AERO-ELASTIC ANALYSIS OF WIND TURBINES USING SYSTEM IDENTIFICATION TECHNIQUES

B. Marrant, P. Lisandrin, Th. van Holten
DAR, Faculty of Aerospace Engineering, Delft University of Technology
Delft, The Netherlands

Abstract

This paper presents a newly developed tool to identify linear models from the non-linear responses as calculated with a software package. The package which is used is able to simulate all kinds of multi-body systems without the derivation of the equations of motion and is applied for the aero-elastic analysis of wind turbines. The identification tool is used to enable stability analysis in the linear domain.

This paper presents the theory behind the identification tool as well as its validation. Therefore the identification tool is applied on the responses of a single wind turbine rotor blade as simulated with the multi-body package. The identified model is in that case time invariant. Also the identification with an added state is explained for the case where dynamic inflow is used. The identification method is also applied on a periodic system.

1 Notations

a	=	axial induction factor
a'	=	rotational induction factor
c_d	=	drag coefficient
c_l	=	lift coefficient
c_m	=	moment coefficient
$\overline{q}$	=	non-dimensional yaw rate
q_i	=	generalized coordinate
r	=	local blade radius
v	=	change in velocity induced by the rotor
w	=	local tangential wind velocity at rotor plane
$\vec{x}$	=	state vector
$\vec{y}$	=	output vector
y	=	response
y_{est}	=	estimated response
A	=	state matrix
A_4	=	axisymmetric flow term
B	=	gravity term
C	=	output matrix
F_i	=	aerodynamic force
K	=	non-dimensional flapping frequency
K_1	=	vertical wind shear gradient
K_2	=	non-dimensional lead-lag frequency
$\overline{U}_0$	=	non-dimensional crossflow
V_0	=	free stream wind speed
$\overline{X}$	=	state matrix
$\overline{Y}$	=	shifted state matrix
β	=	flap angle
γ	=	Lock number
ζ	=	lag angle
ζ	=	damping
θ_p	=	blade pitch angle
ψ	=	azimuth angle
ω	=	frequency
Ω	=	rotor angular velocity

2 Introduction

Aero-elastic stability is an important issue in the design process of wind turbines. Therefore, the software package WOBBE [2], [3] has been developed at Delft University of Technology for the aero-elastic analysis of wind turbines. This tool can automatically simulate the motion of any kind of multi-body system consisting of a chain of rigid bodies interconnected by hinges and springs. The necessary user input is limited to the specification of the geometry of the different bodies by means of dimensions, inertial and aerodynamic properties as well as the angular velocity of the rotor and the wind conditions. WOBBE is based on an alternative formulation of the equations of motion using Hamilton's generalized momenta [1]. This fully non-linear package performs simulations in the time domain and this without the need to derive the equations of motion. This makes it relatively easy for the designer to quickly change the number of degrees of freedom as well as the configuration of the wind turbine. The output consists of the evolution of the generalized coordinates q_i and $\dot{q}_i$ in time, the evolution in time of the x-, y- z-positions of the hinge points in one reference system as well as several aerodynamic data such as the axial induction factor a ($v = aV_0$), the rotational induction factor a' ($w = a'\Omega r$), the axial force on and the power delivered by the wind turbine.

Since aero-elastic characteristics of a wind turbine such as its modal frequencies, modal damping and modal shapes are only defined in linearised theory, linear analysis tools are still used for aero-elastic research besides the non-linear simulation tools. Linear tools are usually based on the linearised equations of motion of a wind turbine. In this case, the choice was made to derive linear models from the package of WOBBE with system identification techniques. Therefore a number of time responses are calculated with WOBBE. With these time responses a "best fitting" linear model can be derived making use of parameter identification methods. After this process, classical vibration theory is available for the determination of the modal frequencies, damping and modal shapes, as well as for stability investigation.

This paper presents the identification of linear models for the aero-elastic analysis of numerical simulations with the NM80 wind turbine. This prototype is a pitch-regulated variable speed wind turbine with a rated wind power of 2.75 MW. This work is part of the STABCON project supported by the European Commission where the previously mentioned turbine is used as a test case for the validation of newly developed aero-elastic stability tools [1]. The paper is structured as follows. The system identification method is described in section 3. For the time-invariant case the combination of the software package WOBBE and the identification tool is validated against the tools of the other STABCON partners in section 4. For this purpose the system identification procedure is applied on the simulations of a single rotor blade. For the time-variant case, the identification method is applied on an example of a linear differential equation with periodic coefficients in section 5. The conclusions are presented in section 6.

