IDENTIFICATION OF CONSTITUTIVE MODEL PARAMETERS FOR CRASHWORTHINESS OF COMPOSITE STRUCTURES USING MULTIOBJECTIVE OPTIMIZATION

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Abstract

A numerical-experimental procedure for the identification of the model parameters which define damage and failure mechanisms in composite materials is here proposed. The identification process is based on vertical crash tests on thin-walled cylinders made of fiber-reinforced material and finite element analyses performed using the commercial code PAM-CRASH. The developed identification method, with the flexibility offered by multiobjective optimization, seems a promising way to tune and calibrate complex models as those used to identify damage and failure mechanisms in composite materials. In fact, the parameters identified by the identification procedure allow an error on the absorbed energy within the 5% against one of the 50% using the parameters carried out only via axial static tests. The identified parameters are then used to predict the crash behavior of conical absorption structures and of a front sacrificial structure for high performance automotive vehicles. Also in this case the results obtained show a good correlation with the experimental tests.

Introduction

In the last years, the concept of crashworthiness design is become a very important aspect of the structural design both in the aeronautical and automotive fields and generally extended to all the other transportation sectors. The crashworthiness has to assure the safety and the survivability of occupants within a certain range of accident and impact conditions.

In the aeronautical field, the most crashworthiness demanding structures are the helicopters where the energy absorption, mainly due to high vertical velocity component, is achieved by the plastic contribution of the landing gears and, in the major part, by the subfloor frame.

Several studies deal with design procedures and optimizations of helicopter subfloor made of aluminum alloy as well as composite materials (Ref 1-3). The crashworthiness design appear of primary importance also for the automotive industries which employed great resources to investigate and to improve the occupants safety developing new absorption structures and using innovative design procedures.

In both these industrial fields an increasing interest has been shown on the use of composite material in spite of the traditional ones such as aluminum alloys.

In fact, if properly designed, composite materials exhibit very good absorption capabilities, with controlled and stable crush forces, as well as very high stiffness-to-weight and strength-to-weight ratios.

In order to limit experimental tests, which are both expensive and time consuming, detailed finite element analyses can be exploited to predict the crash behavior of absorption structures. The changing of boundary conditions, mainly due to contacts between different parts, non-linear behavior of materials and other phenomena related to high deformation rate and meaningful deformations during crashes, suggest the use of explicit non-linear finite element codes such as PAM-CRASH and LS-DYNA. Unfortunately composite structures are characterized by complex failure events (lamina bending, transverse shear and local fiber buckling) and progressive damage modes, such as inter-laminar or intra-laminar cracks (Ref 4). Therefore, analytical models and ad hoc constitutive laws must be defined and tuned to predict failure/damage mechanisms and their evolution.

One of the crucial aspects in using finite element codes to predict the crash behavior of composite structures concerns not only with the definition of the constitutive model but also about the evaluation of the model parameters that, in

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general, cannot be estimated only via static tests. To overcome these difficulties an inverse procedure is proposed in this paper as to determinate the constitutive parameters of the "Bi-Phase" material model used within PAM-CRASH. The use of inverse numerical-experimental approaches to determinate material properties and constitutive parameters is not completely new in structural applications. In 1998 E. Markiewicz *et al.* (Ref 5) used an inverse approach to determinate the Cowper-Symonds parameters in a viscoplastic constitutive model, considering prismatic thin-walled sections in aluminum alloy under vertical crashes. In 1999 M.G.D. Geers *et al.* (Ref 6) identified the parameters of gradient-enhanced damage model for glass-fiber-reinforced polypropylene composites by means of a digital image correlated technique. More recently Rikards *et al.* (Ref 7) identified the elastic properties of laminates composites using response surface methods and eigenfrequencies of composite panels experimentally measured.

Inverse identification method

The foundation of an inverse identification method consists of formulating an optimization problem in which the design variables are the parameters to be identified and the error between a set of experimental tests and the corresponding numerical model is assumed as objective function to be minimized.

A block diagram of the inverse procedure is reported in Figure 1.

In this work vertical crash tests, performed on cylindrical thin-walled shells of carbon fabric fiber-reinforced composite material, are considered as set of experimental tests. The corresponding numerical models are then defined within PAM-CRASH.

Finally, the parameters of the material constitutive law, denoted with $\underline{p}$, in Figure 1, are modified by the optimization algorithm as to minimize the mean square error between the numerical and the experimental load-displacement curves of the vertical crash.

The optimization problem can then be formulated as to find the constitutive parameters $\underline{p}$ that minimize the quadratic error express as:

$$obj = \sum_{i=1}^{n} \left\{ F(\underline{p}, s_i) - y(s_i) \right\}^2 \qquad (1)$$

where 's_i' is the displacement at the time 't_i' and 'y' and 'F' are the experimental and numerical load values respectively.

