HELICOPTER TRIM CALCULATION for CALCULATING PERFORMANCE CHARACTERISTICS TAKING into ACCOUNT MUTUAL INFLUENCE of FUSELAGE, MAIN and TAIL ROTORS

S. Mikhailov, A. Kusuymov, E. Nikolaev, N. Shilova, M. Antoshkina

Kazan Tupolev State Technical University

10, K. Marx Str., Kazan 420111 Russia

jhon@smla.kstu-kai.ru

INTRODUCTION

Purpose of the work – development of the program of performance characteristics calculation combined with the calculation of three-dimensional helicopter trimming, which makes it possible further to consider the mutual influence of helicopter fuselage and main and tail rotors, obtained as the results of studies by CFD software packages.

DERIVATION of BASIC EQUATIONS

Helicopter trimming

To solve the problem of helicopter trimming the traditional equations of helicopter motion are used [1]:

$$
\begin{aligned}
& m\left(\frac{\tilde{d}\bar{V}}{dt}+\bar{\omega}\times\bar{V}\right)=\bar{R}_{\mathrm{MR}}+\bar{R}_{\mathrm{TR}}+\bar{R}_{\mathrm{AF}}+\bar{G};\\
& J\frac{\tilde{d}\bar{\omega}}{dt}+\bar{\omega}\times\left(J\bar{\omega}\right)=\bar{M}_{\mathrm{MR}}+\bar{M}_{\mathrm{TR}}+\bar{M}_{\mathrm{AF}};\\
& J_{\mathrm{MR}}\frac{d\bar{\omega}_{\mathrm{R}}}{dt}=\bar{M}_{\mathrm{EN}}-\bar{M}_{\Sigma},
\end{aligned}
\tag{1}
$$

where m – helicopter mass; $\bar{V},\bar{\omega},\bar{\omega}_{\mathrm{R}}$ – velocity and angular velocity vectors of helicopter and main rotor; $\bar{R}_{\mathrm{MR}}$, $\bar{R}_{\mathrm{TR}}$, $\bar{R}_{\mathrm{AF}}$, $\bar{M}_{\mathrm{MR}}$, $\bar{M}_{\mathrm{TR}}$, $\bar{M}_{\mathrm{AF}}$ – forces and moments on main and tail rotors and helicopter airframe, $\bar{G}$ –vector of gravity; J – helicopter moment of inertia; J_{MR} – equivalent moment of inertia of all units kinematically connected with main rotor $\bar{M}_{\Sigma}$ – moments of aerodynamic forces of all blades of tail and main rotors equivalent to the main rotor shaft; $\bar{M}_{\mathrm{EN}}$ – engine moment.

Usually, the dynamics of main and tail rotors is not taken into account in problems of trimming, assuming that there exists a balance of required and available powers, and a degree of engine throttle $д_{щ}$ is not computed. In our procedure last equation is retained, which allows:

1) to calculate fuel consumption per hour and kilometer out of the equation $\bar{M}_{\Sigma}+\bar{M}_{\mathrm{EN}}=0$ in the trimming mode ($щ_{\mathrm{MR}}=\mathrm{const}$), maintaining the balance of required and available powers by suitable selection of $д_{щ}$;
2) to determine the maximum (minimum) speed of level flight or available climb speeds at the specified horizontal velocity with a fixed value of $д_{щ}$, corresponding to the specified engine behavior (cruising, nominal, take-off);
3) to determine the required flight-path angle according to the known level velocity for calculating the helicopter autorotation performance characteristics under condition $\bar{M}_{\Sigma}=0$ by a suitable selection of a descent velocity.
4) As a result of the solution of extended helicopter trimming problem one can calculate the required performance characteristics
5) The loading of fuselage taking into account the influence of main and tail rotors is simulated in the work using CFD- program.

