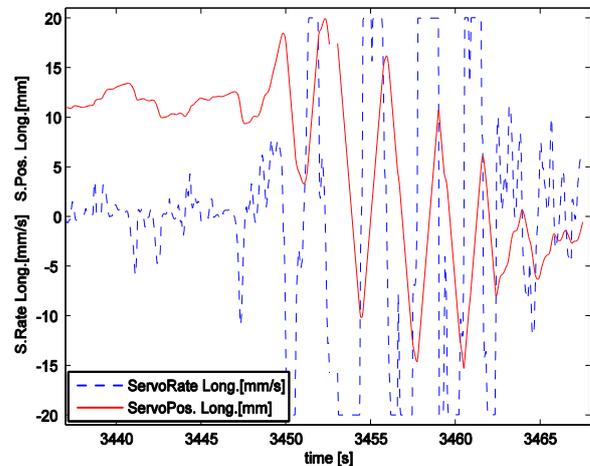


Figure 17 - Acceleration-deceleration task, Test 16 (RC=15 deg/s; BW=5 Hz)

Forward and vertical speed capture task. These two tasks were not judged appropriate to expose potential susceptibility of the vehicle to PIOs. The helicopter did not exhibit any PIOs tendencies, despite the handling qualities degradation induced by the reduction in the actuator's BW and RC.

PIOs prediction criteria and simulation tests: results comparison

The results from the tests carried out using the simulator are in this section compared with those provided by the prediction criteria. Although quite limited, the simulation tests database proved to be useful in giving some insights into the criteria application and the validity of their predictions.

Bandwidth/Phase Delay criterion

In Table 5 the outcomes of the simulation tests which were aimed at investigating PIOs susceptibility due to actuator bandwidth reduction are compared with the results obtained from the application of the Bandwidth/Phase delay criterion. Table 5(a) shows, for each analysed flight condition, the maximum bandwidth values that, according to the criterion, induce the rotorcraft's PIOs sensitivity, while Table 5(b) presents the corresponding bandwidth values detected in the simulator.

Table 5 – Comparison between Bandwidth criterion and simulation results

(a)	
<i>Bandwidth/Phase delay criterion</i>	
<i>Flight condition</i>	<i>Actuator bandwidth [Hz]</i>
Hover	1.43
40 kts	1.38
50 kts	1.40
80 kts	1.37

(b)	
<i>Simulator tests</i>	
<i>Actuator bandwidth [Hz]</i>	
2.5	(Attitude capture task)
1.9	(Acc-dec task)

The data collected in Table 5 show clearly some discrepancies between the criterion and the simulation outcomes: according to the pilot's comments and the simulation time histories, slight tendencies to PIOs appeared during the attitude capture test for $BW \leq 2.5$ Hz and during the acceleration-deceleration manoeuvre for $BW \leq 1.9$ Hz, while the criterion detected PIOs susceptibility for $BW \leq 1.4$ Hz (thus resulting to be non-conservative).

It was deduced that one of the most likely causes of such a mismatch could lie in the several sources of time delay that are associated with simulators, but were not represented in the pilot-rotorcraft model.

A precise figure of the simulator's time delay not being available, three more attempts at applying the

criterion were made, inserting in the mathematical model the following three simulator's time delays:

- $\tau_d = 0.1$ s and $\tau_d = 0.15$ s; according to the FAA Advisory Circular concerning helicopter simulators qualification [25], the delay associated with the visual, motion and cockpit instrument systems must not be greater than 150 ms for Level B simulators and not be greater than 100 ms for Level C and D. As a first attempt, not knowing the specific time delay of the simulator used, the Bandwidth criterion was applied considering these maximum acceptable delays indicated by the FAA Advisory Circular;
- $\tau_d = 0.07$ s; this is the computed value of the simulator total time delay that could guarantee a good correlation between the criterion results and the simulator tests outcomes. Being lower than the maximum value of 100 ms accepted by the FAA, in the Advisory Circular mentioned above, it is a plausible value, and was thus adopted for the last criterion application attempt.

These data are compared with the simulator tests outcomes in Table 6.