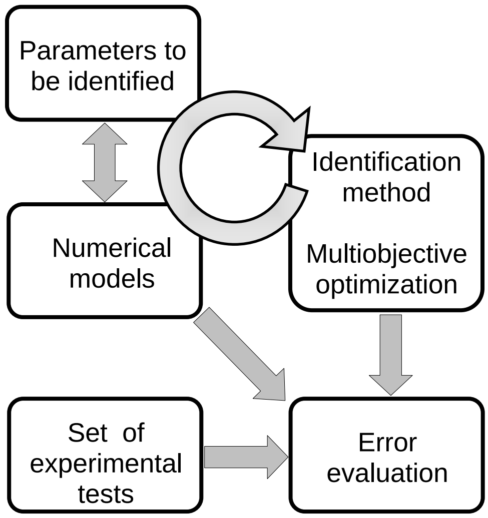


Fig 1. Block diagram of the numerical-experimental method

The choice of the mean square error of the load-displacement curves as objective function is mainly motivated by the need to achieve a good numerical-experimental correlation in terms of the absorbed energy as well as of the force values. In order to show that the identified parameters are intrinsic of the material behavior, they are then used to predict the absorption capabilities of conical absorption structures with elliptical section and of frontal sacrificial structures for high performance automotive vehicles.

Set of experimental tests

All the cylindrical shells used as experimental test set are made of the same carbon fiber-epoxy resin material and have the same nominal geometry. Indeed, they are 300 *mm* height and have a nominal internal diameter of 70 *mm*. Their upper edges are chamfered as so to provide a suitable trigger.

Two different stacking sequences are considered. The first one consists of 4 layers 0°/90°

alternatively oriented where the 0° direction is parallel to the specimen axis. The second one consists of four -45°/45° oriented layers.

The specimens are tested using the drop test facility of the Dipartimento di Ingegneria Aerospaziale of Politecnico di Milano with a mass of 110 *Kg* and an impact vertical velocity of about 8 *m/s*.

Since the vertical acceleration of the drop mass is measured by using an accelerometer located above the mass, the load values are immediately computed.

A displacement encoder monitors the vertical position of the mass during the drop, the impact velocity is measured by derivation.

The final values of the peak force F^d_{max} as well as of the mean crush force are determined after CFC-180 filtering at a sample frequency of 15 *kHz*. The load-displacement curves experimentally obtained are shown in Figure 2 considering both the specimen configurations. Figure 3 shows a typical specimen before and after the vertical crash test.

Numerical model

The explicit dynamic non-linear finite element code PAM-CRASH (Ref 8) is used to numerically analyse the axial crush of the composite cylindrical shells.

The cylindrical shells are modelled with four-node shell elements. A single integration point is defined for each ply through the thickness.

A sensitivity analysis on the element size is carried out as so to prevent mesh effects on the numerical results. The cylinder is then modelled by using 7227 four-node shell elements of dimensions fixed to 3×3 *mm*.

To represent the experimental impact conditions, two different models of the drop mass are compared. In the first one a rigid wall with a finite mass of 110 *Kg* and an impact velocity of 8 *m/s* is used while in the second one a rigid wall with infinite mass and constant vertical velocity of 8 *m/s* is considered. Since the use of a rigid wall with infinite or finite mass doesn't lead important variations on the numerical results, the solution with the constant velocity and infinite mass is used during the simulations in order to reduce the CPU time.

The lower support of the drop test facility is here modelled by using a fixed rigid wall.

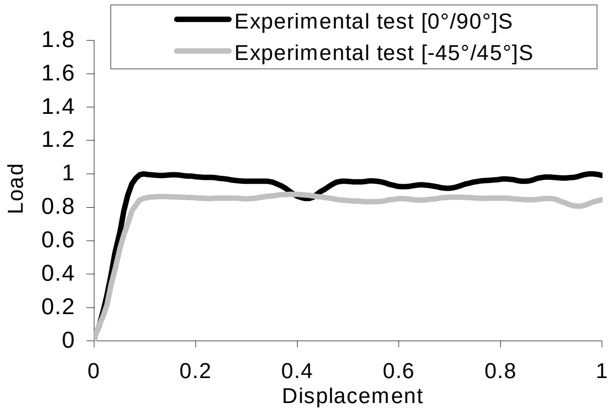


Fig 2. Normalized load-displacement curves of the cylinder typologies with $[0°/90°]_S$ and $[-45°/45°]_S$ stacking sequences

Fig 3. Tested specimen before and after vertical crashes

Sliding motion of nodes with respect to the walls is allowed with a consequent absorption of energy through friction.

The self-contact algorithm 36 (Ref 8) is used to prevent elements penetration. The contact algorithm is based on a penalty formulation, where geometrical interpenetrations between contacting surfaces are penalized by counteracting forces proportional to the penetration depth. The contact

thickness is fixed up to 1.6 *mm*, as the specimen thickness is.
The total CPU time required for each simulation is of about 5 hours on a workstation Visualize HP 4000 considering an analysed time of 0.03 *s* corresponding to a final end shortening of 240 *mm*.

