WEAK COUPLING APPROACH TO STUDY THE EFFECT OF THE CONTROL SYSTEM FLEXIBILITY ON THE HELICOPTER DYNAMIC BEHAVIOR

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ABSTRACT

This work couples a computational structural dynamics (CSD) code to model the control system complex kinematics and a comprehensive analysis code to compute both the rotor aeromechanics and the helicopter dynamic behavior. A weak coupling approach is set-up to exchange the data between the high-fidelity structural calculations LMS Virtual.Lab Motion® and the in-house comprehensive code HOST. The CSD coupling strategy that is set-up in this paper differs from most other works in that the coupled element, in particular the swashplate, is in the middle of the kinematic chain (see Figure 1). Such a coupling helps benefit from the high-fidelity modelling of the control system to account for complex kinematics, swashplate elasticity, and fittings on a flexible fuselage. This initial coupling development helps improve dynamic loads calculations and may then lead to better stability predictions of the helicopter rotor-airframe couplings.

NOTATIONS

A_{1}^{*}, A_{2}^{*}	State matrices (Damping and stiffness
	parts)
b	Number of blades
В	Control matrix
CA	Comprehensive Analysis: computer
	program that calculates the helicopter
	aeromechanical behavior
CFD	Computational Fluid Dynamics
CSD	Computational Structural Dynamics
GHOST	Generalized HOST
HOST	Helicopter Overall Simulation Tool
LTI	Linear Time Invariant
LTP	Linear Time Periodic
MBC	Multi-Blade Coordinates
N_b, N_s	Number of blade and structure modes
\vec{v}, \vec{v}_i	Blade deformations and modal shape
VLM	LMS Virtual.Lab Motion®
α	Relaxation factor
\vec{x}_i^{CA}	Coupled parameter computed by HOST
_	at iteration #i
\vec{x}_i^{CSD}	Coupled parameter computed by CSD at
	iteration #i
$\begin{bmatrix} X \end{bmatrix} \begin{bmatrix} X_i \\ y \end{bmatrix}$	Fuselage deformations (displacements
$\begin{array}{ c c } Y & Y_i \\ \hline T & T \\ \hline T & T \\ \hline \end{array}$	and rotations) and modal shape.
$\begin{bmatrix} Z \\ \phi \end{bmatrix} = \begin{bmatrix} z_i \\ \phi_{ii} \end{bmatrix}$	
$\phi_x \phi_y \phi_y$	
$\begin{bmatrix} r_y \\ \phi_z \end{bmatrix} \begin{bmatrix} r_y \\ \phi_z \end{bmatrix}$	
q_{s_i}, q_{b_i}	Structure and blade modal participation

1. INTRODUCTION

Comprehensive analyses available today have seen dramatic changes since they first appeared in the 1970s. At that time, only limited helicopter and rotor configurations could be modeled, documentation was poor, upgrades and maintenance were tedious. Modularity, versatility and flexibility have been improved in the new generation of the codes in the 1980s. Aerodynamics then became in the focus with the development of advanced wake models available in the comprehensive analyses [1] and with the development of CFD couplings to benefit from code advances (such as OVERFLOW, eLSA, FLOWer, TAU) [2]. CSD couplings were also set-up to better capture the complex blade elastic deformations [3].

2. COUPLING APPROACH

The previous CFD-CSD couplings were set-up for data exchange at the tip of the multibody chain (the blade) [3] [1]. This paper makes further use of CSD codes by modeling the complex kinematic of the control system that lies in the middle of the multibody architecture.



Figure 1: Example of HOST tree structure.

The multibody logic in HOST (see [4]) is based on a kinematic path, which propagates the motion from the upstream model to the downstream model, and a loads path, which computes and then propagates the external and inertial forces from the downstream elements to the upstream model (see Figure 2). Once the kinematic and loads paths have been done, an acceleration unbalance is computed for each model. HOST then makes use of a Newton approach to zero the unbalance of all the models.

Unlike the previous CFD and CSD couplings, the data exchange between the two codes is made in the middle of the classical multibody kinematic and loads paths. This means that the correction made by the coupling has a direct impact both in the kinematic and loads paths of the multibody code, while the CFD coupling on the blade only has a direct impact on the loads paths of the multibody approach.



Figure 2: CSD coupling in the middle of the kinematic chain.