Table 6 – Comparison between simulation results and Bandwidth criterion for different simulator time delays

<i>Bandwidth/Phase delay criterion</i>			
<i>Flight condition</i>	<i>Actuator bandwidth [Hz]</i>		
	<i>Time delay</i>	<i>Time delay</i>	<i>Time delay</i>
	$\tau_d = 0.07s$	$\tau_d = 0.1s$	$\tau_d = 0.15s$
Hover	2.4	3.3	14
40 kts	2.7	3.5	11.5
50 kts	2.5	3.4	12.6
80 kts	2.5	3.5	10.9

By using the calculated simulator time delay, $\tau_d = 0.07$ s, the criterion results obviously achieve good correlation with the simulator tests: PIOs susceptibility is identified for actuator bandwidth values less than or equal to $BW = 2.5$ Hz, as shown during the simulator tests. As expected, for time delays $\tau_d = 0.1$ s and $\tau_d = 0.15$ s, the criterion detects PIOs tendencies for actuator bandwidths greater than the critical value $BW = 2.5$ Hz identified by the simulator tests.

OLOP criterion

As summarized in Table 7, during the tests conducted in the simulator, the pilot detected some rotorcraft PIOs tendencies for rate capability lower than 25 deg/s for the rapid attitude capture task and lower than 15 deg/s for the acceleration-deceleration manoeuvre. These data are now compared with the results provided by the OLOP criterion.

Table 7 – Comparison between OLOP criterion for different simulator time delays and simulation results, hover
(a)

<i>Open Loop Onset Point criterion</i>		
<i>Stick deflection \hat{F}_{es}</i>	<i>Rate Capability RC [deg/s]</i>	
	<i>No time delay</i>	<i>Time delay $\tau_d = 0.07s$</i>
$\hat{F}_{es} = 7.5 \text{ deg}$	38	43
$\hat{F}_{es} = 6 \text{ deg}$	31	36
$\hat{F}_{es} = 4.5 \text{ deg}$	24	27
$\hat{F}_{es} = 3 \text{ deg}$	17	19

(b)

<i>Simulator test</i>
<i>Actuator rate capability RC [deg/s]</i>
25 (Attitude capture task)
15 (Acc-dec task)

Table 7 shows, for four stick deflections, the results associated with the hover condition. The results given for $\hat{F}_{es} = 11 \text{ deg}$ are not considered, because they did not appear to be useful for this analysis, as the criterion predicted PIOs tendencies even for high rate capability configurations.

In order to make the OLOP results uniform with those provided by the Bandwidth method, the OLOP criterion was applied again after the simulation tests, taking into account the added simulator time delay $\tau_d = 0.07s$, which has been introduced previously. For this case, due to the changes in the model dynamics induced by the added delay, the pilot gains were recomputed.

From Table 7 it can be noted that the computed time delay has some influence on the outcomes of the OLOP method, as it leads to an increase of the rate capability for which potential PIOs susceptibility is detected.

The criterion matches with acceptable accuracy the attitude capture task results when the stick deflection is equal to $\hat{F}_{es} = 4.5 \text{ deg}$ and those of the acceleration-deceleration manoeuvre when the stick deflection is equal to $\hat{F}_{es} = 3 \text{ deg}$.

The attitude capture task should be considered the most representative and useful task for the investigation of the criterion reliability, because, as previously mentioned, it is more aggressive and requires higher pilot gain, thus representing a more critical condition for PIOs onset, than the acceleration-deceleration manoeuvre. Thus, it could be concluded that the stick deflection value $\hat{F}_{es} = 4.5 \text{ deg}$ is the most appropriate, because for the attitude capture task it leads to a good agreement between the OLOP and the simulator results.

This conclusion seems to be supported by the fact that, from the time traces of the simulator tests, the mean value of the pilot's longitudinal cyclic control input resulted to be $\bar{\delta}_b = 4.7 \text{ deg}$.

This analysis may indicate that the OLOP should not be used with the maximum stick deflection, as stated in the literature defining the theoretical basis of the criterion [16, 22], but with a stick deflection value more representative of normal conditions pilot control actions.

Real-time PIOs detection method

Recently, since the 90s, a new approach has been introduced in the context of PIOs tendency analysis. With respect to the PIOs criteria, whose aim is to investigate and detect an aircraft's PIOs susceptibility already in the early stage of the design process, the purpose of the new philosophy is to detect and correct potential tendencies of pilot-aircraft couplings in real time during flight.