Material model

This section refers to the definition of the material constitutive law used in the numerical analyses to model the damage of the composite material.
The "Bi-phase" model, denoted with MAT 130 in the PAM-CRASH code (Ref 8), is used.
It includes the effects of directionality in the material stress-strain response allowing a different fibres orientations to be specified at each through thickness integration point for the shell elements.
Indeed, the "Bi-phase" material constitutive law allows to identify two distinct non-physical phases, fiber phase and matrix phase respectively, to which is possible to assign different stiffness and strength properties.
Progressive damage evolution is defined for each phase separately via reduced material stiffness, according to:

$$[R]_{PLY} = [R]_m + [R]_f \quad (2.1)$$

$$[R]_m = [R]_{0m} \cdot (1 - d_m) \quad (2.2)$$

$$[R]_f = [R]_{0f} \cdot (1 - d_f) \quad (2.3)$$

where $[R]^m$ and $[R]^f$ are the matrix and fiber stiffness matrices, with initial undamaged values $[R]_0^m$ and $[R]_0^f$.
The scalar damage parameters, denoted with d^m e d^f respectively for the matrix and fiber phases, range from 0 to 1.
Indeed, the matrix damage is split into two contributions; the first one due to hydrostatic volumetric strain (first invariant of the strain tensor) and the second one due to deviatoric strain (defined as the second invariant of the deviatoric strain tensor). The total matrix damage is the sum of these two distinct contributions.
On the opposite, the fibre damage is due to the sole one-dimensional fibre strain.
The calibration of the model consists of assigning initial stiffness values for $[R]_0^m$ and $[R]_0^f$, deviatoric and volumetric matrix damage, as well as fiber damage.

Since each kind of damage (volumetric and deviatoric damage of the matrix phase, fiber damage) is completely described by 5 parameters in compression and 5 in traction, the total number of parameters required to fully define the material model is equal to 30. Standard calibration methods for unidirectional materials are presented in the PAM-CRASH manuals (Ref 8)

Numerical results using the reference constitutive parameters

The constitutive parameters obtained only via static tests and following the suggestions of PAM-CRASH (Ref 8) are assumed as initial point for the optimization/identification runs. The numerical results obtained with the reference parameters are compared to the experimental ones in Figures 4 and 5 considering both kinds of cylindrical shells.
The results show how the constitutive parameters identified only via static tests are not completely able to predict the crash behavior at the cylinders.
Indeed, the values of the mean crush force numerically predicted are the 40% and the 15% of the experimental values considering the first and the second cylinder topologies respectively.
As far as the stacking sequence of $[45°/-45°]_{2S}$ is concerned, a lateral sliding after 70 *mm* of crushing is observed because of an asymmetric fracture in the lower edge as shown in Figure 6. This probably is due to the main absorption mechanisms of these structures that are related to micro buckling of fiber and delamination.

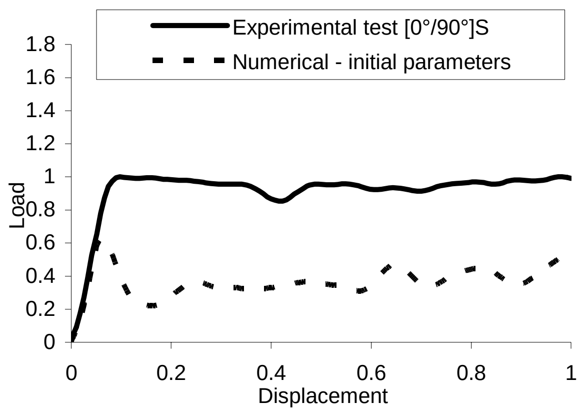


Fig 4. Normalized load-displacement curves of the cylinder typology with stacking sequence of $[0°/90°]_S$

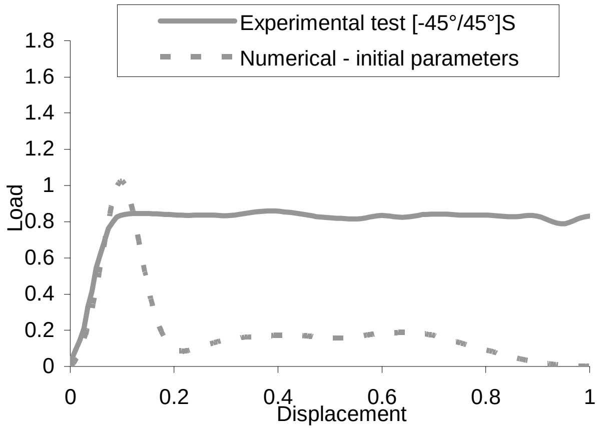


Fig 5. Normalized load-displacement curves of the cylinder typology with stacking sequence of [-45°/45]$_S$

