

# SYSTEM IDENTIFICATIONS OF THREE-AXIS GYRO MODEL AND BASE MODEL OF A RC HELICOPTER WITHOUT STABILIZER BAR

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## Abstract

The three-axis gyro as a black box, always hinders the control development of a helicopter without stabilizer bar. In this paper, a comprehensive research on three-axis gyro model and base model of RC helicopter was concluded. Four-direction frequency sweep flights were conducted on yaw, roll, pitch, and heave channels. For obtaining the dynamic model, the system identification process was completed in CIFER. By using a three-stage method, the yaw dynamic was separated into three parts, which were identified respectively and formed a whole yaw model. The parameters in coupled roll-pitch dynamic were identified, indicating an increase trend from before gyro to after. The derivatives of base model of coupled roll-pitch were closer to the real ones. A single heave model was concluded due to the direct control of heave direction. This work gives a better view for control of helicopter that containing gyro part.

## 1. INTRODUCTION

There has been a significant growth in the use of unmanned aerial vehicles (UAVs) for a multitude of military and civilian applications over the past few years. The RC helicopter is a very important vehicle in this market. There is a remarkable feature of a former single rotor RC helicopter that it always carries a stabilizer bar to compensate its instability. However, with the development of control theory and electronic equipment, the stabilizer bar is able to disappear and the three-axis gyro emerges. Small-scale unmanned helicopter has a considerable high frequency of motion, which leads to problems in developing systematic product. For designing a reliable control system of this helicopter, one method is to assemble an existing gyro product which is certainly not a long-term policy, while the other method is to contain the gyro part in the control design which is always dangerous to have an initial flight test. In this condition, gaining an accurate dynamic model is imperative.

For enhancing the understanding of helicopter dynamics, there are many researches about gaining the dynamic model of the helicopters. The most pragmatic method is to obtain parameters from flight tests by system identifications. The frequency-domain system identification has many advantages in dealing with inherently unstable system [1]. It determines the linearized model by minimizing the error between the model's frequency responses and those derived from measured time-domain data [2]. Bhandari [3,4] identified 6-DoF and 12-DoF dynamic models of a Raptor 50 helicopter. He found a better fitting by using a high-order hybrid model which is useful for high-bandwidth control system. Mettler [5] gave a comprehensive research about model developing and control system setup by using CIFER

and CONDUIT software on CMU's Yamaha R-50. Other successful results are documented in Refs. 6, 7. It is noticeable that these identification tests were conducted on helicopters with stabilizer bar. The tail gyro model was replaced by a first-order or second order linearized model. Liu, G. Q. [8] focused on a specific Futaba gyro to identify its equivalent simplified model, giving a sense of nonlinear. Nevertheless, these are far from enough when it comes to three-axis gyro application.

This paper is to investigate the dynamic model of an existing three-axis gyro product and the base model of a RC helicopter based on the frequency response identification method hold by CIFER. It gives a research way in dealing with this kind of unstable helicopter. The paper concludes the dynamic model of the helicopter (with and without gyro part) and the simplified gyro model with good fitting. The yaw direction, roll direction, pitch direction and heave direction were all considered. Aiming at different performances of directions, different strategies of identification were applied. The influences of gyro were investigated comprehensively.

## 2. FLIGHT TEST INSTRUMENTATION AND DATA



Figure 1. Test platform.

In this work, the JR700 RC helicopter was chosen for the research. The main parameters are shown in Table 1. This version is a single rotor type without a stabilizer bar. So it installs a three-axis gyro to stable the plane. The Futaba CGY750 gyro was chosen. As shown in Figure 1, the STA34 onboard computer was set below the fuselage, which can record the velocity and attitude messages of helicopters changing over time and can also gain the input control signals. The gyro was set on the top, which had some settings to be noted: Flt. Mode=3, Pit.Rate=0.4, Ele.Rate=0.6, and Ail.Rate=0.6. These settings were fixed during experiments. For flight safety reason, there were two safety bars installed on the landing gear, which has 1.5m length and 0.18kg weight for each.