Inertial forces and inertia moments per unit length

Inertial forces and inertia moments per unit length are derived for the blade section perpendicular to feathering axis and intersecting it at the point M which has (x,y,z) coordinates in blade axes

$$
\begin{aligned}
& p_{xi}=-\iint\limits_{(F)}\rho a_x dF,\quad p_{yi}=-\iint\limits_{(F)}\rho a_y dF,\\
& p_{zi}=-\iint\limits_{(F)}\rho a_z dF,\\
& q_{zu}=\iint\limits_{(F)}\rho\left(-a_y\left(x_{\mathrm{sec}}-x\right)+a_x\left(y_{\mathrm{sec}}-y\right)\right)dF,\\
& q_{xi}=-\iint\limits_{(F)}\rho a_z\left(y_{sec}-y\right)dF.
\end{aligned}
\tag{2}
$$

Since the technical problem is solved, some small terms in the resulting formulae are omitted.
Moments of elastic forces in blade section are determined taking into account the effects of untwisting of the rod with initial twist. While deriving the Euler-Bernoulli's hypothesis was used. It's assumed that at one point with coordinates x_0, y_0, r_0 all three hinges with corresponding stiffness C_ξ, C_β, C_{CS} are concentrated.

Aerodynamic load
Aerodynamic load on the blade is calculated with the use of plane-section hypothesis. Normal to the blade axis components of aerodynamic force per unit length are determined in wind axes by formulae

$$\begin{aligned} p_{x_{2a}} &= \left(C_x V_{x_2} n_w + C_y V_{y_2}\right)\frac{b\rho W_n}{2}, \\ p_{y_{2\alpha}} &= \left(-C_x V_{y_2} n_w - C_y V_{x_2}\right)\frac{b\rho W_n}{2}, \\ p_{z_{2\alpha}} &= C_{x_0}\frac{b\rho W^2}{2},\ n_w = \frac{W\left(-V_{x_2}\right)}{W_n^{\,2}}. \end{aligned} \tag{3}$$

where, c_y – section lift coefficient; c_x – section drag coefficient, b – blade chord, ρ – mass density of air.

Torque of aerodynamic forces per unit length

$$\begin{aligned} q_{ka} &= \left(p_{y_{2\alpha}}\cos\varphi - p_{x_{2\alpha}}\sin\varphi\right)x_f - \\ &-\frac{\pi}{4}\left(\frac{3}{4}-\bar{x}_{ec}\right)b^3\rho W_n\left(\dot{\varphi}+\dot{\theta}\right)+m_{zf}\frac{b^2\rho W_n^{\,2}}{2} \end{aligned} \tag{4}$$

Equations of blade deformation can be written as:

$$\begin{aligned} A_1 + \ddot{B}_1 &= F_1, \\ A_2 + \ddot{B}_2 &= F_2, \\ A_3 + \ddot{B}_3 &= F_3, \end{aligned} \tag{5}$$

where A_i and $B_i\ (i=1,2,3)$ are linear integro-differential operators of functions $x(r,t)$, $y(r,t)$, $\theta(r,t)$.
The terms independent of blade deformation are not included in the expressions for A_i. They are assigned to external loads and are included in the expressions for F_i. Moreover, in expressions for F_i the inertia loads, the expressions for which contain the first time derivatives from variables x, y, θ, are included, as well as some inertial and elastic members, quadratic with respect to deformations.

Integration of equations of blade deformation
Values of C_y and C_x are determined depending on the angle α

$$C_y = a_\infty \alpha,\ C_x = const,\ m_{zf} = const. \tag{6}$$

To avoid errors all the equations in the paper were derived using the mathematical (symbolic) software Maple.
The obtained set of equations is a system of nonlinear integral and differential equations with periodical coefficients.

INDUCED VELOCITY

The calculation of induced velocity at an arbitrary point of space is considered based on the example of the typical law of circulation distribution along blade radius [2]. Although this method is approximate, it is very convenient to use it when calculating helicopter trim characteristics as a zero approximation, since it provides for the calculation of the distribution $\bar{\gamma}(\bar{r},\psi)$ over the disk according to the known value C_t (Fig. 1-3)**:**

$$\bar{\gamma}(\bar{r},\psi) = \bar{\gamma}_r + \bar{\gamma}_{s1}\sin\psi,$$

where $$\bar{\gamma}_r = \bar{C}\left(2\bar{r}^2 - \bar{r}^4 - \bar{r}^6\right) = \bar{C}f_r(\bar{r}),$$