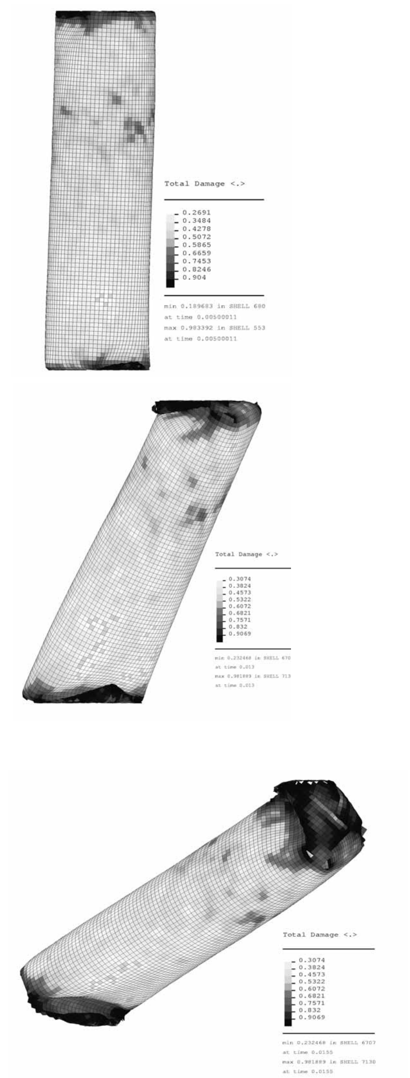


Fig 6. Sequence of the cylinder [-45°/45°]$_S$ numerical deformations

Optimization method

The optimization algorithm used to identify the material law parameters is based on classical Sequential Quadratic Programming (Ref 9). Therefore, assuming ‘n’ parameters to be identified, at each major iteration, the algorithm requires n+1 evaluations of the objective function and of the constraints in order to define the new search direction. Since each evaluation of the objective function requires a non-linear explicit analysis performed with PAM-CRASH, some working assumptions are introduced to limit the number of design variables, i.e. the number of constitutive parameters to be identified.

By assuming:

1. a symmetrical behavior in tension and compression for the matrix volumetric and deviatoric damage,
2. a symmetrical behavior in tension and compression for the fiber volumetric damage,

the number of parameters to be identified is limited up to 15 against the original 30.

A further sensitivity analysis on the values of the matrix volumetric components is carried out to evaluate the effect on the energy absorbed. It is observed that the effect produced by the systematic variation of these 5 parameters is negligible compared to the same variations of the 5 deviatoric components. Thus, the values of the matrix volumetric components are fixed at the reference ones obtained by static tests.

With this further assumption, the total number of parameters to be identified is limited to 10.

As a matter of fact, the 10 selected parameters are not fully independent since they have to describe consistent damage models.

Indeed, the strain values ε_i, ε_l, ε_u together with damage values d_e and d_u at ε_l and ε_u respectively (Ref 8), both for matrix and fiber phases, are subject to the following restrictions:

$$\varepsilon_i < \varepsilon_l < \varepsilon_u \quad d_l < d_u \tag{3}$$

The presence of such kind of inequalities relations among the damage parameters must be carefully considered during the optimization/identification runs in order to achieve feasible solutions.

Following the idea of the standard identification methods, a first single-objective optimization is performed considering as objective the error on the load-displacement curve of the [0°/90°]$_S$ cylinders.
The parameters identified show an improvement of the 50% with respect to the initial ones. Besides, the use of the previously identified parameters used for the cylinder with stacking sequence of [-45°/45°]$_S$ leads an increasing of the mean crush force of about 50% with respect to the initial one, as shown in Table 1.
With the intent to achieve better results, the optimization method is modified in order to account simultaneously both the available cylindrical typologies.
Therefore, a multiobjective optimization problem is formulated in order to consider the presence of different experimental test sets.

Multi-objective implementation

Two different objective functions, F_1 and F_2 are defined as the mean square errors between the load-displacement curves returned by the numerical analyses and the experimental crash tests considering the first and the second cylinder typology separately.
The formulation of a multi-objective optimization (Ref 10-11) allowing to simultaneously consider two or more objectives each other in conflict, is based on the definition of the Pareto optimal points. A point inside the optimization domain is a Pareto's point if it satisfies the imposed constraints and if any further improvement of one of the objectives necessarily implies a lose of at least one of the others.
However, as it happens in many practical problems, the identification of the Pareto points is only a part of the whole optimization process since a single final feasible solution is required. In these cases, decision criteria must be applied to draw out the final solution.
Since the identification of the whole Pareto's points should results very expensive from a computational point of view, herein the decision criteria are a priori fixed and a goal-attainment method is used.