It is worth noting that the center of the onboard computer did not coincide with the center of gravity (CG) of the helicopter. The lever arm effect [9] will influent the accuracy of acceleration velocity. By using suspension method, the CG was gained by capturing high-quality photos. According to Ref. 5, the data were corrected with the measured center offset.

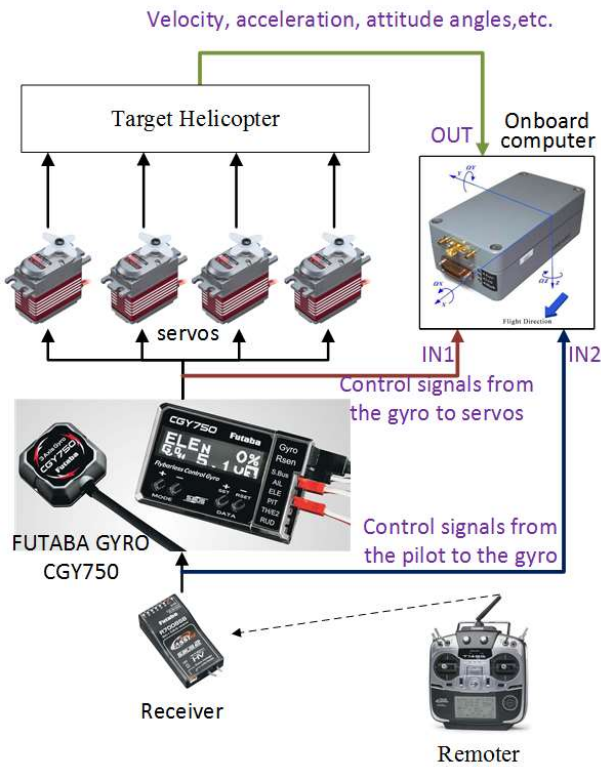


Figure 2. Experiment system illustration.

Table 1. JR700 parameters

| Parameter     | Value                    | Meaning                                              |
|---------------|--------------------------|------------------------------------------------------|
| $R_{mr}$      | 0.767 m                  | Main rotor disc radius                               |
| $R_{tr}$      | 0.137 m                  | Tail rotor disc radius                               |
| $m$           | 8.01kg                   | Helicopter mass                                      |
| $l_f$         | 1332m                    | Fuselage length                                      |
| $c_{mr}$      | 65mm                     | Main rotor blade chord                               |
| $g$           | $9.8015 N \cdot kg^{-1}$ | Acceleration of gravity                              |
| $\rho$        | $1.225 kg / m^3$         | Air density                                          |
| $\Omega_{mr}$ | 1700 rpm                 | Main rotor rotation speed (fixed during experiments) |
| $\Omega_{tr}$ | 360.2 rpm                | Tail rotor rotation speed                            |

As shown in Figure 2, there were four types of control channels generated by the pilot: collective pitch (COL), longitudinal cyclic pitch (LON), lateral cyclic pitch (LAT), and tail rotor collective pitch (PED). These four channels were transmitted to the gyro, tabbed as IN2. By control augmentation of gyro, the four channels were stabled and separated to control four servos, three of the swashplate and one of the tail. These servo signals can also be calculated as the four-direction control, which were tabbed as IN1, [col, lon, lat, ped]. The data of IN1 and IN2 were all unitized to [-0.5,0.5]. The whole plane was fully instrumented and flight-tested for this modeling work. By conducting the pitch, roll, yaw, and heave types of sweep frequency motions at hovering, all the important parameters of the base model of helicopter and gyro model were excited. The response variables of helicopter, such as: attitude angles  $[\varphi, \theta, \psi]$  (roll, pitch, yaw), linear acceleration  $[a_x, a_y, a_z]$ , angular velocity  $[p, q, r]$  (roll, pitch, yaw), and velocity  $[u, v, w]$ , were tabbed as OUT. By analyzing the two pairs of signals: [IN1 OUT] and [IN2 OUT], the dynamic models between them can be obtained. The system identification process was conducted in CIPHER software. The frequency response identification is a quite mature method developed for many years. CIPHER is a recognized reliable software for system identification which has contributed to many researches [10, 11]. For each channel, 10 best sweep flight data were used for identification.