The comprehensive code used in this work for the coupling is HOST and is based on a multibody logic that allows modeling any arbitrary helicopter configuration [4]. The analytical internal model of the control system available in HOST cannot capture the complex reinjection phenomena that were proved to be present on some helicopters [5]:

- Swashplate flexibility: Unlike in HOST, the non-isotropic swashplate stiffness properties may be modeled in the CSD code (see Figure 3). The deformation of the swashplate when subjected to a load can therefore be computed.
- Swashplate reinjection: The cross effects of a swashplate deformation at a pitch link attachment on the other pitch link deformation can be accounted for by the CSD software.
- Fuselage reinjection: In particular, the elastic motion of the fuselage dynamically changes the geometry of the control system; hence the pitch controls that are usually only made of the pilot controls must be tweaked to account for the nonnegligible fuselage deformation orders.

Figure 2 shows the effect of the coupling setup on the multibody logic. The models displayed in the red-dashed boxes are the coupled elements and the arrows depict the kinematic/loads paths. The red/dashed arrows exhibit the part of the kinematic/loads paths that are changed by the coupling correction. From left to right:

- CFD coupling (blades only): the kinematic path is not impacted by the coupling correction.
- CSD coupling (swashplate reinjection only): the kinematic path is impacted for the elements downstream to the swashplate. The loads path is also impacted for all the elements.
- CSD coupling (fuselage and swashplate reinjections): the kinematic path is impacted as well as the loads path.

The coupling set-up in this paper helps benefit from the quick advances in the CSD codes to better predict the rotor dynamic loads. The coupling strategy is presented in a first part of this paper and preliminary sensitivity analyses regarding the dynamic behavior of the main rotor are then analyzed. It is expected that the swashplate flexibility does not have a large impact on the trimmed values of the helicopter states, but the swashplate dynamics may then change the stability properties of the coupled structure-rotor system.



Figure 3: Non-isotropic swashplate deformations.

2.1 WEAK COUPLING

A weak coupling strategy based on a periodic trim (see [4]) over one rotor revolution is applied. Hence the information is exchanged between HOST and VLM for one full rotor revolution.

The strong coupling approach (for which the data between the two codes is exchanged at every azimuth step of the rotor) is not appropriate to compute the steady-state periodic behavior of a helicopter, especially when unstable or low-damped modes are present in the system. It is also more computationally expensive than the weak coupling approach.

Therefore, a weak coupling procedure was implemented to compute the helicopter periodic trim states including a high fidelity representation of the control system flexibility. The stability of this coupled equilibrium is the focus of the next section.

2.1.1 METHODOLOGY

The coupling performed in this study was made by using the python wrapping GHOST of HOST and the CSD code LMS Virtual.Lab Motion®. A master script was also developed to manage the data exchange and to iteratively launch the two codes.

2.1.2 VLM

VLM, which is a multibody modeling software for fast motion simulation of complex systems, is used to model the control chain: the attachment of the rods on the fuselage and on the main gear box, the actuators and the two swashplates (see Figure 4). The finite element models of the rotating and fixed swashplates are condensed into two super-elements for faster analyses. Indeed, the super-element approach limits the number of degrees of freedom in the system by focusing on the boundaries and connections of the subsystems. This CSD code is used to compute the motion of the pitch link attachments on the rotating swashplate when the control chain is subject to loads on the pitch links and to displacements of the different attachment points of the rods on the fuselage.



Figure 4: Control System Model.

VLM makes its calculations in the time domain so, to avoid transient behaviors, several rotations are computed in VLM to get the steady-state motions of the pitch link attachments that are passed to GHOST.



Figure 5: Data exchange with VLM.

The inputs of VLM are slightly changed at each iteration of the coupling process, so the last revolution of the last simulation (motion of the pitch link attachments on the rotating swashplate) is used as a starting point to the next VLM. This approach helps minimize the oscillations and accelerates the VLM convergence of the time simulation toward its equilibrium solution.

2.1.3 GHOST

GHOST is the python wrapping of the inhouse comprehensive analysis HOST that enables the couplings on any part of the helicopter defined in the HOST architecture (such as the fuselage, stabilizers and blades). Those couplings can be based on data exchange of several types: motion, force, control or observations. HOST is used to compute the helicopter aeromechanical behavior including the pilot controls, fuselage deformations, and the blades elastic motion. Both the H155 fuselage and rotor are represented by modal bases. The fuselage modal basis is expressed at the main rotor hub center and at the nine attachments of the rods on the fuselage and main gear box (Figure 6). Typically the airframe deformations are computed as the sum of the modal shapes times their modal participations q_{s_i} , as described by the following equation.