The working hypothesis of the method, shown in Figure 7, consists in the definition of a searching direction in the objective function space.
The original multiobjective problem is then formulated by defining a vector of weight $\underline{w}$ among the desired goals. In order to find the best-compromise solution, the following problem is solved:

$$\min_{z\in\chi} \beta \quad \text{subject to} \quad y_i + \beta w_i \geq f_i(\underline{p}) \qquad (4)$$

with $i = 1,\dots,m$ $w_i > \varepsilon$ and $\sum_1^n w_i = 1$

where β is a scalar variable unrestricted in sign, χ is a feasible solution region. If some $w_i = 0$ exist, it means that the maximum limit of objectives $f_i(\underline{p})$ is y_i. It can be easy shown that the set of non inferior solutions may be generated by varying $\underline{w}$. The optimization procedure is implemented in MATLAB 5.3 (Ref 12). The goal-attainment method offers the advantage to be implemented as a non-linear programming problem and therefore to be treated by using a standard Sequential Quadratic Programming algorithm.
A first optimization is performed considering five independent parameters, ε_{i-m}, ε_{l-m}, d_{l-m}, d_{u-m}, ε_{i-f}, and assuming as initial point the parameters obtained only via static tests.
The optimization is performed by fixing the vector weight equal to one, i.e. assuming that the two error functions have the same importance. The optimization procedure required a total number of 10 iterations, e.g. 160 finite elements runs, with a total CPU time 300 h on a HP Visualize 4000. In Figure 8 the normalized values of the objective functions versus the iteration number are shown.

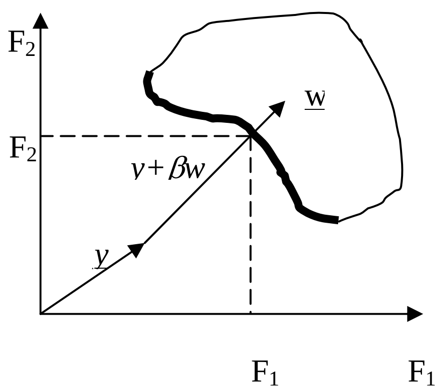


Fig 7. Goal-attainment method with two objective functions

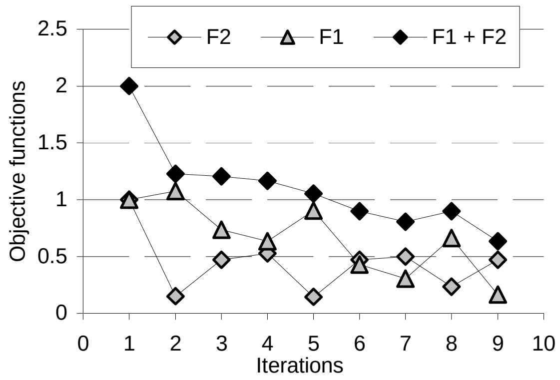


Fig 8. Values of the normalized objective functions versus iterations number

Considering the cylindrical shells with the stacking sequence of $[0°/90°]_S$ the value of the absorbed energy returned by the numerical model is about 80% of experiment tests. Similar results are obtained with the cylinder with the stacking sequence of $[-45°/45°]_S$. Table 1 provides a comparison between the results. Indeed, the first line of the table is obtained by minimizing the error only on the first cylinder typology with stacking sequence of $[0°/90°]_S$.

In the second line results obtained by the multi-objective optimization are reported. It is observed that the mean value of the absorbed energy obtained by a multi-objective optimization is greater then one return by the single-objective optimizations.

Table 1. Comparison between the normalized absorbed energies

	Mean force 0°/90°	Mean force ±45°
Experimental	1	1
Single Objective [0°/90°]	0,923	0,485
Multi Objective	0,85	0,79

A second multi-objective optimization is performed increasing the number of independent parameters to 6: $\varepsilon_{i\text{-}m}$, $\varepsilon_{l\text{-}m}$, $d_{l\text{-}m}$, $d_{u\text{-}m}$, $\varepsilon_{i\text{-}f}$, $d_{l\text{-}f}$.

In this case, the numerical results are within 5% to experiment tests with a meaningful improvement of the results obtained using the parameters evaluated only on the base of static tests, as shown in Table 2. The comparison between the normalized load- displacement curves of the cylinder are reported in Figures 9 and 10.