$$\begin{aligned} \tilde{C} \approx 1.989\Big[&-\tilde{V}\cos\left(\alpha_ц + \delta\right) + \\ &+\sqrt{\tilde{V}^2\cos^2\left(\alpha_ц+\delta\right)+4.827}\Big], \end{aligned}$$

$$\bar{\gamma}_{s1}(\bar{r}) = B\mu_v\bar{\gamma}_r\left(\frac{1}{\bar{r}}-\frac{25}{13}\bar{r}\right) = \tilde{D}f_s(\bar{r}),$$

$$\tilde{D} = \tilde{C}B\mu,$$

$$B = \frac{8\mu_v\left(1+k_\delta^2\right)+a_\infty\sigma_7 k_\delta}{\left(1+k_\delta^2\right)\left(4\mu_v + a_\infty\sigma_7 k_\delta\right)},\ \mu_v = \mu + \bar{\upsilon}_{1x_{cp}}.$$

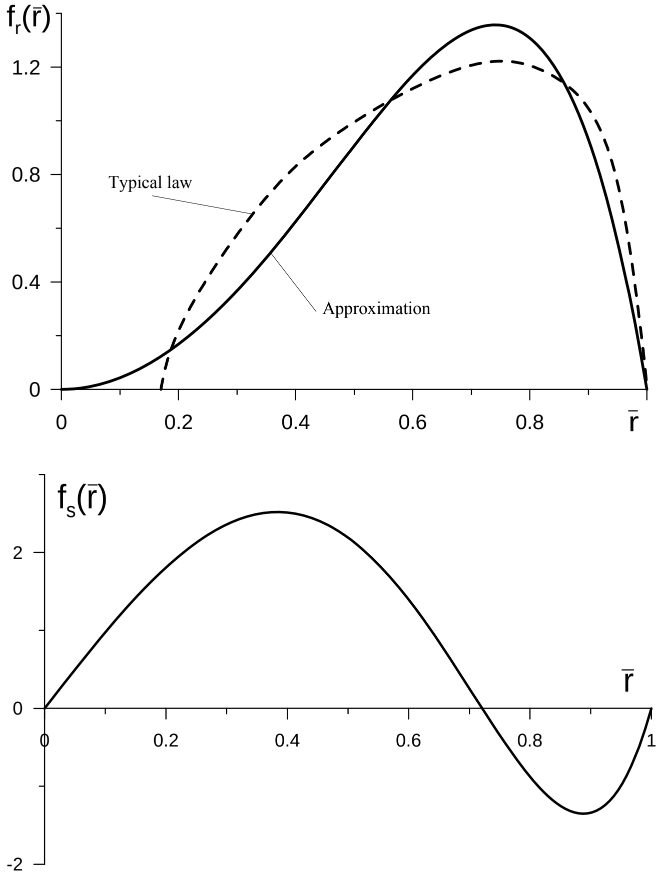

Fig. 1. Basic functions of circulation per unit length

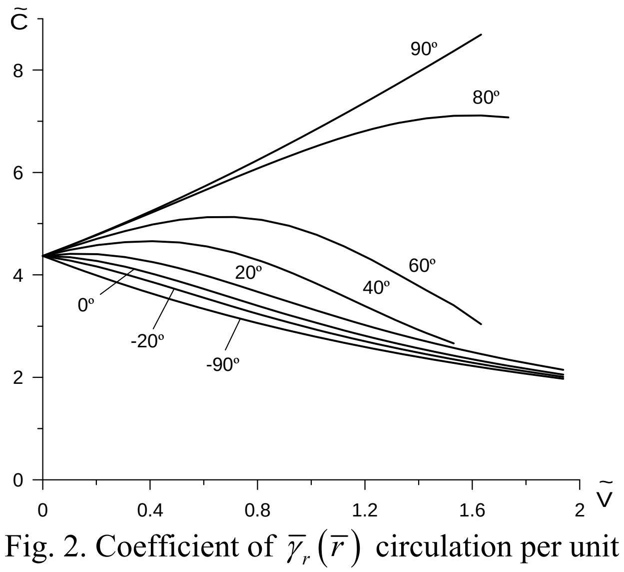

Fig. 2. Coefficient of $\overline{\gamma}_r\left(\overline{r}\right)$ circulation per unit length

