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PRACTICAL COMPUTATION OF UNSTEADY LIFT

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1. Summary

The requirements for the computation of 2D unsteady lift within the context of rotor airloads and performance calculation are shown to be fulfilled by an indicial formulation. It is shown how the indicial formulation may be expanded to provide a more general form of lift transfer function which may be used to evaluate explicitly the response to idealised forcing such as the frequency response and ramp motion. Results from the application of recent developments in the computation of unsteady transonic flow are used to define the indicial lift function which is generalised in an appropriate form for rotor applications and evaluated for idealised forcing. Further evaluation and refinement is provided from comparisons with wind tunnel test data.

Numerical procedures are devised and evaluated in the context of the sampling requirement for rotor calculation. The final form evolved minimises the sensitivity to sampling with the objective of economising on computational demands. Finally, application of the methods evolved is illustrated by comparison with the results of a compressible, time dependent Navier-Stokes calculation for an airfoil subject to an abrupt ramp motion. This comparison highlights the importance of the impulsive lift terms and the development of separated flow, the calculation of which may be superimposed on the formulation presented here.

2. Introduction

The sequence used in most routine rotor airload calculations comprises computation of the local load at a series of radial stations on the blade for a given azimuth followed by radial integration and progressive stepping through the azimuth range. During this procedure the conditions at any given radial station change rapidly through a large angle of attack and Mach number range. Even when separation is not encountered in these cycles there are significant time dependent effects on the airfoil loading. Due to the combined influence of cyclic pitch, blade elastic response and wake structure the local angle of attack may vary in a quite arbitrary manner and any method used to obtain the loading response must be formulated accordingly. The solution must be compatible with the sequential sampled form of computation and, in view of the large number of calculations to be repeated, simplicity of computation is at a premium. The indicial formulation is appropriate to meet these requirements and has been in use for some time. In this paper the unsteady lift calculation is almost entirely concerned with attached (potential) flow but, at the same time, it is formulated to provide a structure for the evaluation of critical conditions for the onset of separated flow and the subsequent required modifications to implement the consequences (see reference 10).

In the period since the indicial formulation for unsteady lift was introduced into the rotor calculation (1975) there have been developments in both theory and computational procedures for calculating unsteady lift at transonic speeds, for example, the application of the LTRAN 2 code presented in reference 3. In spite of the absence of impulsive loading, which is a consequence of the low frequency assumptions of LTRAN, the calculated linear and non-linear indicial response and time stepping solutions for plunging motion provide a valuable basis for refining and substantiating the approximations used in the rotor formulation. The approximations which comprise the specification of the indicial response in the form of exponential functions have formed the basis for the derivation of the lift transfer function from which explicit solutions for the response to idealised forcing may be obtained. These are compared to both the more exact solutions now available and to wind tunnel test results at transonic speeds and enable the approximations to be modified and evaluated.

Another aspect of rotor unsteady lift computation comprises the selection of appropriate numerical procedures to implement the indicial response. Current methods are evaluated and shown to degrade as the sampling interval is increased. Using the lift transfer function to derive an alternative form of response equation, new numerical algorithms have been obtained which are less sensitive and finally, a pseudo half-step lead is introduced which reduces the lag inherent in sampled solutions.

Solutions of the Navier-Stokes equations are becoming feasible at great computational expense. They are held out to be the salvation of the aerodynamicist but for the forseeable future will still be subject to computational limitations and confined to idealised and limited applications. They are, however, of great potential value in assessing the limitations and accuracy of less sophisticated theory. A particular application, presented in reference 12, has been used to compare with the indicial formulation developed in this report and surprisingly close agreement has been demonstrated.

3. Derivation of the indicial lift functions

Practical considerations for the implementing of indicial lift functions suggest that they should be expressed in terms of exponential functions of time. This enables the straightforward derivation of the Laplace transform of the response and hence the lift transfer function. It also facilitates a simple form of numerical solution for use with arbitrary forcing using superposition of a sequence of step inputs. Fortunately, the results of many different theoretical approaches to the calculation of indicial lift response may be approximated to an acceptable degree of accuracy in this manner. Most of the theories that may be used for guidance, however, are subject to limitations in their formulation and consequently it is convenient to make a distinction between the circulatory and impulsive components of the indicial lift response. The scaling normally adopted to generalise time produces the parameter $s = t.2V/c$ which corresponds to the relative distance travelled by the airfoil in terms of semi-chords.

From the results presented in reference 1 for indicial response at varying Mach number it can be deduced that further simplification may be obtained by using the parameter $s' = s(1-M^2)$. Thus the preferred general form for the indicial lift response to pitching motion is given by:

$$C_L(s') = C_{L_\alpha}(M)\,\bar{\alpha}\,\phi_C(s') + \phi_I(s')\,\bar{\alpha} + \phi_q(s')\,\bar{q}$$

where $C_{L_\alpha}(M)$ = lift curve slope at the appropriate Mach number

$\bar{\alpha}$ = step change in angle of attack (in radians) measured at the 3/4 chord location

$\bar{q}$ = step change in non-dimensional pitch rate about the 3/4 chord = $\dot{\theta}\,c/V$

$\phi_C(s')$ = circulatory component of the indicial lift response

$$= 1 - A_1 e^{-b_1 s'} - A_2 e^{-b_2 s'}$$

$\phi_I(s')$ = impulsive component of the indicial lift response

$$= \frac{4}{M} e^{-s'/T_I'}$$

$\phi_q(s')$ = impulsive component of the indicial lift response to pitch rate about 3/4c

$$= \frac{-1}{M} e^{-s'/T_q'}$$

From the above formulation it is apparent that the circulatory component resulting from pitch rate is incorporated into the angle of attack term via definition of angle of attack at the 3/4 chord location. There are many sources from which guidance may be obtained in defining the most appropriate coefficients for the circulatory lift which in general is the most significant component of the total response. For a compressible fluid the total initial response is derived in reference 2 from acoustic considerations as an impulsive force which decays rapidly. The initial variation is given by

$$C_L = \frac{4}{M}\bar{\alpha}\left[1 - \frac{s}{2M}(1-M)\right]$$

Thus

$$\frac{dC_L}{ds} = \frac{-4\bar{\alpha}}{M}\left[\frac{1-M}{2M}\right] \quad \text{or} \quad \frac{dC_L}{ds'} = \frac{-4\bar{\alpha}}{M}\left[\frac{1}{2M(1+M)}\right]$$

If we consider the total lift response to be composed of an exponentially decaying impulsive lift (C_{L_I}) plus an increasing circulatory lift (C_{L_C}) then

$$\frac{dC_L}{ds'} = \frac{dC_{L_I}}{ds'} + \frac{dC_{L_C}}{ds'}$$ and from the above formulation

$$\frac{dC_{L_I}}{ds'} = \frac{-4\bar{\alpha}\,e^{-s'/T_I'}}{M\,T_I'} \quad \text{and} \quad \frac{dC_{L_C}}{ds'} = C_{L_\alpha}\bar{\alpha}\left[A_1 b_1 e^{-b_1 s'} + A_2 b_2 e^{-b_2 s'}\right]$$

Thus equating slopes at t = o (s' = o).

$$-\frac{4\bar{\alpha}}{M}\left[\frac{1}{2M(1+M)}\right] = -\frac{4\bar{\alpha}}{M\,T_I'} + C_{L_\alpha}\bar{\alpha}\,(A_1 b_1 + A_2 b_2)$$

$$T_I' = \frac{4M(1+M)}{2 + C_{L_\alpha} M^2(1+M)(A_1 b_1 + A_2 b_2)} \qquad \text{where } T_I' = T_I(1-M^2).\,2V/c$$

Thus, depending on the coefficients chosen to represent the circulatory component of lift, the time constant for impulsive lift may be adjusted to maintain the correct initial variation of lift. For pitching motion about the 3/4 chord, reference 2 presents the derivation of the indicial response (C_{L_q}). The asympotic value is zero, the decay from the initial value of -1/M is roughly exponential and approximately the same as for the impulsive loading. The problem now remains of how to select the most appropriate coefficients to represent the circulatory response through the Mach number range up to high subsonic speeds.