3 System identification method

3.1 Overview of the identification methods

System identification deals with constructing mathematical models of dynamic systems from experimental data. Basically there exist three types of system identification techniques [5]:

- White-box identification deals with pure physical modeling, based on first principles (i.e. Newton's law, etc.). This means that the modeling is performed without the use of experimental data.
- Grey-box identification corresponds to the case where physical laws are applied to arrive at the model, but where some of the physical parameters in the model are unknown or not exactly known and should be estimated from measured data. In grey-box identification both prior information and experimental data are used.
- Black-box identification refers to the situation where a model is identified purely on the basis of measured data and a given model class. As a consequence of the model being identified on the

[1]The European STABCON project is about the aero-elastic stability and control of large wind turbines. The participants of this project are the Risoe National Laboratory, the Energy research Centre of the Netherlands (ECN), Center for Renewable Energy Sources (CRES), the National Technical University of Athens, Fluid section (NTUA/FS), Technical University Denmark (DTU/IMM/MEK), Universität Stuttgart, Delft University of Technology (DUT) and VESTAS

basis of measured data rather than reflecting the physical structure of a system, the parameters and states in a black-box model might not have a direct physical interpretation.

Therefore, white- and black-box identification can be viewed as extreme cases.

White-box modeling is usually used for the construction of the models for linear aero-elastic analysis tools. In this case, the non-linear equations of motion are derived and linearized afterwards where the linearisations are made such that there are no significant differences with the non-linear equations of motion. The physical parameters in the model are estimated from the properties of the real system.

In an earlier stage of the research, grey-box identification techniques have been used for the identification of linear models from numerical simulations with WOBBE [4]. Although this type of system identification seems very attractive because of the direct physical representation of the models, it also includes a huge disadvantage since in this case the objective function which is to be optimized is generally non-linear. Consequently the minimization of the objective function can only be obtained trough methods of iterative search which leads to difficulties or even failure to identify a representative system. The only possibility is then to improve the initial "guess" for the parameters in the model. Since updating these initial parameters has led to a long and tedious identification process which did not always converge to the optimal solution, it was decided to abandon this approach.

For the identification of linear models it was decided to use black-box identification techniques. The model class which is used is the state-space representation. Usually, the drawback of black-box techniques is that when a state-space system is identified from input and output data the system does not represent a physical structure and the physical meaning of the states is not known directly. This usually requires extra transformations. For the black-box identification method presented in this paper the states are considered to be known. The states for this system consist of the generalized coordinates q_i and their first derivative with respect to time $\dot{q}_i$ since the complete equations of motion for the wind turbine can be written in these two types of variables. The time history of these variables can be retrieved from the output of the non-linear code WOBBE, see figure 4.

3.2 Model identification using least squares

The system which is to be identified is considered to be autonomous (i.e. no inputs) and periodic and looks as follows in state-space notation:

$$\begin{aligned} \dot{\vec{x}}(t) &= A(t)\vec{x}(t) \\ \vec{y}(t) &= C(t)\vec{x}(t) \end{aligned} \tag{1}$$

where $\vec{x}(t)$ is the state vector, $\vec{y}(t)$ is the output vector, $A(t)$ is the periodically varying state matrix and $C(t)$ is the periodically varying output matrix. Since the discrete time representation holds several advantages for the identification the state-space system is rewritten as follows:

$$\begin{aligned} \vec{x}(k+1+iN) &= A_k\vec{x}(k+iN) \\ \vec{y}(k+iN) &= C_k\vec{x}(k+iN) \end{aligned} \tag{2}$$

where $\vec{x}(k)$ is the state vector. A_k and C_k are the unknown periodic state matrix and output matrix respectively at time step k with k ranging from 1 to N. N is the number of azimuth angles at which the states are measured and thus determines the period. i is an integer from zero to n which indicates the number of periods (i.e. revolutions of the rotor). The relation between the states $\vec{x}$ and the state matrix A_k is explained visually in figure 1.

The data for each corresponding time step k for every period can be collected to estimate the A_k matrix.

$$\begin{aligned} &\left[\vec{x}(k+1)\ \vec{x}(k+N+1) \ldots \vec{x}(k+nN+1)\right] = \\ &\quad A_k\left[\vec{x}(k)\ \vec{x}(k+N) \ldots \vec{x}(k+nN)\right] \end{aligned} \tag{3}$$

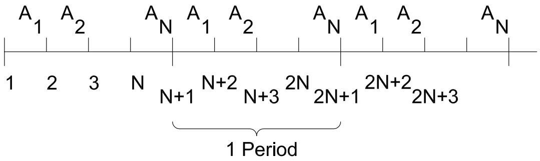

Figure 1: Relation between the state matrix and the states for a periodic system

This can be rewritten in compact notation as

$$\overline{Y}_k = A_k \overline{X}_k \tag{4}$$

where the state matrix $\overline{X}_k$ and the shifted state matrix $\overline{Y}_k$ are

$$\overline{X}_k = \left[\, \vec{x}(k)\ \vec{x}(k+N)\ \ldots\ \vec{x}(k+nN) \,\right]$$

$$\overline{Y}_k = \left[\, \vec{x}(k+1)\ \vec{x}(k+N+1)\ \ldots\ \vec{x}(k+nN+1) \,\right]$$

The state matrix A can be solved from equation (4) using the least squares method which results in the following expression.

$$A_k = \overline{Y}_k \overline{X}_k^T \left(\overline{X}_k \overline{X}_k^T \right)^{-1} \tag{5}$$

The identification therefore leads to a series of n A_k matrices. The discrete A_k matrices should then be transformed into the continuous time domain. This has been done with the zero-order hold method which assumes zero-order-hold on the inputs. This leads to good results when the sample time is small.