### 3. IDENTIFICATION RESULTS AND VALIDATION

#### 3.1. Yaw Identification

On the one hand, the yaw dynamic of helicopter is simpler compared with the roll and pitch, because it has lesser coupling, on the other hand it's more complex for identification, because the gyro adds the tail-locking function. For increasing the accuracy of

the yaw identification, some assumptions should be made toward the gyro dynamic.

In this work, a three-stage identification method was applied on yaw direction. The gyro dynamic can be illustrated as shown in Figure 3.

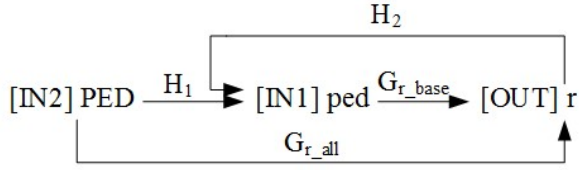


Figure 3. Yaw dynamic illustration.

Stage 1  $G_{r\_base}$ : Helicopter base model of yaw direction.

Stage 2  $H_1$  &  $H_2$ : the linear approximation of gyro model.

Stage 3  $G_{r\_all}$ : Helicopter overall model of yaw direction.

According to the linearization theory [12], the base model can be simply regarded as first-order system.

$$(1) \quad G_{r\_base} = \frac{r}{ped} = \frac{N_{ped}}{s - N_r} e^{-\tau_{r\_base}}$$

While the gyro model can be extracted as PI control system [13].  $k_p$  and  $k_i$  can be obtained by step input on the ground.  $k_a$  was identified by a yaw direction sweep motion, which can also be achieved before taking off.

$$(2) \quad H_1 = \frac{ped}{PED} = (k_p + \frac{k_i}{s}) \cdot k_\delta$$

$$(3) \quad H_2 = \frac{ped}{r} = -(k_p + \frac{k_i}{s}) \cdot k_a$$

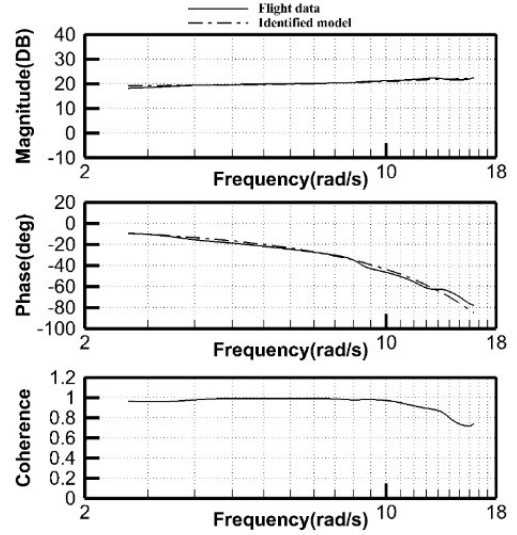
$$(4) \quad G_{r\_gyro}: ped = H_1 \cdot PED + H_2 \cdot r$$

Then by using  $G_{r\_base}$  and  $G_{r\_gyro}$ ,  $G_{r\_all}$  can be calculated as,

$$(5) \quad G_{r\_all} = \frac{r}{ped} = \frac{sk_p N_{ped} k_\delta + k_i N_{ped} k_\delta}{s^2 + (k_p k_a N_{ped} - N_r)s + k_i k_a N_{ped}} e^{-\tau_{r\_all}}$$