A systematic series of calculations have been performed at NASA Ames, principally by W.F. Ballhaus and colleagues, using the LTRAN2 analysis which comprises a solution to the low frequency unsteady transonic small disturbance equation. A consequence of the low frequency constraint is elimination of the impulsive loading which, fortuitously, leaves the loading required to define the component that is labelled here as "circulatory". Thus the results of the LTRAN2 program as presented in reference 3, may be used both as a theoretical source for the indicial response function and as a means of evaluating its application to more general motion.

Figure 1a has been copied as closely as possible from reference 3 and presents the calculated indicial response for the NACA 64A006 airfoil using the LTRAN2 program. For comparison, the LTRAN2 solution with the non linearities suppressed is included together with the 'exact' solution for the linear theory from reference 2. The latter solution is not restricted by the low frequency assumption and consequently the time history is initiated with the non zero (impulsive) force and serves to illustrate the significance of this term.

The solutions have been normalised by the asymptotic values of lift and it is apparent that not only the initial but also the asymptotic behaviour varies between the solutions.

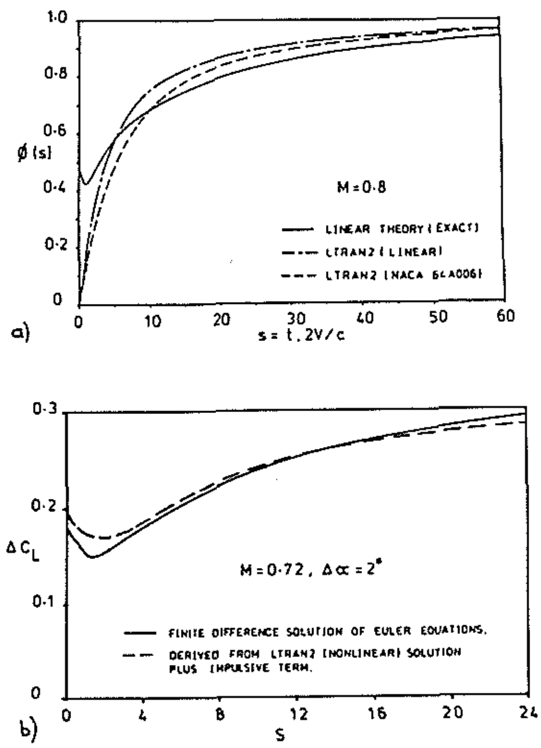


FIG. 1. INDICIAL RESPONSE COMPARISONS

From the two low frequency (LTRAN2) solutions approximations in the form assumed for the circulatory response have been obtained thus:-

a) $\phi_C(s') = 1 - 0.3\,e^{-0.11 s'} - 0.7\,e^{-0.85 s'}$ for the linear solution

b) $\phi_C(s') = 1 - 0.3\,e^{-0.10 s'} - 0.7\,e^{-0.53 s'}$ for the non linear solution

The impulsive force time constants (T_I') associated with the above expressions may then be evaluated.

An independent solution for the non linear indicial response has been obtained from a finite difference solution of the adiabatic Euler equations. This solution, presented in reference 4, is not subject to low frequency assumptions and may be compared with the expression derived from the LTRAN2 non linear solution with the appropriate impulsive term added. The Euler solution was performed for a 2^{o} step change in angle for the NACA 64 A 410 airfoil at M = 0.72 and the result is shown in figure 1(b) along with the derived LTRAN2 result. The asymptotic value of lift for the Euler solution was not available so the LTRAN2 derived result for the NACA 64A006 has been scaled to a lift curve slope of $2\pi/\sqrt{1-M^2}$; the change of Mach number is accommodated by the scaling of s where $s' = s(1-M^2)$. With the impulsive term included the correspondence between the two solutions is quite satisfactory and provides support for the use of the form of generalisation proposed.

4. The Lift Transfer Function

The calculation of unsteady lift may be further generalised by the concept and use of a lift transfer function which expresses in operational form the relationship between the lift response and the forcing. Conventionally, the symbol s is used to represent the Laplace operation; to avoid confusion with the non-dimensional time scale s = t. 2V/c it will be replaced by p Considering first the circulatory lift, the Laplace transform of the indicial response function yields:

$$C_{L_C}(p) = C_{L_\alpha}\left[\frac{1}{p} - \frac{A_1 T_1}{1+T_1 p} - \frac{A_2 T_2}{1+T_2 p}\right]$$

and the transform of the forcing (a step change in angle) is given by $\alpha(p) = 1/p$. Thus the circulatory lift transfer function

$$\frac{C_{L_C}(p)}{\alpha(p)} = C_{L_\alpha}\left[1 - \frac{A_1 T_1 p}{1+T_1 p} - \frac{A_2 T_2 p}{1+T_2 p}\right] = C_{L_\alpha}\left[(1 - A_1 - A_2) + \frac{A_1}{1+T_1 p} + \frac{A_2}{1+T_2 p}\right]$$

When the initial value of the indicial response is zero then $(1 - A_1 - A_2) = 0$ and the circulatory lift transfer function simplifies to

$$\frac{C_{L_C}(p)}{\alpha(p)} = C_{L_\alpha}\left[\frac{A_1}{1+T_1 p} + \frac{A_2}{1+T_2 p}\right]$$

It may be noted that alternative forms of the circulatory lift transfer function may be derived from the indicial response time history but in application the results are much the same.

For the impulsive lift, the Laplace transform of the indicial response is $C_{L_I}(p) = \frac{4}{M}\frac{T_I}{(1+T_I p)}$ and $\alpha(p) = 1/p$ as before. Thus the impulsive lift transfer function is given by $\frac{C_{L_I}(p)}{\alpha(p)} = \frac{4}{M}\frac{T_I p}{(1+T_I p)}$

Similarly the transfer function for pitch rate $q = \dot{\theta}\, c/V$ about the 3/4 chord may be derived as $\frac{C_{L_q}(p)}{q(p)} = -\frac{1}{M}\cdot\frac{T_I p}{(1+T_I p)}$

Thus, for any forcing for which an explicit Laplace transform may be derived the corresponding lift response may be evaluated in an explicit form.