The same identification procedure can be used to identify the output matrix C which transforms the generalized coordinates q_i and their first time derivatives $\dot{q}_i$ to the x-,y-,z-positions in a specified reference frame. This would be done by collecting the generalized coordinates and their first time derivatives in time in the matrix $\overline{X}_k$ and the x-,y-,z-positions in the matrix $\overline{Y}_k$.

4 Identification and analysis of a rotor blade model

This section describes the identification and analysis of a time-invariant model for a single rotor blade from simulations with the software package WOBBE. The results are compared with the results of the other STABCON partners. This way, the combination of the software package WOBBE and the identification tool is validated.

4.1 Automatic simulation of the rotor blade

The identification algorithm is applied to the responses of the hinges of a single rotor blade which are simulated with the software package WOBBE. During the simulations of the single rotor blade, the wind speed is uniform over the rotor area and gravity effects are neglected. This means that the model to be identified is not periodic and only one A and C matrix need to be obtained. This results in a simplified identification scheme. The properties of the rotor blade, which is used in these simulations, are based on the blades of the NM80 wind turbine.

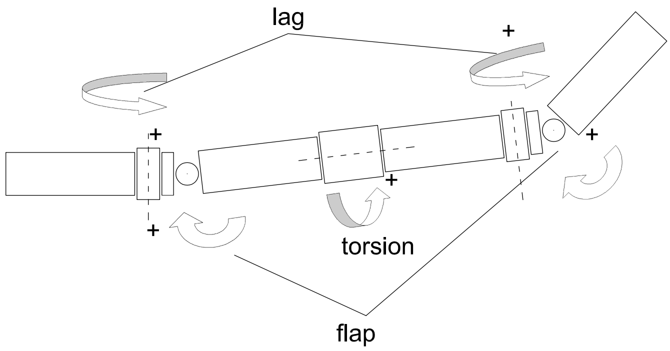

Figure 2: Deflections of a super element

The structural properties of the blade are modeled into WOBBE with three super-elements. Models with super-elements need less degrees of freedom than lumped mass models for the same accuracy and were first introduced by Rauh en Schielen [6], [7]. The super-element which was used for the simulations can be seen in figure 2. Every super element is divided into three large bodies interconnected with two lag hinges, two flap hinges and one torsion hinge in the middle. A dummy element of small dimension is applied between the lag and the flap hinges because the simulation package only allows a rotation in one direction between two successive bodies. Therefore the entire blade model blade contains 15 degrees of freedom consisting of 6 lag hinges, 6 flap hinges and 3 torsion hinges. The structural model is simplified with respect to the real blade in the sense that both structural twist and structural damping of the blades are neglected.

For the calculation of the aerodynamic forces WOBBE uses classical BEM theory. In order to calculate these forces over the blade, look-up tables for the c_l, c_d and c_m values are used for each blade segment. The axial induction factor a is assumed to be constant over the rotor plane which turned out to be a good approximation of the mean value of the distributed axial induction factor as used in the tools of the other STABCON partners. WOBBE has also the option to simulate the dynamic inflow effect. The dynamic inflow model is based on a first order differential equation as described in references [8] and [9]. So far it was not possible to simulate dynamic stall effects in WOBBE.

After a simulation with WOBBE it is possible to identify a linear model from the time responses in the generalized coordinates.

4.2 Identification of a rotor blade model

4.2.1 Model identification without dynamic inflow

When the wake is frozen (induced velocity is constant in time), the differential equations determining the equations of motion can be described with only the generalized coordinates and their first and second time derivatives. Therefore the state vector for the state-space system can be written as:

$$\vec{x} = \begin{Bmatrix} \vec{q} \\ \dot{\vec{q}} \end{Bmatrix}$$

Simulations of a single rotor blade have been done with WOBBE over a time length of 40 seconds. For these simulations, the integrations are performed with the fourth-order Runge-Kutta method. In total 80000 steps are used to keep the integration stable. Since the main interest of this research goes to a stability analysis of the system, only the homogeneous differential equation needs to be identified, i.e. without the forcing part at the right hand side. Therefore for each data set the mean values have been subtracted so that only the perturbations with respect to the steady state remain. The highest frequency of the system is $81Hz$, which is well below the Nyquist frequency of $1000Hz$. Therefore the sample frequency is sufficient for the identification.

In order to identify a good linear model from the responses as simulated with the package WOBBE these responses should be as linear as possible. Therefore the initial perturbations with respect to the steady-state should be kept small enough in order to keep the calculated responses in the linear region. On the other hand the initial perturbations should be large enough in order to excite all the modes so that they can be identified. Since the Coriolis forces introduce non-linear effects in the lag and torsional degrees of freedom, the initial flap disturbances should be small enough. Therefore the perturbations with respect to their steady-state values should be $\pm 10 - 20\%$ for the flapping hinges, $\pm 60\%$ for the torsion hinges and $\pm 80 - 100\%$ for the lead-lag hinges.