Table 2. Values of the energy returned by experimental tests and numerical analyses for the cylindrical shells

	Initial parameters	Identified parameters
Stacking sequence $[0°/90°]_{2S}$	44%	96%
Stacking sequence $[-45°/45°]_{2S}$	23%	103%

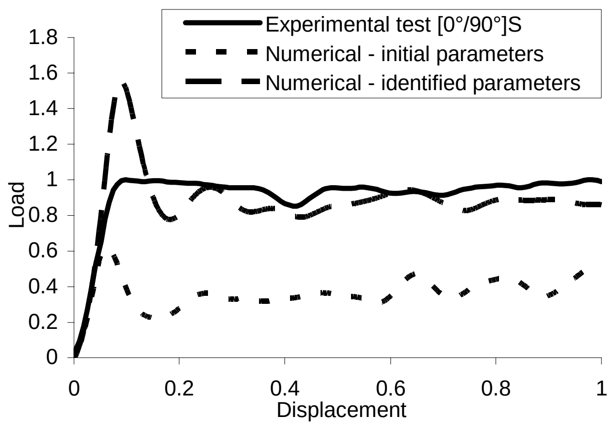


Fig 9. Comparison between the normalized load-displacement curves of the cylinders with stacking sequence $[0°/90°]_S$

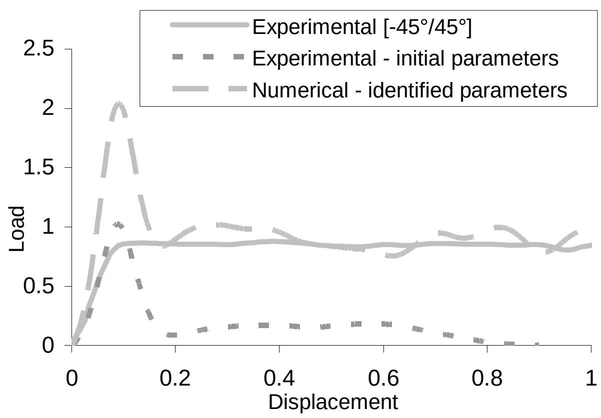


Fig 10. Comparison between the normalized load-displacement curves of the cylinders with stacking sequence $[-45°/45]_S$

Analysis of other structural typologies

The damage parameters identified by means of the inverse identification procedure are then used to analyze two different energy absorption structures made of the same composite material. In particular, numerical analyses are used to predict the crash behaviour of conical structures with varying elliptical section and of a frontal sacrificial structure for high performances automotives.

Conical structure

The specimens are fabric fiber-reinforced composite cylinder with elliptical cross section variable in length. They are realized using a total number of 7 layers, with a stacking sequence of [0°/45°/0°/0°/0°/45°/0°]. The total length of the structures is of 360 *mm*.
The experimental tests are performed using the same drop test facility used to test the reference cylindrical shells. A drop mass of 110 *kg* is used and the impact velocity is increased up to 10 *m/s*.
The model consists of 10344 4-nodes shell elements. The contact algorithm type 36 is used to prevent penetration between different structural parts.
The load-displacement curves obtained by the finite element analyses using the initial and the identified parameters are compared to the experimental ones in Figure 11. The results seem particularly interesting if compared to the ones obtained by using the original damage parameters based only on static tests.
Some details regarding the deformed shape evolution of the conical structures are reported in Figure 12.

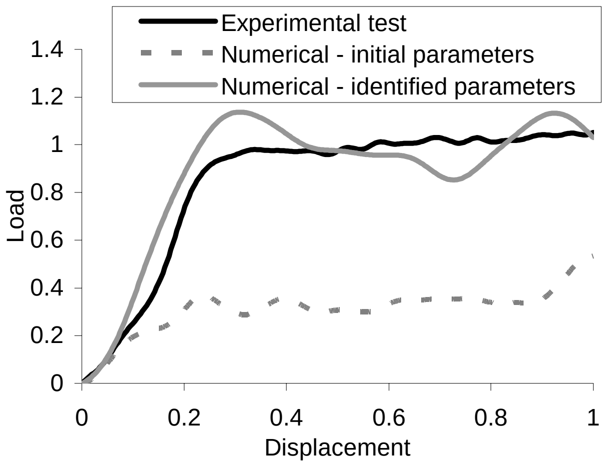
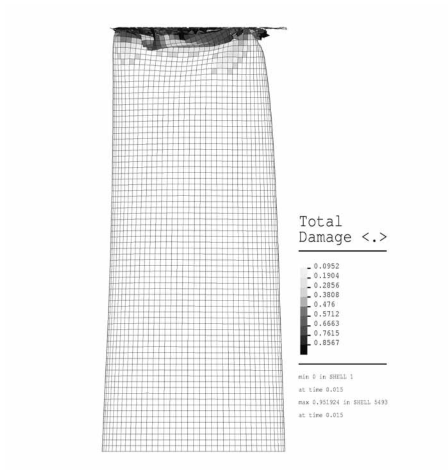
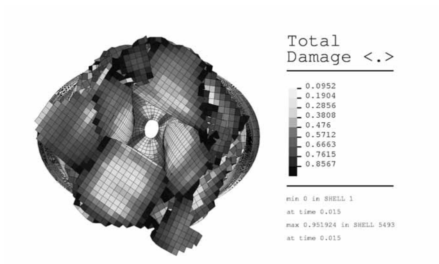


Fig 11. Comparison between the normalized load-displacement curves of the conical absorption structure