The ultimate objective of this line of development is to provide an adequate numerical procedure for application to forcing of an entirely arbitrary nature. To substantiate such a procedure requires a check against experiment and/or more rigorous theory. Because it is practical to produce experimental or theoretical results only for idealised motion then it is useful to have a means of evaluating the structure of the model independent of the numerical procedures. Additionaly, since it is physically impossible to produce experimental verification of indicial lift response, then such motion that may be utilised experimentally or theoretically may be evaluated via the transfer function and any discrepancies related back to the indicial response function. For these reasons it is useful to develop explicit solutions for idealised motion from the lift transfer function such as the response to sinusoidal motion about an arbitrary pitch axis.

5. Frequency Response to Pitching Oscillations

For sustained pitching oscillation $\theta = \sin \omega t$ the Laplace transform of the angle of attack defined at the 3/4 chord is:

$\alpha(p) = \frac{1}{\omega}\frac{1+Bp}{1+p^2/\omega^2}$ where $B = \bar{x}.c/2V$ and $\bar{x}$ = distance of the 3/4 chord aft of the pitch axis, in semi chords.

Thus $C_{L_C}(p) = \frac{C_{L_\alpha}}{\omega}\left[\frac{1+Bp}{1+p^2/\omega^2}\right]\left[\frac{A_1}{1+T_1 p} + \frac{A_2}{1+T_2 p}\right]$ for which the steady state response is given by

$$C_{L_C}(t) = A_1\left[\frac{1+B^2\omega^2}{1+T_1^2\omega^2}\right]^{1/2}\sin(\omega t+\phi_1) + A_2\left[\frac{1+B^2\omega^2}{1+T_2^2\omega^2}\right]^{1/2}\sin(\omega t+\phi_2)$$

where $\phi_n = \tan^{-1}(B\omega) - \tan^{-1}(T_n\omega)$

The impulsive lift contribution is represented by

$C_{L_I}(p) = \frac{4\,T_I p}{M\omega}\frac{(1+Bp)}{(1+T_I p)(1+p^2/\omega^2)}$ for which the steady state response is

$$C_{L_I}(t) = \frac{4\,T_I\omega}{M}\left[\frac{1+B^2\omega^2}{1+T_I^2\omega^2}\right]^{1/2}\cos(\omega t+\phi_I)$$

where $\phi_I = \tan^{-1}(B\omega) - \tan^{-1}(T_I\omega)$

For oscillatory pitching $q(p) = \frac{c}{\omega V}\left[\frac{p}{1+p^2/\omega^2}\right]$

Thus $C_{L_q}(p) = -\frac{c\,T_q}{M\omega V}\,\frac{p^2}{(1+T_q p)(1+p^2/\omega^2)}$ and the steady state

$$C_{L_q}(t) = \frac{c\,\omega^2 T_q}{M\,V}\,\frac{1}{(1+T_q^2\omega^2)^{1/2}}\sin(\omega t - \phi_q) \quad \text{where} \quad \phi_q = \tan^{-1}(T_q\omega)$$

To evaluate the above contributions to the frequency response for pitching oscillation about the 1/4 chord we may substitute $\bar{x} = 1$, thus $B = c/2V$, $k = \omega c/2V$ and express the time constants in terms of the primed values

thus $T_1\omega = \frac{k}{b_1(1-M^2)}$, $T_2\omega = \frac{k}{b_2(1-M^2)}$, $T_I\omega = \frac{T_I' k}{1-M^2} = T_q\omega$

and $B\omega = k$. The resultant frequency response is then obtained by summing the various contributions to the in phase and quadrature components.

6. Frequency response to plunge oscillation

The frequency response to plunge oscillation may be obtained in a similar manner or by simplifying the pitch frequency response by virtue of the different boundary conditions at the 3/4 chord and absence of the pitch rate term;

i.e. $\alpha(p) = \frac{1}{\omega}\,\frac{1}{(1+p^2/\omega^2)}$ thus $B=0$ and $C_{L_q}(t) = 0$

7. Response to an idealised ramp

Just as a step input is physically unrealisable but nevertheless a useful concept, so too is an ideal ramp which implies that for $t < o$ then $\dot{\theta} = o$ and for $t > o$ then $\dot{\theta}$ has some constant finite value. For $\dot{\theta}(t) = K_\theta$ then $\theta(p) = K_\theta / p^2$
and for pitch about an axis $\bar{x}$ forward of the 3/4 c then $K_\alpha = K_\theta(1 + \bar{x}\,c/2V)$

Thus $C_{L_C}(p) = C_{L_\alpha} K_\alpha \left[\frac{1}{p^2} - \frac{A_1 T_1}{p(1+T_1 p)} - \frac{A_2 T_2}{p(1+T_2 p)}\right]$ which results in

$$C_{L_C}(t) = C_{L_\alpha} K_\alpha \left[t - A_1 T_1(1 - e^{-t/T_1}) - A_2 T_2(1 - e^{t/T_2})\right]$$

now $K_\alpha t = \alpha(t)$ thus the above equation may be re-expressed in the form

$$C_{L_C}(t) = C_{L_\alpha}\left[\alpha(t) - \Delta\alpha_E(t)\right] = C_{L_\alpha}\,\alpha_E(t)$$ where $C_{L_\alpha}\,\Delta\alpha_E(t)$ may be viewed as a lift deficiency; i.e.

$\Delta\alpha_E(t) = K_\alpha\left[A_1 T_1(1 - e^{-t/T_1}) + A_2 T_2(1 - e^{-t/T_2})\right]$ In this form the expression may be used as the basis for a numerical form of solution.

Also $C_{L_I}(p) = \frac{4}{M} K_\alpha \frac{T_I}{p(1+T_I p)}$ $\therefore$ $C_{L_I}(t) = \frac{4}{M} K_\alpha T_I (1 - e^{-t/T_I})$

Due to pitch rate about the 3/4 chord. $q = \dot{\theta} c/V = K_q$

$$C_{L_q}(p) = -\frac{K_q}{M}\,\frac{T_q}{p(1+T_q p)} \qquad \therefore \qquad C_{L_q}(t) = -\frac{K_q}{M} T_q (1 - e^{-t/T_q})$$

8. Response to a real ramp

It is possible to attempt ramp motion in experiment by means of an actuator but obviously the initial value of $\dot{\theta}$ must be zero. In practice the resultant motion may be represented operationally as

$$\theta(p) = \frac{K_\theta}{p^2(1+T_F p)}$$ where T_F represents the time constant for the mechanical hydraulic system

i.e. $\theta(t) = K_\theta\left[t - T_F(1-e^{-t/T_F})\right]$. As before $K_\alpha = K_\theta(1+\bar{x}c/2V)$