It is very important for each identification that the state matrix $\overline{X}$ is not (close to) singular. This would make the identified responses go to infinity. Typical singular values of the state matrix are shown in figure 3. A number which is very important in this respect is the condition number, which is the ratio of the largest and the smallest singular value. Large condition numbers indicate an almost singular matrix. Typical values which have been encountered with the identification are in the order of $10^4 - 10^5$, but no problems did occur.

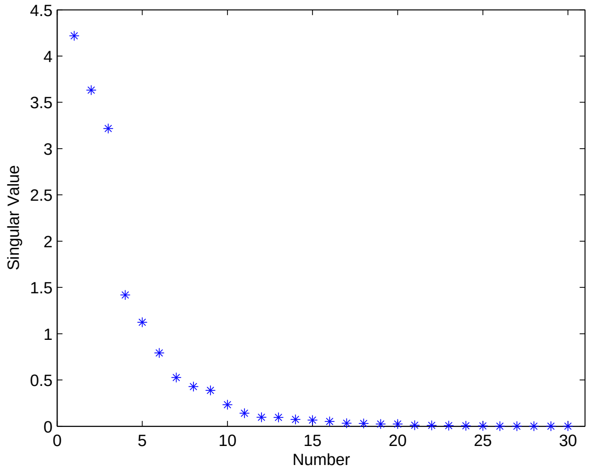

Figure 3: Singular values of the state matrix for a simulation at a wind speed of 7 m/s

A comparison of the responses as calculated with WOBBE and as determined with the identified model for the first three hinges is shown in figure 4, where q_1 is the first lead-lag hinge, q_2 is the first flapping hinge and q_3 is the first torsion hinge. The model fit is expressed in the *Variance Accounted For* (VAF).

$$VAF = 100 \left(1 - \frac{variance(y - y_{est})}{variance(y)}\right) [\%]$$

where y is the real response and y_est is the estimated response from the identification. When y is equal to y_{est} the Variance Accounted For is 100%. As can be seen in figure 4 the identified linear model represents the responses as calculated with WOBBE extremely well. There is no difference between the original and the identified responses as the model fit approaches 100%. The model fits for the other degrees of freedom are similar and were typically above 99%. For some degrees of freedom a model fit of 88% or 95% resulted which is still very acceptable. The most difficult part was to obtain linear responses for the lead-lag and torsion hinges at the outboard section and therefore the smaller models fits occurred at these hinges.

After the identification of a linear model classic vibration theory can be used for the determination of the modal frequencies, damping and shapes.

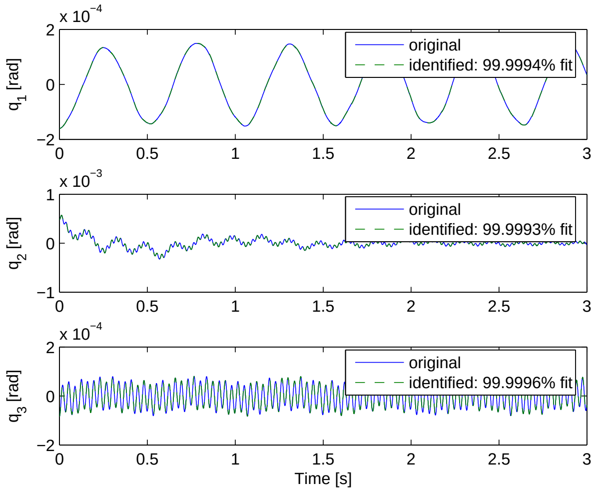

Figure 4: Comparison of the responses of the first three hinges of the single rotor blade. The responses as calculated with WOBBE are compared with the responses as calculated with the identified model for a wind speed of $7m/s$.

4.2.2 Model identification with dynamic inflow

When the dynamic inflow effect is used for the simulation of a rotor blade the axial induction factor a should also be taken into account for the identification because this variable will change in time according to a first order differential equation. The influence of the rotational induction factor a' on the aerodynamics is neglected in the identification because it is very small. Therefore the new the state vector will be:

$$\vec{x} = \begin{Bmatrix} \vec{q} \\ a \\ \dot{\vec{q}} \end{Bmatrix}$$

Including the axial induction factor a in the identified model usually increases the fit between the identified responses and the original responses to values of 99% when dynamic inflow is involved. This means an increase of the model fit for some responses with 3%

with respect to the identified model without this added state. Figure 5 shows the variation of the axial induction factor in time, where only the induction factor with respect to its steady value has been shown. The identified axial induction factor does not completely fit the original signal with a model fit in the order of $75-90\%$, see figure . However it must be remembered that only the frequencies, damping and shapes of the real modes (i.e. from the hinges) are of interest.