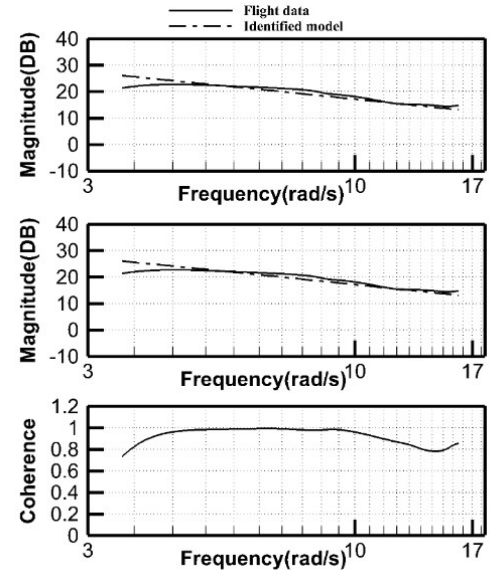
As shown from the equation,  $G_{r\_all}$  is a second-order system, which can also be identified by signal pair [IN2 OUT]. While  $G_{r\_base}$  and  $G_{r\_gyro}$  can also be solved out by system identification method, from signal pair [IN1 OUT] and [IN2 IN1] respectively. Finally, all the parameters in the equations above were collected.

The identification results:



Cost: 4.372

Figure 3. Yaw identification by data [IN2 OUT] corresponding to model  $G_{r\_all}$ .



Cost: 44.54

Figure 4. Yaw identification by data [IN1 OUT] corresponding to model  $G_{r\_base}$ .

Figure 3 and 4 show a high matching degree between the flight data and the identified model. It is worth noting that the cost of the data [IN2 OUT] is lower than [IN1 OUT], which is because the data IN2 was the sweep input signal controlled by the pilot and it was easier to maintain the sweep frequency request. All the parameters of the yaw dynamic were obtained as shown in Table 2. The main reference indicates the main identification data that deciding the parameter because of their high credibility.

**Table 2. Parameters of yaw identification results**

| Parameter        | Value   | Main reference |
|------------------|---------|----------------|
| $N_r$            | -0.7512 | [IN1 OUT]      |
| $N_{ped}$        | 75.9459 | [IN1 OUT]      |
| $k_f$            | 17.70   | Ground test    |
| $k_p$            | 0.8646  | [IN2 OUT]      |
| $k_\delta$       | 2.68    | [IN2 IN1]      |
| $k_a$            | 0.2941  | [IN2 OUT]      |
| $\tau_{r\_all}$  | 0.072   | [IN2 OUT]      |
| $\tau_{r\_base}$ | 0.0505  | [IN1 OUT]      |

Finally, the three-stage model of the yaw control dynamic are decided:

$$(6) \quad G_{r\_base} = \frac{75.9459}{s + 0.7512} e^{-0.0505s}$$

$$(7) \quad G_{r\_gyro}: ped = (k_p + \frac{k_f}{s}) \cdot (k_\delta PED - k_a r) \\ = (0.8646 + \frac{17.7}{s}) \cdot (2.68 PED - 0.2941 r)$$

$$(8) \quad G_{r\_all} = \frac{r}{ped} = \frac{175.9690s + 3602.6}{s^2 + 20.0636s + 395.3770} e^{-0.072s}$$

### 3.2. Coupled roll-pitch identification

The coupled roll-pitch equation is [5]:

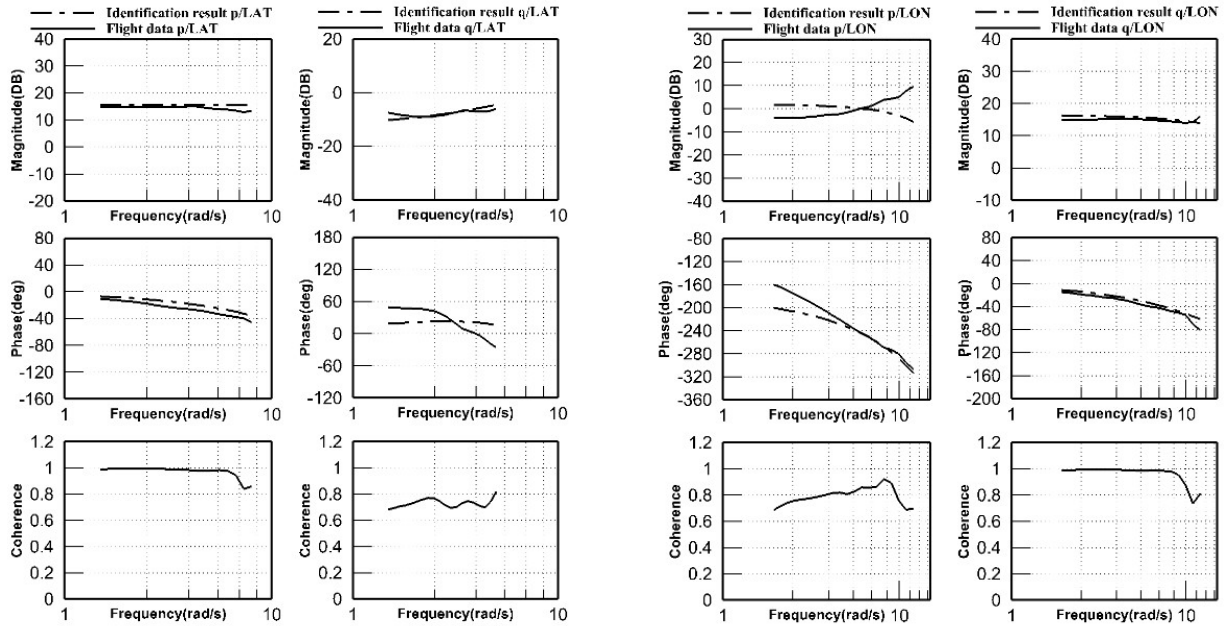
$$(8) \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \tau_f & 0 \\ 0 & 0 & 0 & \tau_f \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{a} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & L_b \\ 0 & 0 & M_a & 0 \\ 0 & -\tau_f & -1 & A_b \\ -\tau_f & 0 & B_a & -1 \end{bmatrix} \begin{bmatrix} p \\ q \\ a \\ b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ A_{lat} & A_{lon} \\ B_{lat} & B_{lon} \end{bmatrix} \begin{bmatrix} \delta_{lat} \\ \delta_{lon} \end{bmatrix}$$

Observation equation:

$$(9) \quad \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ a \\ b \end{bmatrix}$$

Where  $a$  and  $b$  are the longitudinal and lateral flapping angle respectively,  $\tau_f$  is the rotor time constant.

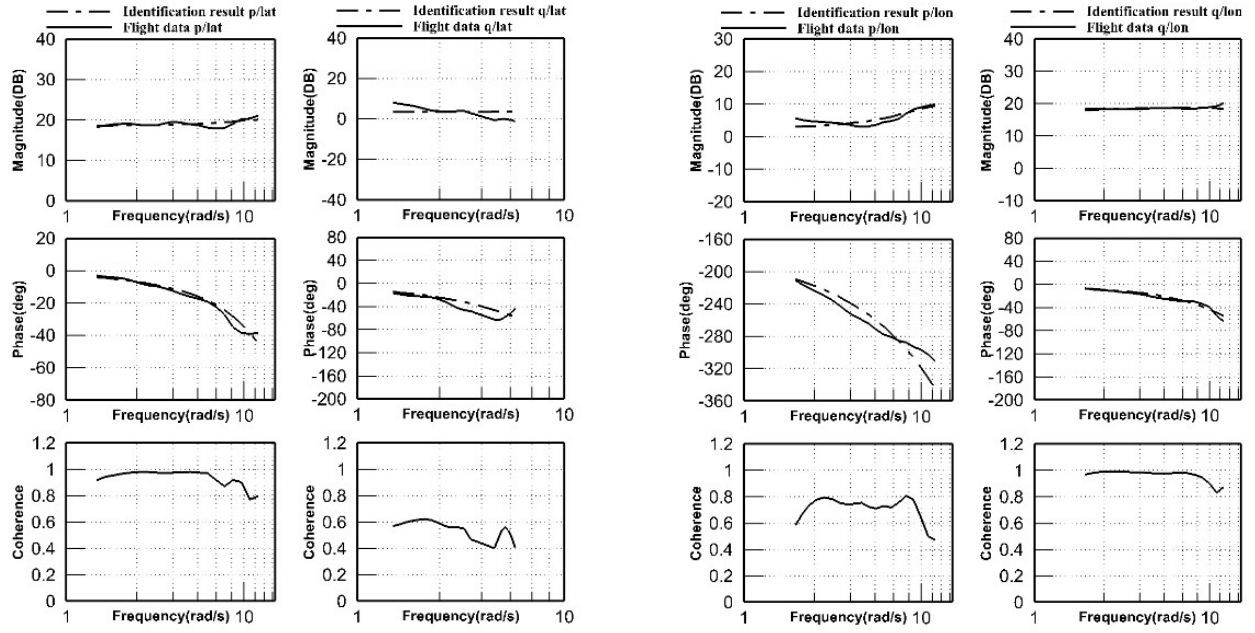
The identification results:



a) p/LAT & q/LAT

b) p/LON & q/LON

Figure 5. Coupled roll-pitch identification by data [IN2 OUT].



a) p/lat & q/lat

b) p/lon & q/lon

Figure 6. Coupled roll-pitch identification by data [IN1 OUT].

Table 3. Parameters of coupled roll-pitch identification results

| Derivative   | [IN2 OUT] |         |                  | [IN1 OUT] |         |                  |
|--------------|-----------|---------|------------------|-----------|---------|------------------|
|              | Value     | CR(%)   | Insensitivity(%) | Value     | CR(%)   | Insensitivity(%) |
| $\tau_f$     | 0.0323    | 18.423  | 1.245            | 0.04456   | 23.12   | 2.876            |
| $L_b$        | 439.2     | 12.29   | 2.737            | 539.5     | 13.9    | 4.733            |
| $M_a$        | 310.8     | 13.537  | 2.155            | 315.1     | 7.54    | 2.157            |
| $A_b$        | -0.2261   | -       | -                | -0.3514   | 26.15   | 10.00            |
| $B_a$        | 0.2261    | 18.63   | 5.608            | 0.5340    | 8.659   | 2.45             |
| $A_{lat}$    | 0.00845   | 7.796   | 3.569            | 0.06571   | 5.58    | 2.757            |
| $A_{lon}$    | 0.2109    | 3.945   | 1.467            | 0.3697    | 3.914   | 1.498            |
| $B_{lat}$    | 0.2155    | 3.960   | 1.646            | 0.3778    | 3.894   | 1.893            |
| $B_{lon}$    | -0.04006  | 8.479   | 2.868            | -0.06070  | 7.005   | 3.022            |
| $\tau_{lat}$ | 0.00848   | 40.685  | 10.11            | 0.0115    | 35.781  | 7.954            |
| $\tau_{lon}$ | 0.00516   | 38.341  | 2.185            | -0.0083   | 32.168  | 8.231            |
| Average cost |           | 98.6534 |                  |           | 40.1776 |                  |

As shown in Figure 5, 6 and table 3, the identification results fit the flight data very well. The average costs are less than 100. It is worth noting that model from [IN1 OUT] is better than [IN2 OUT]. During this identification procedure, it was discovered that the cost from IN2 was very difficult to lower, especially the off-axis response. The off-axis information was so limited that it was hard to decide the coupled derivatives, such as  $A_b$ ,  $B_a$ ,  $A_{lat}$ , and  $B_{lon}$ . The relation  $A_b = -B_a$  was added in the procedure. Because equation 8 and 9 represent the base roll-pitch model of helicopter, the identified model of [IN1 OUT] is close to the real model and the derivatives are closer to the real value. Containing the gyro model in [IN2 OUT], the derivatives of identification

result do not have real meaning, which can explain the difficulty of identification from IN2 compared with IN1.

The variation trends from the two models are shown. After the data pass through gyro, the rotor time constant is increased of 37%.

$$\frac{A_{lon\_IN2}}{A_{lon\_IN1}} \approx \frac{B_{lat\_IN2}}{B_{lat\_IN1}} \approx 0.6,$$

which matches with the gyro settings. The rotor moment (flapping spring) derivatives  $L_b$  and  $M_a$  are increased in a certain degree.  $L_b$  is greater than  $M_a$ . The increasing trend is also fitted for  $A_{lat}$  and  $B_{lon}$ .

The validations in time domain are shown in Figure 7-8.