Thus $$C_{L_C}(p) = \frac{C_{L_\alpha}K_\alpha}{(1+T_F p)}\left[\frac{1}{p^2} - \frac{A_1T_1}{p(1+T_1p)} - \frac{A_2T_2}{p(1+T_2p)}\right]$$

$$\therefore C_{L_C}(t) = C_{L_\alpha}\left(\alpha(t) - K_\alpha\left[A_1T_1 + A_2T_2 + T_F e^{-t/T_F}\left(\frac{A_1T_1}{T_1-T_F}+\frac{A_2T_2}{T_2-T_F}\right) - \frac{A_1T_1^2e^{-t/T_1}}{T_1-T_F} - \frac{A_2T_2^2e^{-t/T_2}}{T_2-T_F}\right]\right)$$

$$C_{L_I}(p) = \frac{4K_\alpha}{M}\frac{T_I}{p(1+T_Fp)(1+T_Ip)} \qquad \therefore \quad C_{L_I}(t) = \frac{4K_\alpha T_I}{M}\left(1 + \frac{T_Fe^{-t/T_F} - T_Ie^{-t/T_I}}{T_I - T_F}\right)$$

$$C_{L_q}(p) = -\frac{K_q T_q}{M}\frac{1}{p(1+T_Fp)(1+T_qp)} \qquad \therefore \quad C_{L_q}(t) = -\frac{K_qT_q}{M}\left(1 + \frac{T_Fe^{-t/T_F} - T_qe^{-t/T_q}}{T_q - T_F}\right)$$

9. Application of explicit solutions

The above collection of explicit lift response functions may be used for comparison with both theoretical and experimental results, literally or as an aid for interpreting variation in possibly inconsistent results. As an example and as an evalution of the approach, the LTRAN2 results for plunge motion may be examined. In addition to generating the indicial response function for M = 0.8, reference 3 presents the results of a timewise integration solution for the lift response to plunging motion using the linear form of the program. As noted previously, the indicial response calculation may be utilised to provide the coefficients for the corresponding derived circulatory lift transfer function i.e. from $\phi_C(s') = 1 - 0{\cdot}3\,e^{-0{\cdot}11s'} - 0{\cdot}7\,e^{-0{\cdot}85s'}$
As the solution incorporates the low frequency assumption the impulsive term should be excluded from the comparison. The timewise integration solution is presented in reference 3 in the form of a frequency response; i.e. amplitude and phase versus reduced frequency, and is compared in figure 2 with the explicit solution for frequency response derived from the transfer function. Bearing in mind the limitation involved in approximating the indicial response, the frequency response derived from the circulatory lift transfer function is in quite good agreement with the results of the timewise integration.

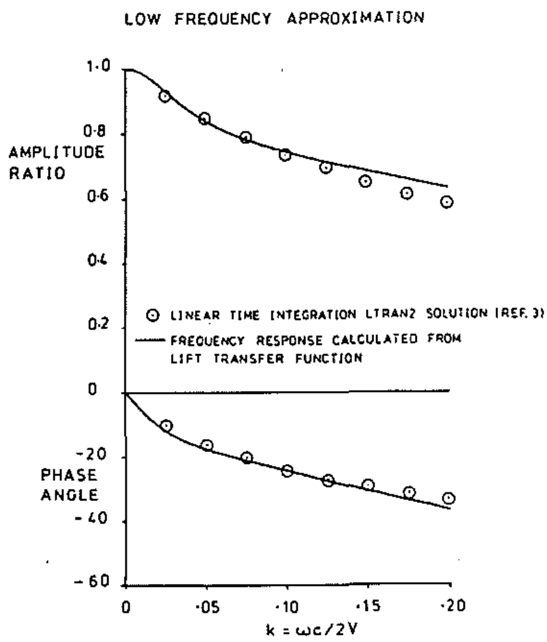
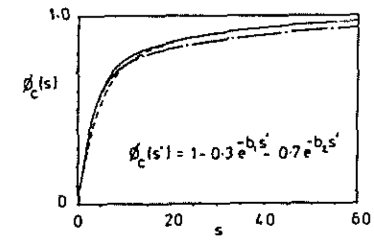


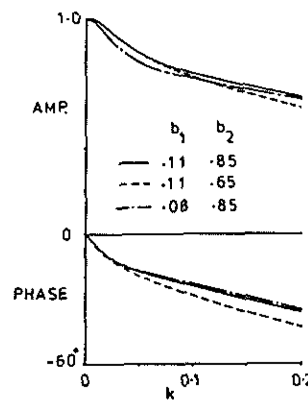
FIG. 2. PLUNGE FREQUENCY RESPONSE, M=0·8.

In reference 3 the timewise integration is used to evaluate application of the indicial solution but in a different manner; i.e. by using a superposition integral.

The consequences that variation in the indicial response impose on the frequency response may be evaluated via the explicit relation. Based on the above example two aspects are illustrated in figure 3 where variation in the initial and asymptotic behaviour is quantified by modifying the time constants b_2 and b_1 from the baseline values of 0.85 and 0.11 respectively. Modifying b_2 from 0.85 to 0.65 shows the sensitivity of phase at relatively high (at M = 0.8) reduced frequency to small changes in the initial response. Conversely, modifying b_1 from 0.11 to 0.08 (asymptotic behaviour) primarily affects the amplitude response at low frequency. It will be shown that the experimental variations in frequency response may be related back in a similar manner to imply required modifications in the assumed indicial response and hence the lift transfer function.



(a) INDICIAL FUNCTION (M=0.8)



(b) PLUNGE FREQUENCY RESPONSE

FIG. 3. SENSITIVITY TO INDICIAL LIFT FUNCTION

Experimental data are available from many sources and for various dynamic modes of motion which comprise pitch oscillation, plunge oscillation and ramp motion. Ramp results for a range of Mach numbers are available from one source only; the plunge oscillation results in the available literature tend to support theory but are limited in extent and exhibit more scatter than desirable particularly with regard to phase angle. Pitch oscillation tests are more numerous and on that account provide the best likelihood for a meaningful evaluation of theory. To put the results for different airfoils and from different wind tunnels on a common basis the amplitude response for normal force has been expressed as an amplitude ratio by normalising with respect to $2\pi/\sqrt{1-M^2}$ in accordance with linear theory. For pitch oscillation about the 1/4 chord, figure 4 presents test results through a range of Mach number at a reduced frequency $k = \omega c/2V$ of 0.2.

From the nonlinear LTRAN2 results the indicial response function

$$\phi_c(s') = 1 - 0.3\,e^{-.1 s'} - 0.7\,e^{-.53 s'}$$

was obtained for the low frequency or circulatory response. The corresponding deduced oscillatory response is included in figure 4 and exhibits a phase lag significantly in excess of the test values. Addition of the full impulsive component results in improved phase matching at low Mach number but not at high subsonic Mach number. Attenuating the impulsive loading term by the factor $(1-M^2)$ produces a much more consistent trend in the variation of phase angle by comparison with experiment. Amplitude ratio is much less sensitive to the impulsive term at this reduced frequency. Although the impulsive term will become more important at higher reduced frequencies, at M = 0.8 a reduced frequency of 0.2 is already high for rotary wing applications and at a lower Mach number the modification becomes progressively less significant.