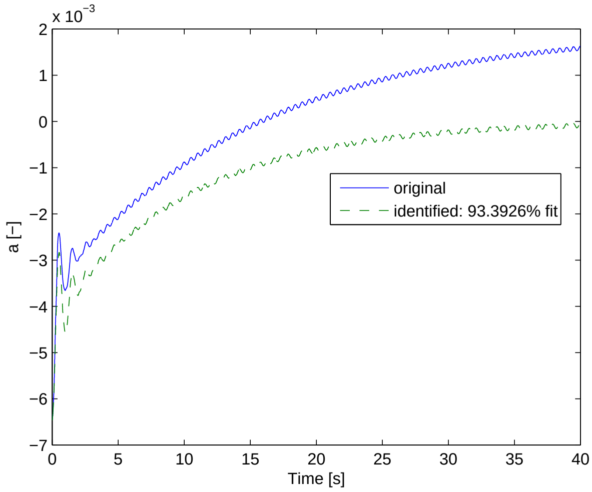

Figure 5: Comparison of the evolution of the identified and original axial induction factor in time

A sensitivity analysis has been done to estimate the accuracy of the predicted damping with the identification method. For this reason a simulation has been done with WOBBE over a time frame of 80 seconds instead of 40 for a wind speed of 7 m/s. Analysis of the model obtained from the identification with this longer data set revealed that there are no differences in the predicted frequencies ω. There are no differences in the predicted damping ζ either except for the fifth mode with a difference of 6% which is still acceptable.

Also the sensitivity to larger initial perturbations has been investigated. In this case the responses become non-linear. For this investigation the simulations which showed linear responses at the different nodes for a wind speed of $7m/s$ (see figure 4) have been taken as a reference case. For the first case the initial perturbations on top of the steady-state have been taken 4 times as large as for the reference case. The changes in the identified frequencies are negligible but the identified modal damping decreases within a range of $2-4\%$ except for the damping of the 4^{th} mode which decreases with 38% with respect to the reference case. For the second case the initial perturbations are taken 16 times as large as for the reference case. The changes in identified modal frequencies are still negligible but the damping decreases with $10-30\%$ except for the 5^{th} mode which changes by 450% with respect to the reference case. The large decrease in damping of the 5^{th} mode which occurred in both cases is still acceptable since this involves a lead-lag mode which already has a very small damping for the reference case. When the initial perturbations are increased the non-linearities first show in the lead-lag and torsion hinges at the outboard sections. Therefore the model fit between the identified and original responses is less good (20-30%) than for the reference case.

4.3 Analysis of the rotor blade model

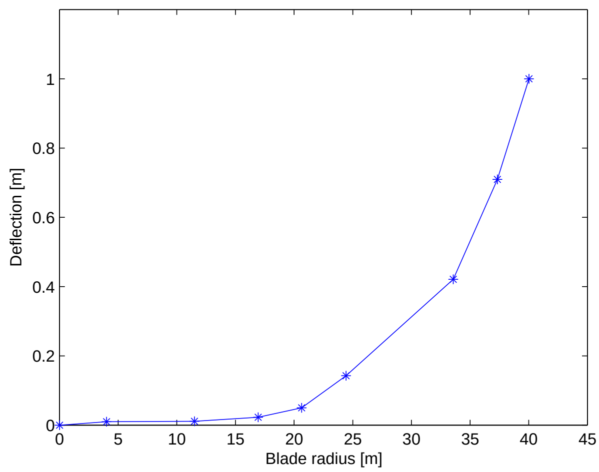

Figure 6: The first flapwise structural bending mode for the rotor blade scaled to a tip deflection of 1 meter.

Linear models have been identified from simulations of the single rotor blade with WOBBE at wind speeds of 7, 10, 13, 16 and $19m/s$ covering the complete wind speed range from somewhat above cut-in wind speed to somewhat below cut-out wind speed. The simulations have been done with the dynamic inflow effect included.

The first two structural mode shapes are shown in figures 6 and 7. Their shape corresponds very well with the modes as determined by the other STABCON partners [2]. There is a small amount of coupling in these modes with a maximum of 10%.

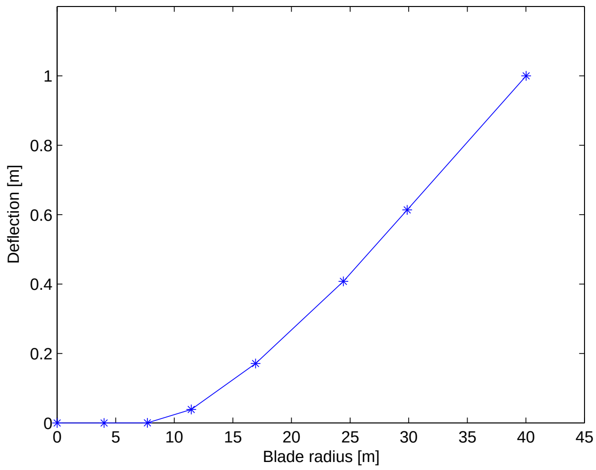

Figure 7: The first lagwise structural bending mode for the rotor blade scaled to a tip deflection of 1 meter.