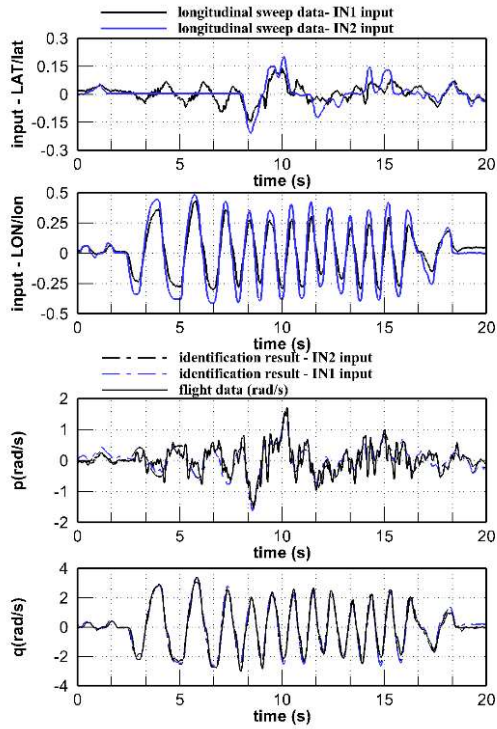


Figure 7. Time domain validation on longitudinal direction of coupled roll-pitch identification.

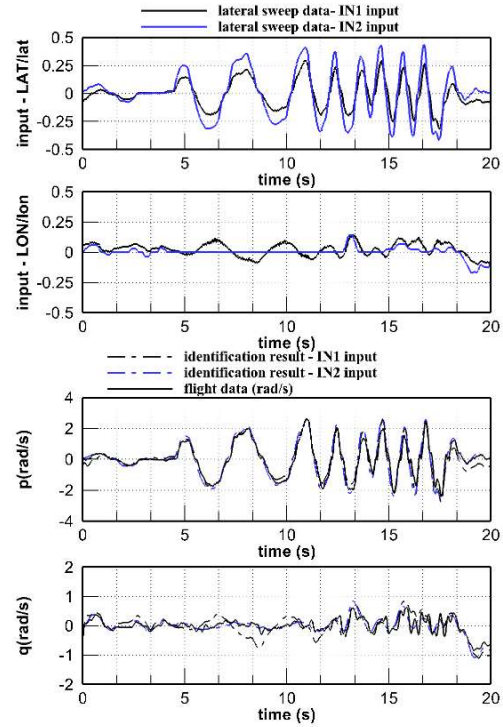


Figure 8. Time domain validation on lateral direction of coupled roll-pitch identification.

### 3.3. Heave identification

The gyro does not have function of augmentation upon the heave direction. So  $G_{h\_base} = G_{h\_all}$ . The coupling influence of other directions on heave was not considered in this paper. The transfer function of heave velocity to collective input is

$$(10) \quad \frac{w}{\delta_{col}} = \frac{Z_{col}}{s - Z_w}.$$

The identification results of heave direction are shown in Table 4.

**Table 4. Parameters of heave identification results**

| Parameter | Value    |
|-----------|----------|
| $Z_{col}$ | -41.8503 |
| $Z_w$     | -0.9403  |
| Cost      | 40.62    |

## 4. CONCLUSIONS

In this paper, the dynamic models of a RC helicopter, before and after three-axis gyro, were investigated by using identification method from frequency responses in CIDER. There models in yaw, roll-pitch, and heave direction were considered.

1. In the yaw identification, a three-stage identification method was applied. By combining a PI feedback assumption and three-stage identification result, the paper obtained a good yaw analysis of

gyro mode, base model, and overall dynamic model.

2. The coupled roll-pitch models with and without gyro part were achieved. The both models match the flight data very well, while the derivatives of base model were closer to the real dynamics of base helicopter. All the parameters in these identifications increased a certain degree, resulting from the augmentation function by gyro.

3. The heave direction was identified with low cost as a single first-order model. The gyro makes little influence on the collective input.

This work deals with identification problem of three-axis gyro in a more comprehensive way. It will provide a better reference for control development on helicopter without stabilizer bar.

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