The ROVER method [1, 26] is based on the assumption that PIOs cannot be prevented in real time, because no specific conditions that could precede the PIOs onset exist, thus the purpose of ROVER is to detect it as early as possible in order to minimize the effects on the pilot-aircraft system.

The criterion checks for oscillatory signals in stick inputs and angular rate outputs and compares the characteristics of the two sets of signals.

In this analysis, a Matlab® code was developed for the implementation of the ROVER. It can be integrated in a Simulink® model and is able to receive and process in real time the model data output, but to date it was only used to post process the time history data obtained from the simulator tests.

The code receives the input data, i.e. pitch angular rate and longitudinal cyclic control, computes their time derivatives and, through the identification of the derivatives sign change, detects the signals' peaks.

Then, four parameters are computed, namely the angular velocity frequency, the angular velocity peak-to-peak amplitude, the phase delay between cyclic command and angular velocity, the cyclic command peak-to-peak amplitude. For each parameter a threshold value is set, so that when one of them is exceeded a flag is set in the outputs. Table 8 shows the threshold values used.

Table 8 – Threshold values imposed in ROVER

Processed Variables	Threshold
Pitch rate q frequency: f_q [rad/s]	$1 \leq f_q \leq 8$
Pitch rate q amplitude: A_q [deg/s]	40
Longitudinal cyclic amplitude: A_{δ} [deg]	15
Pitch rate q and longitudinal cyclic phase delay: $\Phi_{q-\delta}$ [deg]	$83 \leq \Phi_{q-\delta} \leq 97$

The code generates as output a chart with time on the x-axis and the flags on the y-axis. According to the criterion, the occurrence of the four flags within a small time range represents the detection of a PIO onset.

The time histories data recorded during the simulator tests were post-processed with the ROVER. The ROVER results proved to be in good agreement with the pilot's comments. As an example, Fig. 19-20 show the ROVER results for two runs, Test 9 and Test 13. The method was able to discriminate the PIOs episodes from other oscillatory responses and to detect the onset of a PIO.