(1)
$$\begin{pmatrix} X \\ Y \\ Z \\ \phi_{x} \\ \phi_{y} \\ \phi_{z} \end{pmatrix} = \sum_{i=1}^{N_{s}} q_{s_{i}} \begin{pmatrix} X_{i} \\ Y_{i} \\ Z_{i} \\ \phi_{xi} \\ \phi_{yi} \\ \phi_{zi} \end{pmatrix}$$

The blade flap, lag and torsion deformations \vec{v} are computed using:

(2)
$$\vec{v}(t,r) = \sum_{i=1}^{N_b} q_{b_i}(t) \cdot \vec{v}_i(r)$$

The modal shapes are inputs to the code while the modal participations are used as internal states in HOST.



Figure 6: Control chain attachments on the elastic fuselage

In the current study, the fuselage modal base contains three modes that were chosen from the finite element analysis of the H155 and for which the modal shape has large displacements at the rotor head or at the positions depicted in Figure 6. The rotor is also modelled by a modal basis made of the modes up to the first torsion mode. In the end, the rotor is represented by six blade modes (two lead-lag modes, three flap modes and one torsion mode). The torsion and bending modes are decoupled in the modal basis. The calculations are made up to the sixth harmonic. Indeed the loads of a b-bladed rotor need be computed up to the b+1 harmonic in the rotating frame to get the right loads that are transmitted to the fuselage (in the fixed frame).

Modifications were made in HOST to allow forces, motions and controls to be robustly used as exchange data in the coupling process, anywhere in the kinematic and loads paths. Indeed, the previous CFD couplings required only force data to be robustly imported in the comprehensive analysis.

2.1.4 MASTER SCRIPT

A master script was developed to build the coupling process that iteratively calls GHOST and VLM to find a coupled trimmed and periodic full helicopter solution.

Coupling specifications

A data exchange based on Fourier coefficients would save memory and would also suppress the azimuthal interpolation necessary for the two codes to operate at their own rates, but it was chosen to exchange the data as time series. Indeed, such a format is a general approach that could also be used for a strong coupling strategy that requires data exchange at each time step.

Weak Coupling Strategy

The coupling calculation is initialized with a HOST estimation of the motion of the pitch link attachments on the swashplate, using a fully rigid swashplate model. This results in a periodic trim solution that is the starting point of the coupling iterations.

Once HOST has found a trim solution, the motion of the control elements and the forces on the pitch links are used as inputs to the CSD. The periodic solution is set to be found up to a given harmonic number (six in this study).



Figure 7: HOST-CSD weak coupling procedure

The coupling procedure iteratively replaces the comprehensive analysis motions of the pitch link attachments by the outputs of the CSD code. In fact, the comprehensive calculations at iteration #i are corrected by the difference between the CSD and the CA computations at iteration #i-1. This corrected variable is used for the CA calculation at the next coupling iteration.

$$(3) \qquad \vec{x}_i^{cpl} \leftarrow \vec{x}_i^{CA} + \left(\vec{x}_{i-1}^{CSD} - \vec{x}_{i-1}^{CA}\right)$$

Convergence Criterion

The previous equation, describing the correction of the CA by the CSD code, can also be written as:

(4)
$$\vec{x}_{i}^{cpl} \leftarrow \vec{x}_{i-1}^{CSD} + (\vec{x}_{i}^{CA} - \vec{x}_{i-1}^{CA})$$

This shows that the comprehensive analysis coupled parameter is equal to the CSD parameter when the coupling process has converged:

(5)
$$\lim_{i \to \infty} \vec{x}_{i-1}^{CSD} + \left(\vec{x}_i^{CA} - \vec{x}_{i-1}^{CA} \right) = \vec{x}^{CSD}$$

Therefore, convergence is reached when the CA calculations are identical between two iterations. However the CA coupled parameter may not be equal to the CSD parameter, even once the coupling process has converged.

Hence, the convergence of the coupling process set-up in this paper is based on the evolution of the coupled parameters, computed by the CA, as a function of the coupling iteration number. In our case the motions of the pitch link attachments on the swashplate are therefore analyzed to determine the convergence of the coupling process.