Lateral view - state t=0,015 s



Top view – state t=0,015 s

Fig 12. Deformed shape of the conical absorption structure with elliptical section

Frontal Sacrificial Structures (FSS)

The second structural typology analyzed is a frontal sacrificial structure for high performance automotives.
The frontal sacrificial structure is 660 *mm* long and presents a variable cross-section along its axis.
It is an honeycomb aluminum structure with several composite layers on the inner and outer surface. The composite material used is the same by which the reference cylindrical shells are made.
The experimental tests are performed using the horizontal crash facility of the Dipartimento di Ingegneria Aerospaziale of Politecnico di Milano. The structure is fixed upon a main horizontal sled accelerated by means of a smaller driving sled wire trained. The training system mainly consists of a series of wire rope pulleys and of an air-compressed piston.
The run covered by the main horizontal sled is of about 45 *m* as shown in Figure 13.
Just few meters before the impact, the driving sled finishes its run while the main one goes on until the specimen crashes against a rigid mass. Similarly to the vertical drop tests, the sled deceleration is measured by means of accelerometers.

Fig 13. Horizontal sled

The measured impact velocity is approximately equal to 12 *m/s* and the slide mass is of 390 *Kg* accounting the mass of the sacrificial structure.
As far as the numerical model is concerned, it was decided to model only half of the whole structure because of the symmetry both of the structure and of the impact conditions. Thus, symmetric boundary conditions are introduced.
The proposed solution allows to significantly reduced the total CPU time.
The final model, represented in Figure 14, consists of 23450 4-nodes shell elements.
The elements dimensions are chosen observing the residual shape of the structure after the crash. Indeed, the characteristic dimensions of the element in the tip of the structure are of about 3×3 *mm* while, going away from the tip, the elements dimension are increased.
The nodes of the base are constrained to a rigid wall with infinite mass, while a rigid wall of finite mass equal to 390 *Kg* is used to simulate the main horizontal slide.
The same material properties and the same contact algorithms used for the previous structures are here considered.
The aluminium alloy material is modelled as a isotropic layer of different thickness inside the composite ones using a multi-layered shell formulation. The acceleration-time curve of the frontal sacrificial structure numerically obtained is compared with the experimental one in Figure 15 after CFC 600 filtering.

Also for this second structural typologies a good correlation is achieved both in terms of the acceleration values and of the deformed shape evolution shown in Figure 16.