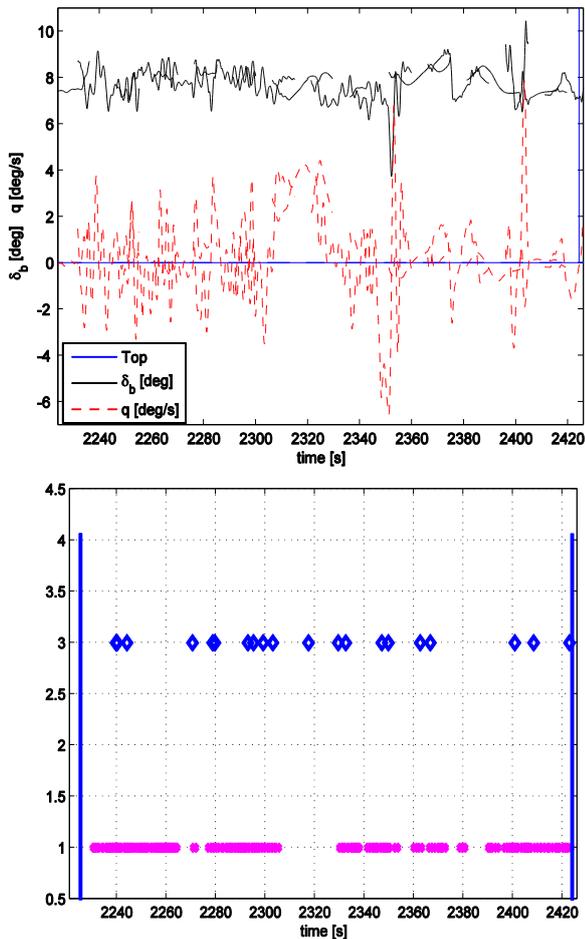


Figure 19 – ROVER applied to Test 9

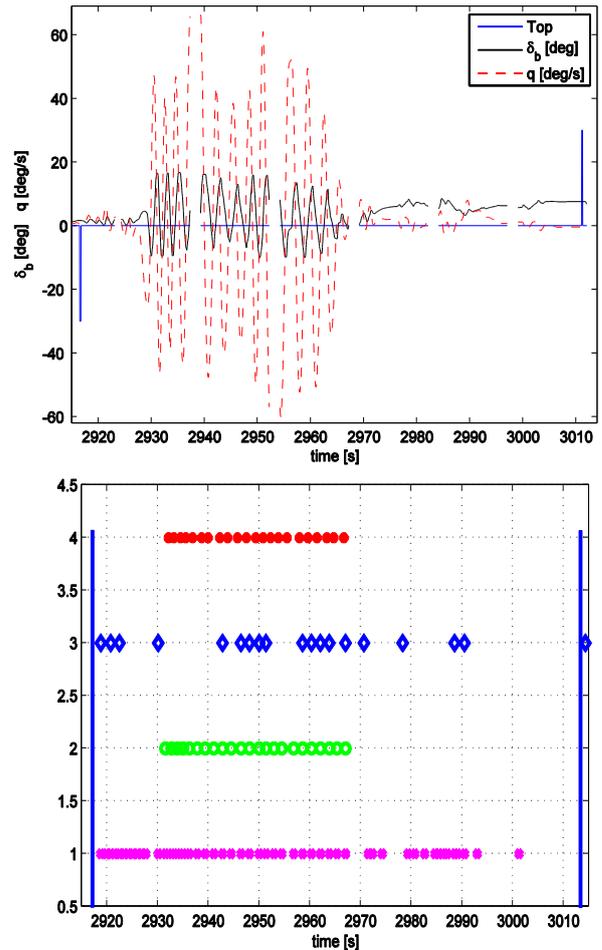


Figure 20 – ROVER applied to Test 13

Conclusion

This paper has presented an attempt to apply to rotorcraft fixed wing aircraft PIOs prediction criteria, namely the Bandwidth/Phase Delay, the Open Loop Onset Point (OLOP) and the Realtime Oscillation Verifier (ROVER).

The application of both the Bandwidth/Phase Delay and the OLOP criteria required a comprehensive pilot-rotorcraft analytical model; a pure gain pilot model was used. This choice was made on the basis of literature references, in fact, as declared by analytical studies and experimental data, the pure gain pilot showed to be a good approximation of the human pilot dynamics during fully developed PIOs.

As required by the criteria, an aircraft linear model was needed; the linear model of an example medium class helicopter, provided by AgustaWestland, was used. The linear model was derived from a full non-linear model, for four different trim flight conditions, namely hover and forward flight at 40, 50 and 80 kts.

The Bandwidth/Phase Delay criterion was applied to the defined pilot-rotorcraft model, for decreasing bandwidth of the actuators on the longitudinal and lateral cyclic control channels.

The OLOP method, developed to investigate potential PIOs tendencies caused by rate limiters activation, was applied to the pilot-rotorcraft model, for decreasing rate capability of the rate limiters associated with the control actuators.

Simulator tests with a pilot were performed in a fixed-based simulator facility with the purpose of investigating the suitability of some manoeuvres in exposing PIOs susceptibility and of collecting a database for the comparison with the results previously provided by the Bandwidth/Phase Delay and OLOP off-line criteria.

The results obtained by the application of the PIOs prediction methods did not show significant discrepancies with respect to the simulation tests outcomes.

With respect to the Bandwidth/Phase Delay criterion, the difference could be reduced by taking into account the effective global time delay associated with simulators, which was at first neglected. It was calculated that a good correlation between the simulation and the method results can be achieved if the simulator global delay amounts to 70 ms, which would be a plausible value for a Level D simulator, according to the FAA advisory circular.

The OLOP results showed to be dependent upon the value of the stick deflection adopted in the application procedure (\hat{F}_{es} parameter). When using the maximum stick deflection, the method appeared to be strongly conservative and to over predict PIOs susceptibility. The analyses carried out in this investigation indicated that a stick deflection value representative of the normal pilot control action should be a more appropriate choice.

Simulator tests and pilot comments were used to identify the manoeuvres most suited to detect rotorcraft PIOs tendencies: the attitude capture test and the acceleration-deceleration manoeuvre.

A Matlab code was written to implement the ROVER, a method aimed to detect PIOs in real time. The code, used to post-process the time histories obtained from the simulator tests, showed high agreement with the pilot's comments.

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