The frequencies ω of the first five modes of the single blade can be seen in figure 8. The modes in order of occurrence from low to high frequency are the first flapwise, first lagwise, second flapwise, third flapwise and second lagwise mode. The identified frequencies are almost constant over the range of wind speeds. The frequencies for mode 1, 2 and 4 differ $\pm 4\%$ with the other STABCON partners, mode 3 and 5 differ by $\pm 15\%$.

The non-dimensional damping ζ for the first five modes of the single blade can be seen in figure 9. The same trend for the damping of the five modes can be seen in the results of the other STABCON partners. The reduction of the damping at rated wind speed (i.e. $13 m/s$) is clearly visible. This is due to the reduced lift gradient occurring at these high angle of attacks. A comparison with the predicted damping of the other partners revealed that the other partners predicted a larger damping for all the flapwise modes (mode 1, 3 and 4). The mean differences with the other partners are in the order of 40% for the first and the third mode and even 70% for the fourth mode. The reason for the smaller predicted damping is the difference in the aerodynamic wake model used. This can be explained as follows. The axial induction factor a which is a measure for the induced velocity of the rotor is taken constant over the radius in the WOBBE calculations where the other STABCON partners have an axial induction factor which is distributed over the rotor radius. This constant axial induction factor is approximately the mean value of the distributed axial induction factor as calculated by the other partners. The distributed axial induction factor as calculated by the other STABCON partners is larger at the outboard sections than the constant mean axial induction factor as calculated by WOBBE. Therefore the local angle of attack at the outboard section is smaller and the local lift gradient is larger when the axial induction factor is distributed over the radius. This explains the smaller damping of the flapwise modes identified from WOBBE calculations with respect to the damping as predicted by the other STABCON partners, since the

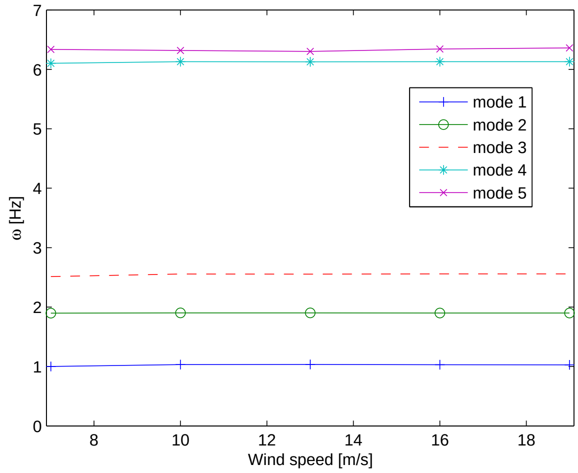

Figure 8: The natural frequencies of the first five modes of a rotor blade with respect to the wind speed.

[2] Note that the results of the other STABCON partners are preliminary results and have consequently not been published yet.

magnitude of the lift gradient is proportional to the amount of damping. Since both the distributed and constant axial induction factor are two extreme approximations of the real rotor inflow, the real damping of the flapwise modes will be in between the damping identified from WOBBE and the damping predicted by the other STABCON partners. The damping of the edgewise modes (mode 2 and 5) lies in between the results of the other partners with differences in the order of 10%. The lagwise damping is very small (around zero) and becomes even slightly negative in between $13m/s$ (rated wind speed) and $19m/s$ (almost at the cut-out wind speed). For the real blade the lagwise damping will be positive since the structural damping is not taken into account in the automatic simulations with WOBBE.

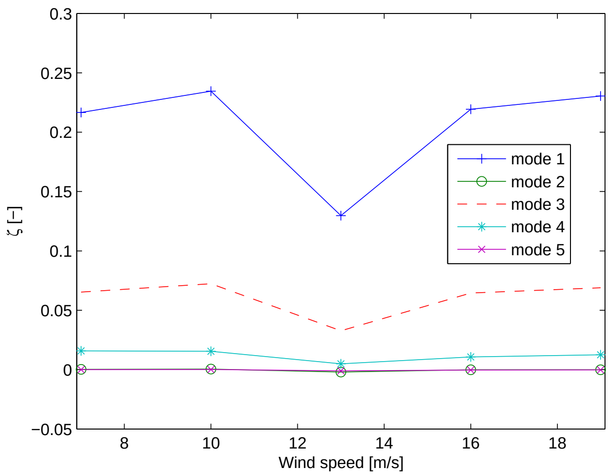

Figure 9: The damping of the first five modes of a rotor blade with respect to the wind speed