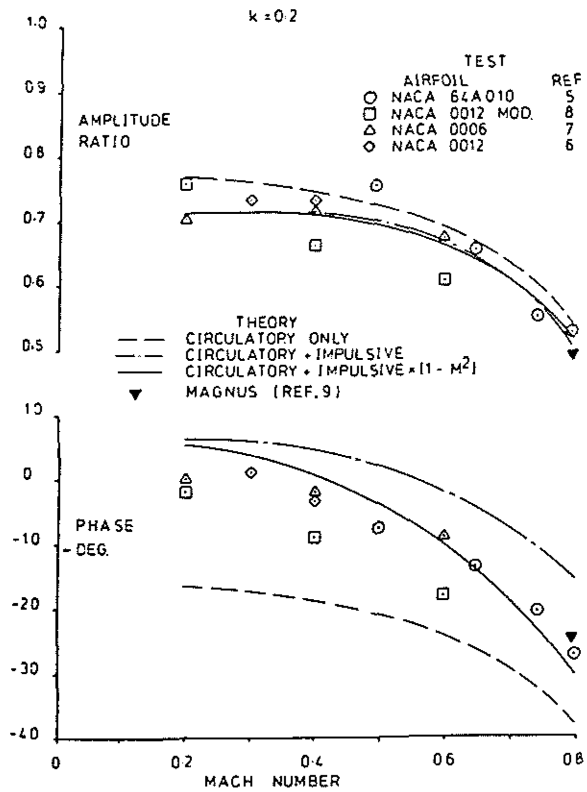


FIG. 4 OSCILLATORY DATA COMPARISON

The significance of the trends illustrated by this example are considered sufficient to justify application of the attenuation to the impulsive loading derived from piston theory.

Also shown in the comparison of figure 4 is the theoretical result obtained by Magnus (reference 9) for the NACA 64A010 airfoil from a finite difference solution of the unsteady Euler equations. The quasi steady value of lift curve slope obtained in ref. 9 is considerably in excess of the linearised value and has been used to normalise the amplitude of C_N at k = 0.2.

Some possible improvement in the coefficients of the circulatory indicial lift response function is suggested by the experimental results. The highest test Mach number available for attached flow and a range of frequency is M = 0.8 from reference 5 (NACA 64A010 airfoil). Some results are shown in figure 5 together with data for M = 0.5. A modification of the coefficient b_1, representing the asymptotic behaviour, from 0.10 to 0.14 produces a small but significant improvement all round; the modified impulsive response was included in the calculations.

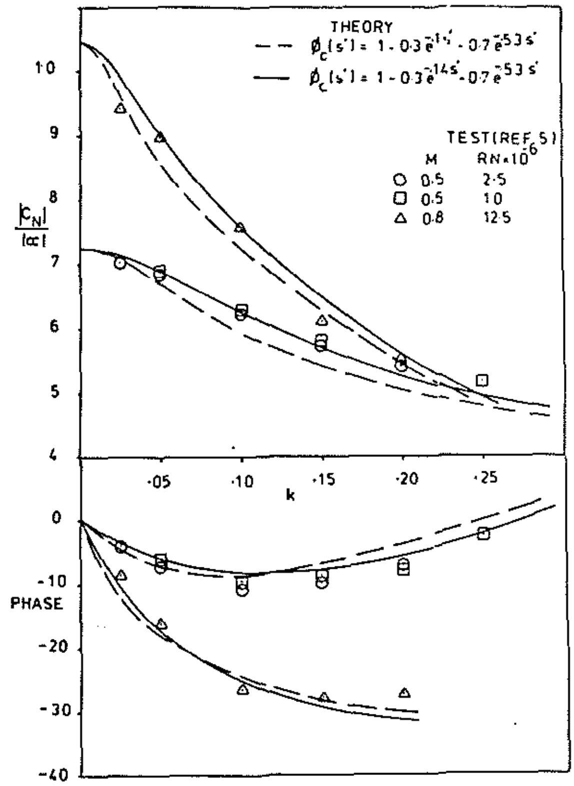


FIG. 5. FREQUENCY RESPONSE COMPARISON

A wind tunnel rig capable of producing ramp motion has been constructed at the Aircraft Research Association. Increase of pitch angle at varying rate is produced by an electro-hydraulic actuator; the buildup in rate occupies a short period and can be modelled in reproducing the motion time history. The results of some test on the NACA 0012 are shown in figure 6 for M = 0.61, 0.71 and 0.76; the nominal rates are $\dot{\theta}c/2V$ = .0068, .0059 and .0054 respectively. For comparison, the quasi static lift curve slope and the calculated response are shown using

$$\phi_c(s') = 1 - 0.3\, e^{-.14 s'} - 0.7\, e^{-.53 s'}$$

and the modified impulsive term. Apparent in the test results but not included in these calculations are the effects of flow separation.

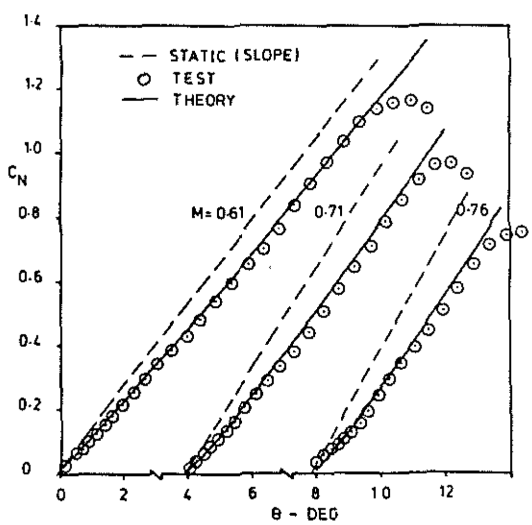


FIG. 6. RAMP DATA - COMPARISON

One of the objectives behind the work presented in this paper has been to provide a basis for the assessment of the effects of flow separation. To accomplish this requires the ability to model with some degree of certainty the characteristics of dynamic behaviour appropriate to the attached flow condition and to project them into the range where flow separation is occuring experimentally. This aspect is pursued in reference 10.

Application of the indicial formulation to arbitrary motion in the manner outlined above requires not only that the mathematical aspects of superposition are valid but also that the physical aspects of the implied linearity can be justified. The latter problem has been addressed in a series of experiments conducted at NASA by Davis and Malcolm (reference 11) which substantiate linearity even for super-critical flow as long as the flow remains attached.