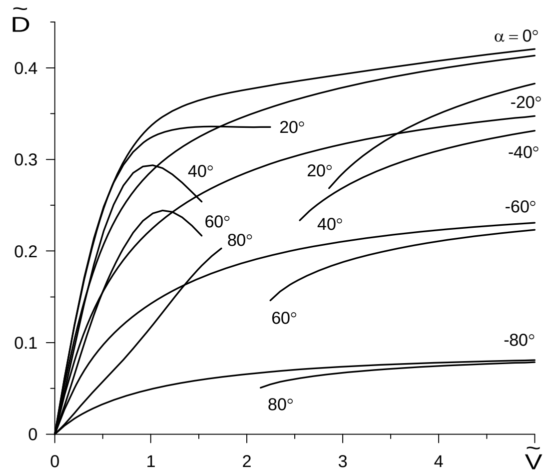

Fig. 3. Coefficient of $\overline{\gamma}_{s1}\left(\overline{r}\right)$ circulation per unit length

The tilt angle of vortex cylinder δ and the induced velocity averaged over the main rotor disk and divided by hover average induced velocity (ideal hover induced velocity) are calculated by formulae [3]:

$$\tilde{\upsilon}_{1y_{av}} = sign\left(\delta\right)\frac{1}{2}\Big[-\tilde{V}\cos\left(\alpha_r+\delta\right)+ +\sqrt{\tilde{V}^2\cos^2\left(\alpha_r+\delta\right)+4sign\left(\delta\right)}\Big] \quad (7)$$

$$\tilde{\upsilon}_{1x_{av}} = sign\left(\delta\right)\tilde{\upsilon}_{1y_{av}}tg\left(\frac{\pi}{4}-\frac{|\,\delta\,|}{2}\right) \quad (8)$$

$$\tilde{V} = 2\left(1-sign\,|\,\delta\,|\right)\left(sign\delta\sin^2\alpha_{av}+2\cos\left(\alpha_{av}+\delta\right)\sin\alpha_{av}+ +\cos^2\left(\alpha_{av}+\delta\right)\sin\delta\left(2-\sin|\,\delta\,|\right)\right)^{-\frac{1}{2}} \quad (9)$$

Consider the part of the cylindrical vortex sheet [4], limited on two sides by generatrix and leaving one end to infinity (Fig. 4). Let isolate the vortex element ds and the circulation element dΓ at the point $M\left(\xi,\eta,\zeta\right)$ of vortex sheet

$$\mathrm{d}\Gamma = \gamma d\eta\,, \quad (10)$$

where γ – circulation per unit length in the direction of the generatrix of the cylinder.

Determine the induced velocities from this element at the point $A\left(x_1,0,z_1\right)$.
By Biot-Savart formula

$$d^2v = \frac{d\Gamma}{4\pi}\frac{d\overline{s}\times\overline{l}}{\left|l\right|^3} \quad (11)$$

where $\overline{l}$ – radius-vector connecting the point A with the point M ; $d\overline{s}$ – vortex element.
Modify the expressions for induced velocities obtained in [4]

$$v_x = \frac{\gamma}{4\pi}\frac{1}{\sin\delta}\int_S \left\{ \left(\zeta - z_1\right) J_\xi \cos\delta \frac{d\xi}{ds} - \right.$$
$$\left. -\left[\left(\xi - x_1\right) J_\xi \cos\delta - J_\zeta\right] \frac{d\zeta}{ds} \right\} ds \qquad (12)$$

$$v_y = \frac{\gamma}{4\pi}\frac{1}{\sin\delta}\int_S \left\{ \left(\zeta - z_1\right) J_\xi \frac{d\xi}{ds} - \right.$$
$$\left. -\left[\left(\xi - x_1\right) J_\xi - \cos\delta J_\zeta\right] \frac{d\zeta}{ds} \right\} ds \qquad (13)$$

$$v_z = \frac{\gamma}{4\pi}\sin\delta \int_s J_\xi \frac{d\xi}{ds} ds \,, \qquad (14)$$

where J_ξ , J_ζ – tabulated integrals depending on the distance to the point A .
Derivatives $\frac{d\xi}{ds} = \cos\tau$ and $\frac{d\zeta}{ds} = \sin\tau$ are the cosine and sine of τ , the angle of inclination of the arc element ds to the axis Ox_1 . Then the integration with respect to the angle ϑ can be replaced by the integration with respect to the arc s which is much easier.