5 Identification of a time varying, periodic system

The identification procedure described in section 3 is now tested on a system described by differential equations with periodic coefficients. The system consists of an isolated, rigid blade rotor, having a flap and lag degree of freedom, and rotates at a given angular speed. The structural model for the rotor consists of lumped mass and springs for the structural part, whereas the aerodynamic model is steady. The equations are taken from reference [10] (see pages 165, 183, 140) and the periodicity of the coefficients comes from the fact that the rotor is submersed in a gravitational field, withstands a crosswind not perpendicular to the rotor plane and has a yawing motion. The equations of motion can be written in the following compact form:

$$\begin{aligned}
&\zeta'' + C_0\zeta - X_{\beta'}\beta' + X_\beta\beta = F_1 \\
&\beta'' + C_1\beta' + C_2\beta = F_2
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
&C_0 = 2Bcos(\psi) + K_2 \\
&X_{\beta'} = 2\beta - \frac{\gamma}{2}A_4 + \frac{\gamma}{2}cos(\psi)[\frac{K_1\bar{V}_0}{2} + \frac{\theta_p}{3}(\bar{U}_0 + \bar{q}')] \\
&+\frac{\gamma}{2}sin(\psi)[\frac{\bar{q}}{2} + \frac{2}{3}\beta\bar{U}_0] \\
&X_\beta = \frac{\gamma}{2}\bar{U}_0(\lambda + \frac{\theta_p}{3})sin(\psi) \\
&C_1 = \frac{\gamma}{8}[1 - \frac{4}{3}cos(\psi)(\bar{U}_0 + \bar{q}')] \\
&C_2 = K + 2Bcos(\psi) + \frac{\gamma}{6}\bar{U}_0 sin(\psi)
\end{aligned} \tag{7}$$

for the meaning of the different coefficients see reference [10]. The equations are non-linear in the variables β and β' due to the term $X_{\beta'}\beta'$. Besides the coefficients and the forcing terms are periodic with the azimuth angle ψ. The problem definition is completed providing the initial conditions on the variables and their first derivative. The system of equations is solved numerically using a second order finite difference scheme. In the derivation of the numerical model the equations are linearised evaluating the β terms in the coefficient $X_{\beta'}$ at the time j.

5.1 Test Case

The system identification procedure is tested on the numerical example of page 211 of reference [10]. First

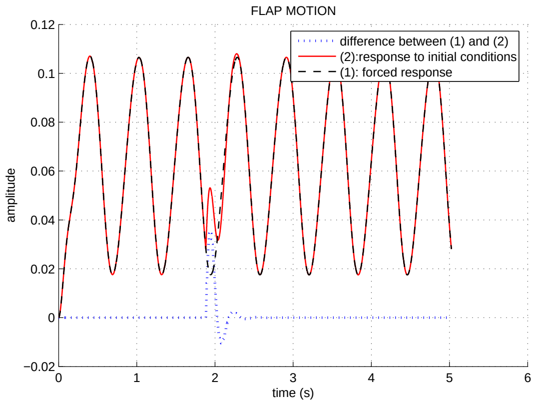

Figure 10: Time histories of the flap degree of freedom for the forced response, curve (1), initial condition response, curve (2), and difference between (1) and (2)

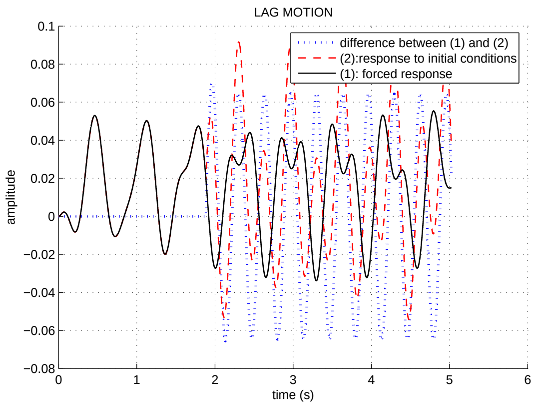

Figure 11: Time histories of the lag degree of freedom for the forced response, curve (1), initial condition response, curve (2), and difference between (1) and (2)

of all, the system, defined by the differential equations (7), must be excited in order to extract the signals used for the identification. The system of equations is solved in two steps:

- In the first step a steady, periodic solution, is found solving the system 7 with zero initial conditions. This means waiting for the system to adjust to the excitations coming from the aerodynamic and gravity forces, until a steady periodic condition is reached.
- In the second step, a perturbation in terms of the lagrangian variables β and ζ and their derivatives is assigned, making sure that the response of the system lies within the linear range, i.e. the perturbations are small compared to the steady, periodic condition.

Finally, subtracting the steady response from the initial conditions response, the perturbed transient response is obtained, suitable to be used for the identification procedure. Figure 10 and 11 show the time-histories of the flap and lag degrees of freedom for the forced case, curve (1), forced with initial conditions, curve (2), where the initial conditions are applied after 5 revolutions (periods) of the rotor, and the difference between the two curves that provides the response of the system to the initial conditions only, after the steady, periodic equilibrium condition has been reached.

The time history of the flap and lag degree of freedom perturbation with respect to the steady periodic condition are shown in figures 12 and 13 respectively.