10. Numerical Methods

The indicial formulation for unsteady lift response and the associated numerical methods have been chosen to meet the requirements of the helicopter rotor loads and structural response calculation. No constraint is placed on the formulation of the structural response or the variation of inflow and non repeatable values may be accomodated, i.e. successive revolutions of the rotor may exhibit different response and encounter a modified wake configuration. For a given step change in angle of attack (α) the indicial response for circulatory lift in the time domain is given by $C_{L_C}(t) = C_{L_\alpha}.\alpha.\phi_C(t)$ thus the product $\alpha.\phi_C(t)$ may be viewed as an equivalent angle of attack $\alpha_E(t)$; i.e.

$$\alpha_E(t) = \alpha\,(1 - A_1 e^{-t/T_1} - A_2 e^{-t/T_2})$$

Thus $\alpha_E(t)$ is composed of the instantaneous value of α minus two exponentially decaying terms. Initially, the sum of the decrements equals α. In any subsequent period they decay by a constant amount. This is also true when further step inputs are introduced, thus a numerical algorithm may be constructed to solve for a sampled system, i.e.

$$\alpha_{E_n} = \alpha_n - X_n - Y_n$$

where $X_n = X_{n-1} e^{-\Delta t/T_1} + A_1(\alpha_n - \alpha_{n-1})$

$$Y_n = Y_{n-1} e^{-\Delta t/T_2} + A_2(\alpha_n - \alpha_{n-1})$$

where $T_1 = \left[b_1(1-M^2)\,2V/c\right]^{-1}, \quad T_2 = \left[b_2(1-M^2)\,2V/c\right]^{-1}$

It is significant that in the above formulation all the prior history of the circulatory lift decrement is contained within the two terms Xn and Yn. In the particular case where the indicial lift function may be expressed in terms of exponential functions, Duhamel's integral for the superposition of successive increments in indicial lift reduces to this simple form.

For a single step input, the impulsive lift $C_{L_I}(t) = \alpha \frac{4}{M} e^{-t/T_I}$

For the sampled system of a series of step inputs $C_{L_{I_n}} = \frac{4}{M} I_n$

where $I_n = I_{n-1} e^{-\Delta t/T_I} + (\alpha_n - \alpha_{n-1})$

Similarly, for the pitch rate term

$$C_{L_{q_n}} = \frac{-1}{M} Q_n \quad \text{where} \quad Q_n = Q_{n-1} e^{-\Delta t/T_q} + (q_n - q_{n-1})$$

To evaluate the implementation of the above algorithms, together with subsequent developments, the numerical solutions for sampled forcing have been compared to exact (continuous) solutions for oscillatory motion through a range of frequencies and to ramp motion of increasing rate. In order to illustrate the sensitivity to sampling rate in a simplified though representative form an idealised forcing in the form of a doublet may be used; i.e.

$$\alpha(t) = 23{\cdot}34\left[t'(t'-1)\right]^2 \sin(2\pi t') \qquad \text{where } t' = t/T_D \text{ and}$$

T_D is a characteristic time used to control the timescale; for $t' > 1$, $\alpha(t') = 0$. The resulting time history is illustrated in figure 7 and demonstrates features of both ramp and oscillatory forcing. For the current application the doublet is scaled to occupy a time period equivalent to 15 semi chord lengths of travel (s). Together with the amplitude of 1 radian used the impulsive forces produced are similar in magnitude to the circulatory values at the Mach number of 0.3. The indicial function for circulatory lift used in the examples shown is

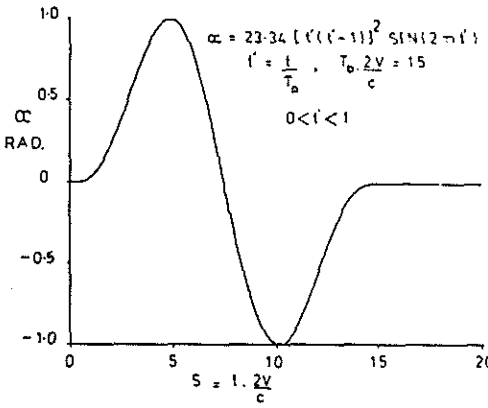


FIG. 7 DOUBLET

$\phi_C(s') = 1 - 0{\cdot}3\, e^{-{\cdot}08 s'} - 0{\cdot}7\, e^{-{\cdot}65 s'}$ which corresponds to the classical linear theory.

The objective in developing the numerical algorithm is to minimise the number of samples required to produce an acceptable degree of accuracy when compared to an exact solution. In this discussion the number of samples is related to the value t' = 1 or s = 15; i.e. a sampling N = 10 corresponds to intervals of $\Delta s = 1.5$; Using the algorithms derived above (labelled for convenience as "step" algorithm) the consequences of reducing the sampling are shown in figure 8. For the circulatory lift the reduction in sampling produces an artificial lag in the response which can approach 1 time step for the transient and tends to 1/2 time step for sustained harmonic forcing. However, the amplitude of the response is not severely degraded. For the impulsive lift component, reduction of sampling results in severe degradation of the amplitude of the response which would be unacceptable in the sample shown for N = 20.

Having established transfer functions for the circulatory and impulsive lift it is possible to derive the response to a change in α which varies linearly within the sampling period. This corresponds (up to the end of the interval) to the response to an idealised ramp as derived earlier. Thus, based on the relations

$C_{L_C}(t) = C_{L_\alpha}\alpha_\varepsilon(t)$, $\alpha_\varepsilon(t) = \alpha(t) - \Delta\alpha_\varepsilon(t)$ and

$\Delta\alpha_\varepsilon(t) = K_\alpha\left[A_1T_1(1 - e^{-t/T_1}) + A_2T_2(1 - e^{-t/T_2})\right]$ then for a sampled system $K_{\alpha_n} = (\alpha_n - \alpha_{n-1})/\Delta t$ and $\Delta\alpha_{\varepsilon_n} = A_1T_1(K_{\alpha_n} - K'_{1_n}) + A_2T_2(K_{\alpha_n} - K'_{2_n})$

where $K'_{1_n} = K'_{1_{n-1}}\, e^{-\Delta t/T_1} + (K_{\alpha_n} - K_{\alpha_{n-1}})$, $K'_{2_n} = K'_{2_{n-1}}\, e^{-\Delta t/T_2} + (K_{\alpha_n} - K_{\alpha_{n-1}})$

Similarly we may derive the impulsive lift $C_{L_{I_n}} = \frac{4}{M} T_I (K_{\alpha_n} - K'_{I_n})$

where $K'_{I_n} = K'_{I_{n-1}} e^{-\Delta t/T_I} + (K_{\alpha_n} - K_{\alpha_{n-1}})$

also $C_{L_{q_n}} = -\frac{T_q}{M} (K_{q_n} - K'_{q_n})$ where $K'_{q_n} = K'_{q_{n-1}} e^{-\Delta t/T_q} + (K_{q_n} - K_{q_{n-1}})$

and $K_{q_n} = \frac{c}{V} \left(\frac{\dot\theta_n - \dot\theta_{n-1}}{\Delta t} \right)$

Labelling the above as 'ramp' algorithms the results are compared with the 'step' algorithm for a sampling of N = 10 in figure 9. For circulatory lift the results have deteriorated, a phase lead has been substituted for a (somewhat smaller) lag. The impulsive load time history is improved significantly, the amplitude is now correct at the expense of some phase shift.