A harmonic approach is used by HOST to find the periodic trim states and the convergence criterion is therefore applied to the Fourier decomposition of the coupled parameters.

The comprehensive analysis trims the helicopter including the CSD corrections. The free parameters of the trim law and all the internal forces and displacements may be changed by the coupled system. As in Ref. [6] the convergence criterion was therefore extended to other parameters.

Relaxation Factor

A relaxation factor can be implemented to avoid divergence and accelerate convergence. Indeed, if the CSD correction is large, the CA may not be able to find a new trim in one step. Relaxation may also accelerate the convergence of the coupled calculations by avoiding overshoots. It must be noted, however, that the relaxation factor of the converged calculation must be 1 for the coupled parameter to be equal to the CSD calculations.

(6)
$$\vec{x}_i^{cpl} \leftarrow \vec{x}_i^{CA} + \alpha(i) \cdot \left(\vec{x}_{i-1}^{CSD} - \vec{x}_{i-1}^{CA} \right)$$

Therefore, the relaxation factor must follow a law such that $\lim_{i\to\infty} \alpha(i) = 1$. An example of relaxation factor as a function of the iteration number is given in Figure 8.

Indeed, if the relaxation factor is constant and not equal to one, the value of the coupled parameter once the coupling process has converged is not equal to the CSD value:

(7)
$$\vec{x}_{\infty}^{cpl} \leftarrow (1-\alpha)\vec{x}_{\infty}^{CA} + \alpha\vec{x}_{\infty}^{CSD} \\ \neq \vec{x}_{\infty}^{CSD}$$

In practice the use of a relaxation law helped converge faster (up to 30% or 5 iterations) and save a few iterations, which individually takes about one minute.



Figure 8: Relaxation factor as a function of iteration number.

Post-processing

MatLab® routines were used to post-process the HOST-VLM coupling results. In particular, the loads and motion (position, speed and acceleration) of the coupled elements can be plotted as a function of the coupling iteration number. The harmonic content of all the data exchanged in the coupling process can be plotted to check convergence.

2.2 VALIDATION

This section shows that the coupling environment, the data exchange and the models are operational and validated.

2.2.1 RIGID SWASHPLATE

This section deals with the validation of the complex control chain kinematics used in VLM. Such a validation was performed by checking that both the VLM and HOST models match when they are set to rigid and they both receive the same pilot inputs. In practice the pilot controls are given, and the motion of the pitch-link attachments are analyzed. Figure 9 (top graph) shows that both the stand-alone HOST simulation and the VLM results perfectly match as the errors in the pitch link vertical position is of the order of $10\mu m$. The same results were observed on all the pitch link attachments on the swashplates. Therefore, the complex geometry of the control chain is validated in VLM. The next section addresses the influence of higher fidelity modelling on some parameters such as the pilot controls and the blade motion.



Figure 9: Pitch link #1 vertical position as a function of azimuth.

2.3 **RESULTS AND ANALYSES**

The following section presents results that were obtained during some specific maneuvers of the H155 in particular a 160kt level flight for which the rotor asymmetry is large enough to trigger dynamic responses.

2.3.1 H155 BLUE EDGETM

Some dynamic issues were encountered during the development of the Blue EdgeTM blade on H155, in particular large torsion deformations and high control loads were measured during the development of such a blade [5]. The stability of the flap/torsion mode also proved to be a challenge in the development of this double swept blade that was flight tested on H155 [5]. Therefore, the H155 was chosen to analyze the sensitivity of the rotor dynamic behavior (including torsion and control loads) to the control chain flexibility.

The following sections assess the effect of the fuselage reinjection and eventually both reinjections (fuselage and swashplates) on the control loads and on the torsion deformations.

2.3.2 FUSELAGE REINJECTION

Figure 10 and Figure 11 show that the effect of the fuselage reinjection only slightly changes the dynamic behavior of the helicopter (FUS in the following figures means 'FUSelage reinjection'). Furthermore, the impact on the actuators is negligible and the pilot controls to achieve a high speed flight are not changed due to the fuselage elasticity.



Figure 10: Torsion at blade tip: effect of fuselage reinjection.



Figure 11: Pitch link force: effect of fuselage reinjection.