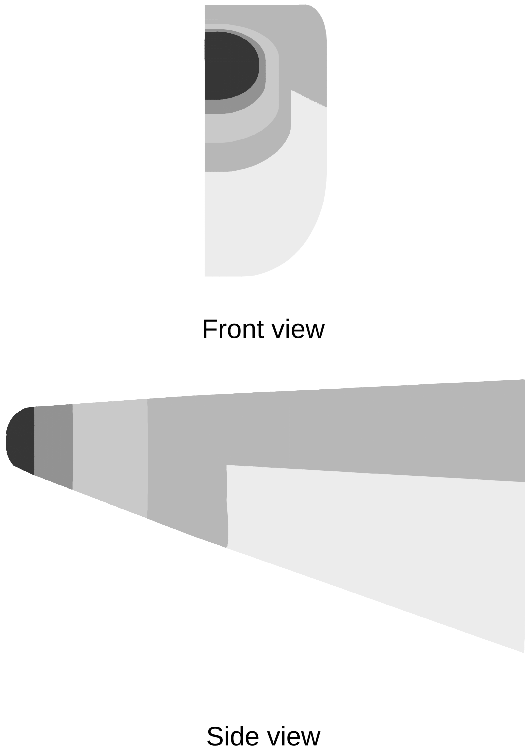


Fig 14. Numerical model of the Frontal Sacrificial Structure

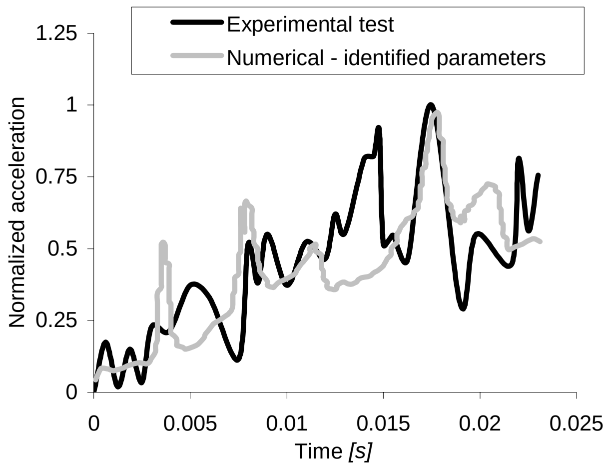


Fig 15. Numerical and experimental acceleration-time diagrams of the FSS

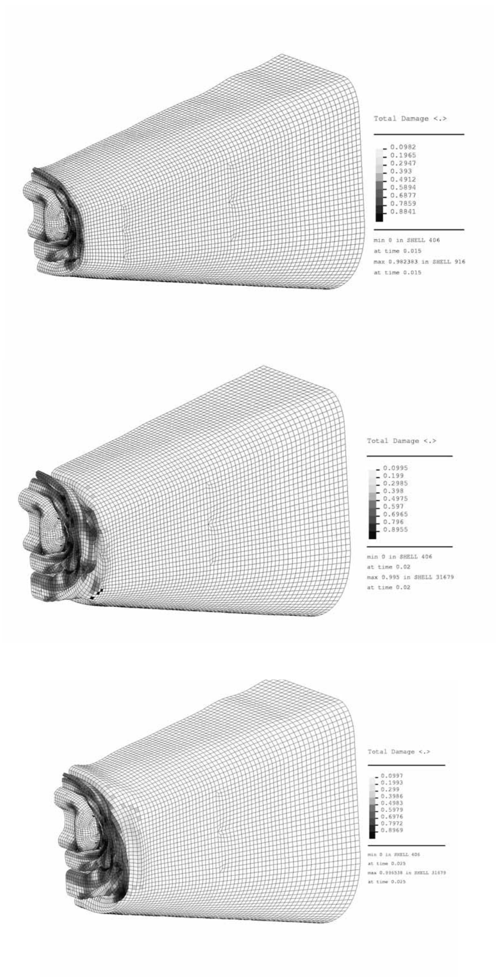


Fig 16. deformed shape evolution of the Frontal Sacrificial Structure

Conclusions

The knowledge and the definition of the damage parameters of constitutive models is essential to numerically predict the absorption capabilities of composite structural elements. This research propones an helpful approach for the identification of the damage parameters of the "Bi-Phase" material model used within PAM-CRASH.

The traditional formulation of inverse-identification methods is here modified as so to simultaneously consider different set of experimental data. Therefore a multi-objective optimization is formulated using a goal-attainment method.

The identified parameters allow to obtain an important improvement of the numerical-experimental correlation and to contain the errors on the absorbed energy within the 5% considering cylindrical shell structures.

In order to evaluate the quality and the reliability of the previously identified parameters, crash simulations are performed considering two different absorption structures: conical structures with elliptical section and frontal sacrificial structures for high performances automotives. The numerical results are compared with the corresponding experimental tests showing a good correlation in terms both of the deformed shapes and of the acceleration values.

Thus, the use of inverse identification methods, with emphasis on the flexibility offered by multi-objective optimizations, seems a promising way to tune and calibrate complex models as those used to identify damage and failure mechanisms in composite materials. Besides, the procedure seems very flexible and applicable to a large spectrum of industrial problems.

References

1. Giavotto V, Caprile C, Sala G., The Design of Helicopter Crashworthiness - Energy Absorption of Aircraft Structures as an Aspect of the Crashworthiness. Proceeding AGARD 66th Meeting of the Structures and Material Panel. Luxembourg, 1988. p.6.1-6.9;

2. Kindervater C, Kohlgruber D, Johnson A. Composite Vehicle Structural Crashworthiness: a Status of Design Methodology and Numerical Simulation Techniques. Proceeding of International Crashworthiness Conference. Dearbon, Michigan, 1998. p. 444-460;

3. Bisagni C, Lanzi L, Ricci S. Optimisation of Helicopter Subfloor Components under Crashworthiness Requirements using Neural Networks. Journal of Aircraft 2002; 39(2): 296-304;

4. Farley L, Jones M. Crushing characteristics of continuous fiber-reinforced composites tubes. Journal of Composite Materials 1992; 26(1); 37-50;

5. Markiewicz E, Ducrocq P, Drazetic P. An inverse approach to determine the constitutive model parameters from axial crushing of thin-walled square tubes. Int. J. Impact Engineering 1998; 21(6); 443-449;

6. Geers MGD, de Borst R, Peijs T. Mixed numerical-experimental of non-local characteristics of random-fiber-reinforced composites. Composite Science and Technology 1999; 59; 1569-1578;

7. Rikards R, Chate A, Steinchen W, Kessler A, Bledzki AK. Method for identification of elastic properties of laminates based on experiment design. Composites Part B: Engineering 1999; 30(3); 279-289;
8. PAM-CRASH Users Manuals and Reference Manuals Version 2000. Paris: ESI/PSI Inc., 2000;
9. Fletcher R. Pratical methods of optimization. New York: Wiley-Interscience, 1980;
10. Wolfram S. Multicriteria optimization in engineering and in the sciences. New York: Plenum Press, 1998;
11. Miettinem K. Nonlinear multiobjective optimization. Boston: Kluwer Academic Publisher, 1999;
12. The MATH WORKS, Inc. Optimization Toolbox – User's Guide MATLAB 5.3. Natick, MA:1999.