The above evaluation suggests that it would be beneficial to use the 'step' algorithm to calculate the circulatory lift and the 'ramp' algorithm to calculate impulsive lift. The advantage of doing this is avoidance of significant amplitude error but nevertheless a phase lag remains which it would be desirable to eliminate. One method to reduce the lag incurred by finite sampling is to introduce a step or half step lead in the forcing function. However, for arbitrary forcing, this requires an extrapolation of α based on the prior samples and for sparse sampling the procedure may introduce some large overshoot errors. All the algorithms incorporate a deficiency term in one form or another which is allowed to decay exponentially at each pass and is updated by addition of the new increment in forcing; i.e. from the 'step' algorithm for circulatory response:

$$X_n = X_{n-1} e^{-\Delta t/T_1} + A_1 (\alpha_n - \alpha_{n-1})$$

If the decay over the next increment in time is anticipated, the second term on the right hand side of the equation would also be factored by $e^{-\Delta t/T_1}$. Thus an effective half step lead may be introduced by factoring the term by $e^{-\Delta t/2T_1}$

i.e. $X_n = X_{n-1} e^{-\Delta t/T_1} + A_1 (\alpha_n - \alpha_{n-1}) \; e^{-\Delta t/2T_1}$

The above method for introducing a half step lead does not involve any prediction of the forcing over the next sampling period and avoids the consequent error. Thus a 'hybrid' series of algorithms may be constructed to incorporate the best features of those so far derived and yet minimise the phase lags involved.

For circulatory lift $\alpha_{E_n} = \alpha_n - X_n - Y_n$

where
$$X_n = X_{n-1} e^{-\Delta t/T_1} + A_1 (\alpha_n - \alpha_{n-1}) \; e^{-\Delta t/2T_1}$$
$$Y_n = Y_{n-1} e^{-\Delta t/T_2} + A_2 (\alpha_n - \alpha_{n-1}) \; e^{-\Delta t/2T_2}$$

For impulsive lift $C_{L_{I_n}} = 4 \frac{T_I}{M} (K_{\alpha_n} - K'_{I_n})$

where $K'_{I_n} = K'_{I_{n-1}} e^{-\Delta t/T_I} + (K_{\alpha_n} - K_{\alpha_{n-1}}) \; e^{-\Delta t/2T_I}$

and for pitch rate about the 3/4 chord $C_{L_{q_n}} = -\frac{T_q}{M} (K_{q_n} - K'_{q_n})$

where $K'_{q_n} = K'_{q_{n-1}} e^{-\Delta t/T_q} + (K_{q_n} - K_{q_{n-1}}) \; e^{-\Delta t/2T_q}$

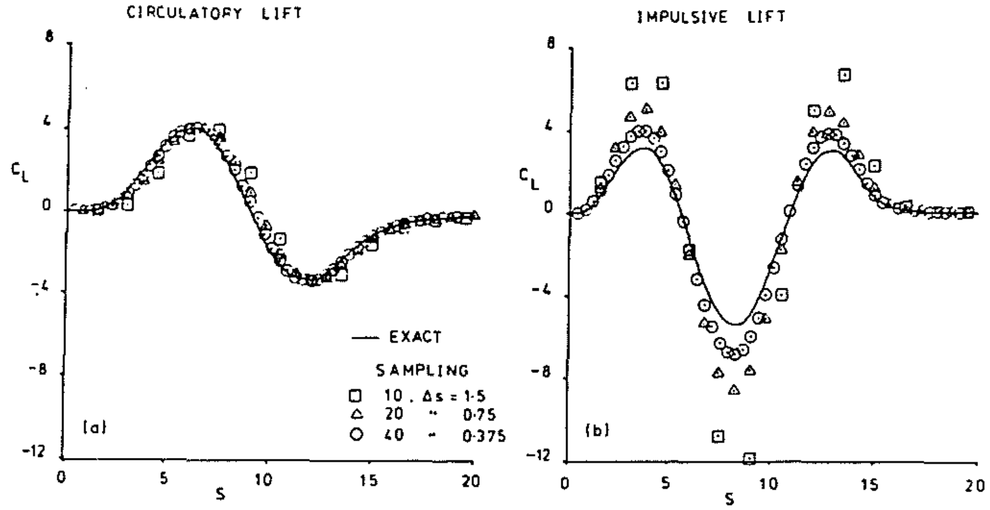


FIG. 8. DISCRETE SAMPLING - STEP ALGORITHM

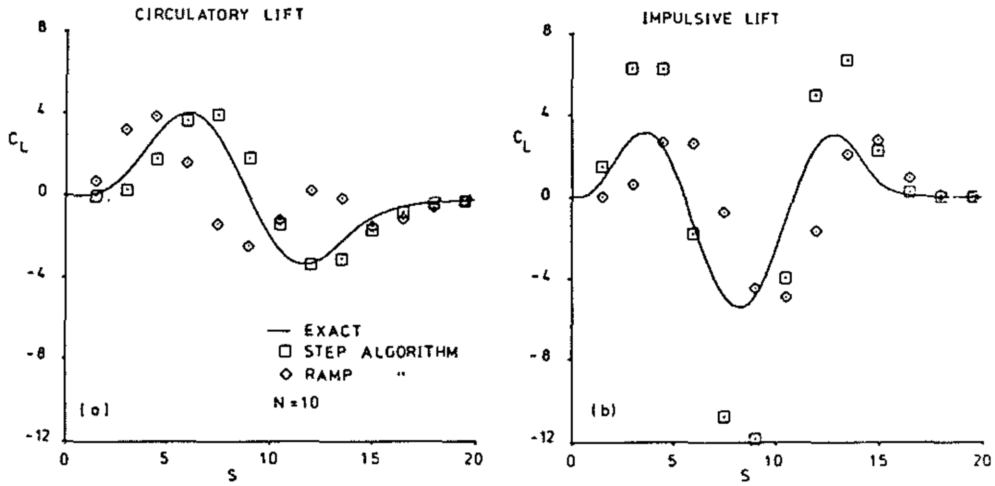


FIG. 9. DISCRETE SAMPLING - RAMP ALGORITHM

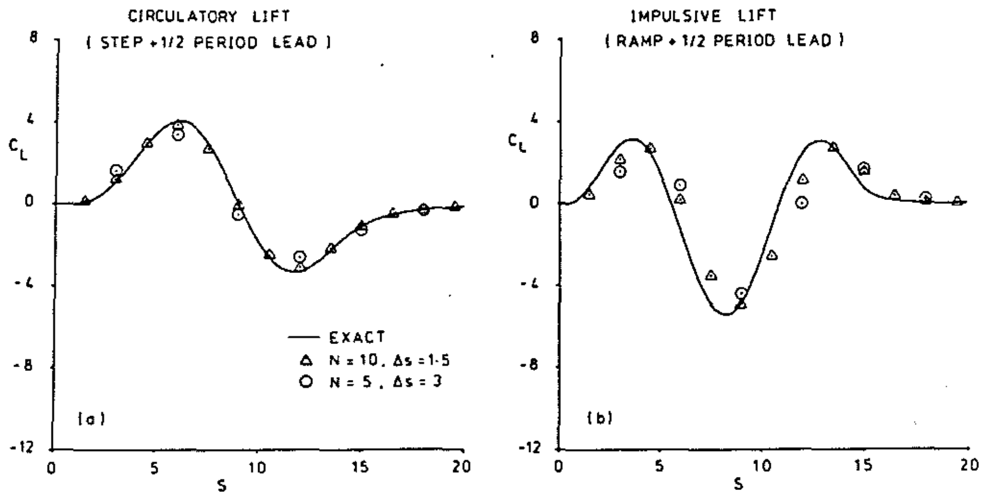


FIG. 10. DISCRETE SAMPLING - HYBRID ALGORITHM

Implementation of these 'hybrid' algorithms is illustrated in figure 10. For circulatory lift, at the previously used sampling of N = 10 there is negligible error. When the sampling rate is halved again (N = 5) the error is still small. For the impulsive loading, at N = 10 there is a small lag which increases slightly for N = 5. For most practical cases the proportion of impulsive lift will be very much smaller and, overall, it may be anticipated that the above 'hybrid' algorithms will be adequate for application to rotary wing calculations.