If we divide the vortex space under the disc into n vortex cylinders and assume that the circulation within each volume is constant, then we can define an influence matrix of the vortex disc. The induced velocities, calculated at each calculation point of each contour with the assumed unit circulation, will be the elements of the influence matrix. With the known distribution $\overline{\gamma}\left(\overline{r},\psi\right)$ over the disk having influence matrices for three components of induced velocities it is possible to calculate induced velocity distribution.

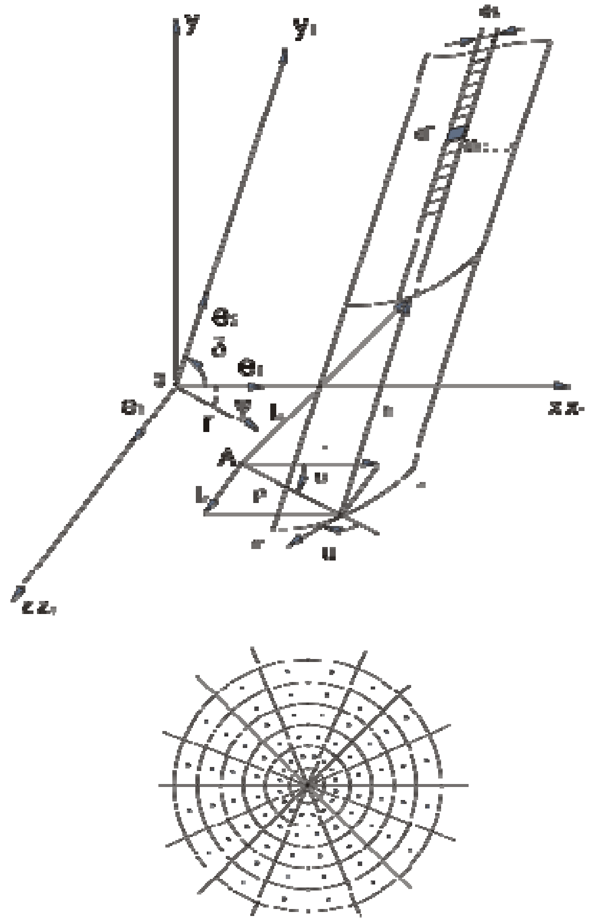

Fig. 4 Coordinate systems

The main rotor with the following parameters is considered for an example: $\mu = 0.15$ $\frac{c_t}{\sigma} = 0.188$; $\sigma = 0.089$.