2.3.3 FULL MODEL

Figure 12, Figure 13 and Figure 14 show the effect of the fuselage and swashplate reinjections on the input controls and on the blade elastic deformations for a trim state at 160kts (FUS-SWP in the figures means both the FUSelage and SWashPlate reinjections are turned on). The modification of the controls displacements) (servo-actuators is quite negligible, but the pitch-link forces have a richer dynamic content that can also be seen on the torsion at the blade tip. In particular this dynamic torsion may be changed by as much as 0.2deg.



Figure 12: Servo-actuators positions.



Figure 13: Torsion at blade tip: effect of swashplate reinjection.



Figure 14: Pitch link force and position computed by HOST.

The harmonic content of the blade and rotor loads is much richer when the flexibility of the control chain is turned on. As a result, the blade torsion mode (frequency, damping and modal shape) may significantly be changed by the swashplate elasticity and fuselage reinjection. The next section sets-up a linearization approach for the dynamic analyses of a HOST-VLM coupling.

3. LINEARIZATION

Equivalent linear systems are used to study the stability and handling qualities of helicopters. In our case the rotor, and in particular the blade torsion, are in the center of the stability analyses [4].

3.1 PROBLEM DEFINITION

In the previous section, a coupled trim was found thanks to a weak coupling strategy. In this section a methodology is defined to find a coupled HOST-CSD linear system. In our case the stability of the coupled torsion mode is of interest, based on the large torsion deformations and controls loads encountered during the development of B2005 [5].

3.2 LINEARIZATION IN HOST [4]

Once a trim calculation has been performed for fixed flight conditions, the linearization process can be done. In HOST this linearization process is based on the perturbations of all the internal states, input controls and movement components. In general the evolution of a perturbation is analyzed over a complete rotor revolution to study the stability of periodic systems. In HOST, the stability is calculated using the periodic components of the state variables. In the case of a periodic system, the equivalent linear system is azimuth-dependent:

(8)
$$\Delta[\ddot{X}] = [A_{1,\psi}^{H}] \cdot \Delta[\dot{X}] + [A_{2,\psi}^{H}] \cdot \Delta[X] + [B_{\psi}^{H}] \cdot \Delta[U]$$

To obtain a linear system with constant coefficients, the state vector is replaced by its periodic components:

(9)
$$[X] = [X_0] + \sum_{i} [[X_{ic}] \cos(i\psi) + [X_{is}] \sin(i\psi)]$$

where the harmonic components of the states are functions of time. In the end, we get the following state-space system. It should be noted that the A and B matrices defined below are constants.

3.3 LINEARIZATION IN VLM

In VLM, linearization is computed at each time step, and the state space system is azimuth dependent:

(11)
$$\begin{aligned} \Delta \begin{bmatrix} \ddot{X} \end{bmatrix} &= \begin{bmatrix} A_{1,\psi}^{CSD} \end{bmatrix} \cdot \Delta \begin{bmatrix} \dot{X} \end{bmatrix} + \begin{bmatrix} A_{2,\psi}^{CSD} \end{bmatrix} \cdot \Delta \begin{bmatrix} X \end{bmatrix} \\ &+ \begin{bmatrix} B_{\psi}^{CSD} \end{bmatrix} \cdot \Delta \begin{bmatrix} U \end{bmatrix} \end{aligned}$$

3.4 PROBLEM SET-UP

It can be seen that the state space systems computed by VLM and HOST are of different natures: in HOST the system is LTI (one state space for the whole rotor revolution) while in VLM the system is LTP (as many state space systems as time steps in a rotor revolution). The phenomena to be investigated involve the rotor motion coupled with the airframe, swashplate control and dynamic inflow. It is known that the use of MBC being crucial for the study of such interactions [7], the HOST state space format is therefore chosen as the format for the equivalent state space system.

3.5 STRATEGY

The coupling strategy set-up in this section is developed to get an equivalent coupled HOST-VLM state-space system.

3.5.1 FLOQUET APPROACH

The Floquet theory is useful for the dynamics analysis of systems for which the periodicity has non-negligible effects [8]. This theory is based on the calculation of the monodromy matrix, which represents the evolution of the states after one rotor revolution when the basis of those states is perturbed (perturbation of independent combinations of the states). Computing the monodromy matrix can be time consuming as it requires as many time simulations over one rotor revolution as there are degrees of freedom in the system. Therefore computing the monodromy matrix of a coupled CA-CSD system requires a number of strong coupling simulations that cannot be afforded. Non-linear effects that may change the modal properties of the monodromy matrix depending on the azimuth of the perturbation would also further increase the complexity of the analysis. Moreover, the internal states of the code could not be reached by the master script and a steady-state assumption would be necessary to get the coupled state-space system. All those drawbacks made us choose a simpler approach to compute the equivalent state-space system of the coupled HOST-VLM helicopter model.