11. Comparison with a Navier Stokes solution

In reference 12 a compressible, time dependent Navier-Stokes calculation procedure is applied to ramp motion of an NACA 0012 airfoil. The initial angle of 6^o increases to a final angle of 19^o in a period of time equivalent to 0.63 chord length of airfoil travel and is held constant thereafter. The freestream Mach number is 0.147 and the Reynolds number equals 10^6. From the pressure distributions presented for successive time intervals a time history of C_N has been constructed and is shown in figure 11. During the initial stage of airfoil motion extremely large values of C_N are generated by the impulsive components of the loading which is distributed more or less uniformly along the chord. At the cessation of motion the impulsive loading decays rapidly and the circulatory loading completes its buildup. Viscous effects become significant and separation begins immediately due to the large angle of attack and C_N which exceeds the sustainable steady state value of about 1.2. Leading edge suction decays and at a time $\tau = t.V/c$ of about 2.5 a perturbation in the pressure distribution may be associated with the presence of a leading edge vortex. Subsequent pressure distributions indicate convection of the pressure perturbation downstream and by the last sample at $\tau = 4.2$ it is nearing the trailing edge.

The value of C_N does not decline significantly towards the end of the period which indicates that although extensive flow separation exists the vortex lift is still present.

The above detailed calculations are somewhat unique in the published literature and provide another opportunity for evaluation of the indicial methods under development. For the comparison, the evolved circulatory indicial lift function $\phi_C(s') = 1 - 0{\cdot}3\, e^{-{\cdot}14 s'} - 0\,7\, e^{-{\cdot}53 s'}$ has been used in conjunction with the corresponding impulsive force time constant. As a baseline, an inviscid solution was generated; i.e. no allowance for flow separation and a value of $C_{L\alpha} = 2\pi$. The result which is shown in figure 11 exhibits an asymptotic value of lift which is too high but the initial (mainly impulsive) loading is matched remarkably closely.

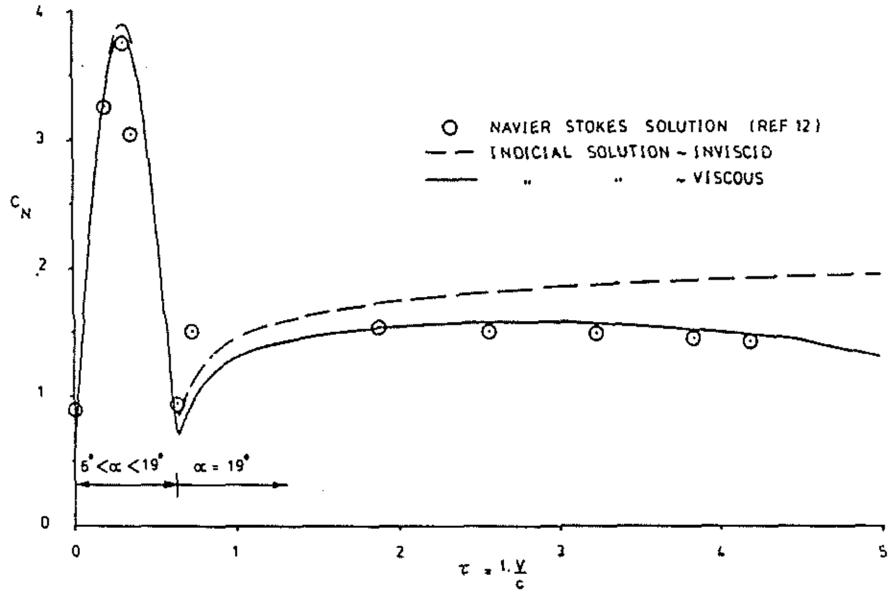


FIG. 11. COMPARISON WITH NAVIER-STOKES SOLUTION

In reference 10 a method is presented which enables the onset and progress of flow separation to be modelled for time dependent lift in a manner compatible with an indicial formulation. Critical values of the pressure and the progression of gross trailing edge separation are modelled using first order lags involving the time constants T_P and T_F which are expressed in terms of semi chord lengths of airfoil travel. The relationship between the additional loading (pressure) and trailing edge separation under steady conditions is obtained from test data (in this case from reference 13) and the onset of leading edge separation from inviscid calculations of the peak velocities and gradient in the leading edge region. In this case a critical C_N (static) for leading edge separation of 1.5 was deduced (allowing for the reduced lift curve slope of 0.10/deg) which is greater than that achieved statically (1.2) at R.N. = 10^6.

The consequence of including the flow separation (viscous) model is illustrated in figure 11, predominantly affecting the later stages of circulatory lift. Using values of T_P and T_F of 1.7 and 2, respectively, the model predicts a rapid growth of separation at around τ = 2.5 but the total lift is not immediately affected due to the build up of vortex lift which ceases, however, at τ = 4.5. The only exception to the good agreement between the indicial and Navier-Stokes solution occurs at τ = 0.73, just after cessation of the motion where an extremely rapid and complex change in the chordwise distribution of loading is taking place. An additional point at τ = 0.63, not included in reference 12, was obtained from a pressure distribution subsequently provided by the author and emphasises the rapid changes occuring at this time.

12. Conclusions

Rotor load calculations require a versatile method for evaluating the unsteady lift response to angle of attack forcing which may vary in a discontinuous or almost arbitrary manner. At the same time the local freestream Mach number is varying rapidly. The indicial formulation is appropriate to meet these requirements and has been in use for several years. The continued development of more refined analyses for unsteady transonic flow, their application to idealised motion and availability of wind tunnel test data, again, for idealised motion, has provided an opportunity to assess and refine the indicial formulation.

From the theoretical solution for the indicial response to a step change in angle of attack it is possible to produce an approximate and generalised indicial lift function. From the generalised indicial lift function it is possible to derive an even more general transfer function relating unsteady lift response to angle of attack and this may be used to derive explicit expressions for the idealised forcing used in experimental and theoretical studies, for example, the frequency response to harmonic forcing.

Using the above approach, use of the generalised indicial lift function has been validated and refined. Comparison with both theory and test is favourable.

Application to rotor calculations requires numerical procedures that do not impose undue limitations on the sampling interval for azimuthal stepping. This is for economy in computation. The unsteady lift transfer function has been used to derive improvements in the current numerical procedures which have been evaluated for sensitivity to sampling.

The analysis and procedures developed above have been applied for comparison with a compressible, time dependent, Navier-Stokes solution for an extremely abrupt idealised ramp and hold motion. The indicial solution has been extended to include the separation effects which are present in this case and, again, the resulting comparison is favourable.

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