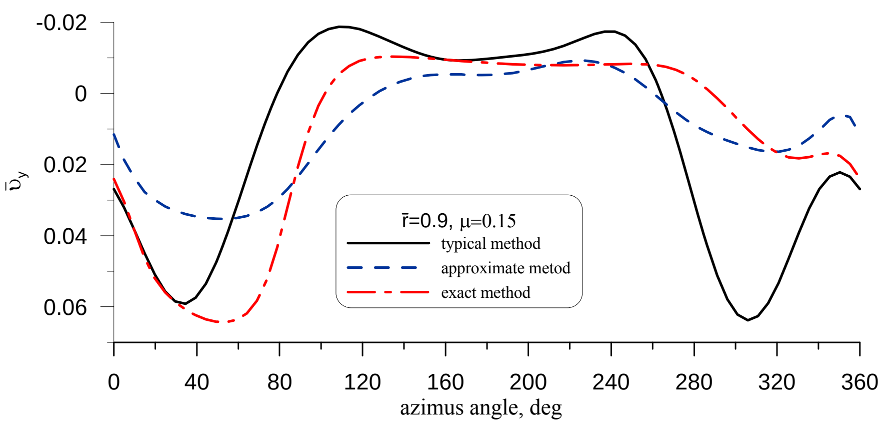

Fig. 5. The normal component of induced velocity

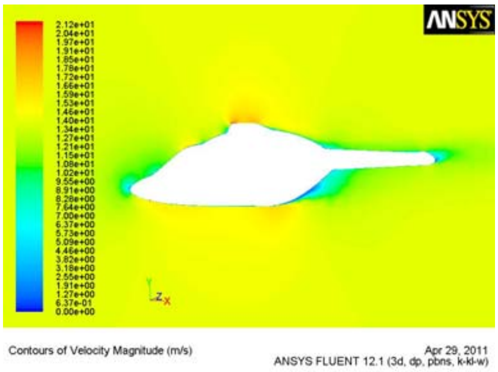

Fig. 6. Velocity magnitude in the symmetry plane for isolated fuselage without the actuator: α=0 degrees, V_∞=14 m/s

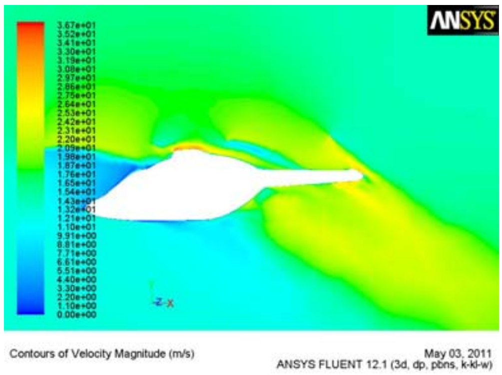

Fig. 7. Velocity magnitude in the symmetry plane for fuselage with the actuator: α=0 degrees, V_∞=14 m/s

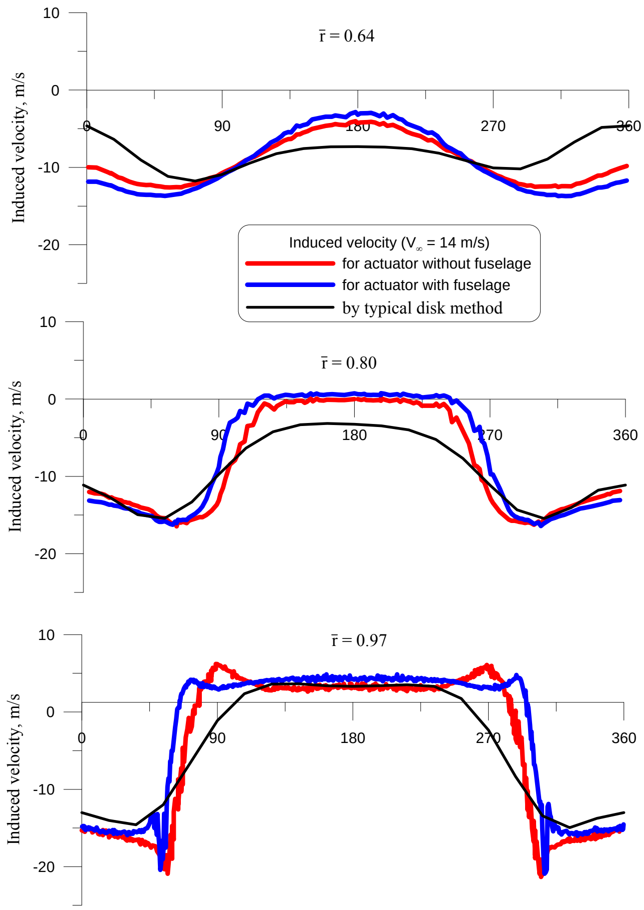

Fig. 8. Induced velocities distribution over azimuth

Dotted line is the curve of the normal component of induced velocity $\bar{\upsilon}_y(\bar{r},\psi)$, calculated by the method of Shaidakov for the approximate calculation of induced velocities with the typical law of circulation distribution; solid line plots the curve of induced velocities, obtained by the proposed method of the induced velocities determining at any point of space, the law of circulation distribution γ is assigned typical.

CONCUSIONS

The fuselage aerodynamic characteristics are calculated as a result of simulation by CFD-program taking into account main and tail rotors airflow. Procedure and algorithm of the calculation of helicopter trim characteristics n the three-dimensional space with the use of a disk vortex conception of main and tail rotors is developed. The results of the calculations of trim and performance characteristics of helicopter “Ansat” are presented.

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