Figure 15: Linearization approach.

3.5.2 STATE SPACE APPROACH #1

It was previously explained that the HOST linearization process was well adapted to the analysis of the rotor motion coupled with the airframe. It was therefore chosen to expand the HOST linearization capabilities to the couplings.

In this approach, the equivalent HOST-VLM linear system is built in a two-step approach:

1- VLM model state-space system:

The state-space system of the VLM model is created by GHOST by perturbing the input connections of the external model and by analyzing its connection outputs to HOST.

2- Coupled state-space system:

The equivalent linear system of the VLM model is then used within the classical linearization process from HOST to correct

the effect of perturbations of the HOST states on their accelerations.

3.5.3 STATE SPACE APPROACH #2

In this approach, the first step of the previous methodology is replaced by the calculation of the VLM state-space system by the CSD code itself. This supposes the CSD code modelling a rotating part can compute a state space system with multi-blade coordinates, such as described by equation 8.

3.6 CONCLUSIONS

- 1- A weak coupling procedure is applied for trim calculations of a comprehensive analysis coupled with a CSD code.
- 2- The coupled element (swashplate and/or airframe) is in the middle of the HOST multibody kinematic chain.
- 3- The coupling was performed simultaneously on two different elements (airframe and swashplate).
- 4- The convergence criterion is based on the evolution of the CA values of the coupled parameters and of other variables that depend on the trim law.
- 5- Relaxation improved the convergence
- 6- The coupling of HOST with VLM (complex control chain kinematics) did not significantly change the trimmed pilot controls.
- 7- On the other hand, the dynamic oscillations of the blade torsion are increased by the swashplate and fuselage reinjections.
- 8- Better predictions of the controls loads and torsion mode deformations are expected.
- 9- A linearization coupling strategy is set-up and remains to be tested in order to study the stability of the rotor when subject to swashplate and fuselage reinjections.

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REFERENCES

- [1] W. Johnson, "Rotorcraft Aerodynamics Models for a Comprehensive Analysis," *Journal of the American Helicopter Society*, 1998.
- [2] M. Costes, J. Raddatz, S. Borie, L. Sudre, A. D'Alascio, M. Embacher and P. Spiegel, "Advanced rotorcraft aeromechanics studies in the French-German SHANEL project," in 35th European Rotorcraft Forum, 2009.
- [3] M. Potsdam, H. Yeo and W. Johnson, "Rotor airloads prediction using loose aerodynamic/structural coupling," *Journal* of Aircraft, vol. 43, pp. 732-742, 2006.
- [4] B. Benoit, "HOST, a General Helicopter Simulation Tool for Germany and France," in *American Helicopter Society* 56th Annual Forum, Virginia, 2000.
- [5] Rauch, "Blue Edge: The Design, Development and Testing of a New Blade Concept," *AHS*, 2011.
- [6] Altmikus, Wagner, Beaumier, Servera, "A Comparison: Weak Versus Strong Modular Coupling for Trimmed Aeroelastic Rotor Simulations," *American Helicopter Society*, 2006.
- [7] W. Johnson, "Milestones in Rotorcraft Aeromechanics," Ames Research Center, Mofett Field, California, 2011.
- [8] S. L. D.A. Peters, "Significance of Floquet Eigenvectors for the Dynamics of Time-Varying Systems," *American Helicopter Society*, 2009.
- [9] A. A. Zaki, "Using tightly-coupled CFD/CSD simulation for rotorcraft stability analysis," 2012.
- [10] Kufeld, R. M. ; Johnson, W., "The effects of control system stiffness models on the dynamic stall behavior of a helicopter," *Journal of the American Helicopter Society*, vol. 45, pp. 263-269, 2000.
- [11] A. Abhishek, A. Datta and I. Chopra, "Prediction of UH-60A structural loads using multibody analysis and swashplate dynamics," *Journal of Aircraft*, vol. 46,

pp. 474-490, 2009.

[12] K. Nguyen and W. Johnson, "Evaluation of dynamic stall models with UH-60A airloads flight test data," in *Annual Forum Proceedings*, vol. 54, American Helicopter Society, 1998, pp. 576-588.