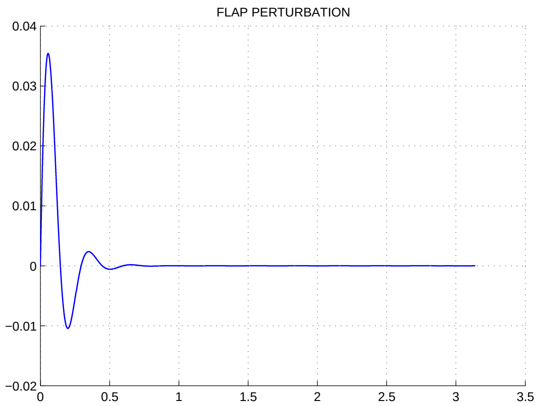

Figure 12: Time history of the flap degree of freedom perturbation with respect to the steady periodic condition

Finally the system identification procedure described in section 3 is applied to the perturbation signals eliminating the part of the signal before the initial condition was applied. The results are presented in figure 14 and 15.

As it can be seen the system is identified correctly and there is no difference between the measured and reconstructed time histories of the flap and lag degrees of freedom.

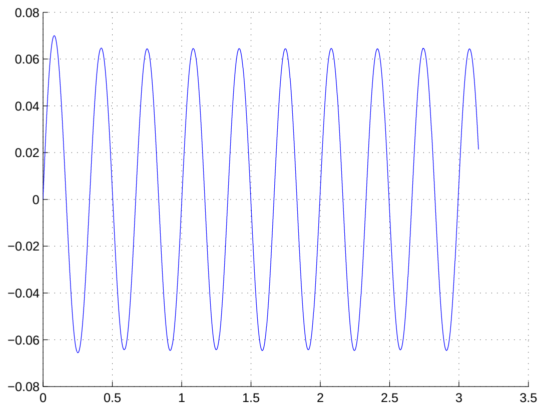

Figure 13: Time history of the lag degree of freedom perturbation with respect to the steady periodic condition

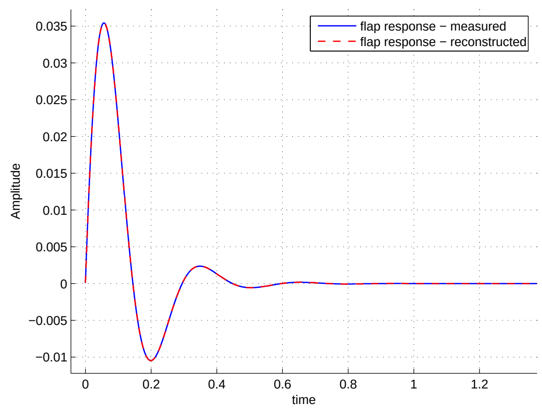

Figure 14: Comparison between the measured and reconstructed flap evolution.

6 Conclusion

A method is obtained to identify linear models with periodic coefficients from the non-linear responses which are calculated with the software package WOBBE. These models can then be used for stability analysis in the linear domain using classic vibration theory. Since the identification method is easy to use, it does not compromise the general applicability of the simulation tool WOBBE. It has been found that an increased accuracy can be obtained for the identified model when the axial induction factor is included as an added state in the case that dynamic inflow is included in the simulations with WOBBE.

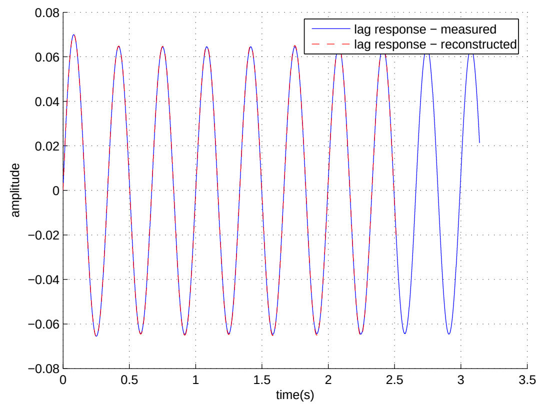

Figure 15: Comparison between the measured and reconstructed lag evolution.

The identification tool is applied to rotor data which were simulated with WOBBE. There is good correlation between the responses from the identified linear models and the original responses calculated with WOBBE. Analysis of the identified single rotor blade models shows good agreement in the trends of the modal frequencies and modal damping with the other STABCON partners. However, the identified damping for the flapwise modes is smaller when compared with the other STABCON partners due to differences in the wake model. The use of an annular wake model leads to higher damping than for the uniform induced velocity model. For the lagwise modes the overall agreement is good. The identification method also works for periodic systems since there is no difference between the time histories of the original and identified model responses for a representative example.

References

[1] Th. van Holten: *On the use of Hamiltonian Concepts in Rotor-Dynamics*, pp. II.3.5.1-6, 16th European Rotorcraft Forum, 1990, Amsterdam

[2] K. van Overbeek Th. van Holten: *WOBBE Automatic Simulation of Non-linear Dynamic Systems*, Delft University Press, October 1999

[3] J. Holierhoek Th. Van Holten Th. J. Mulder, *Automatic Simulation to Determine the Aeroelastic Stability of Large Scale Wind Turbine Rotors*,

26th European Rotorcraft Forum, The Hague, The Netherlands, September 2000.

[4] B. Marrant Th. Van Holten, *System Identification for the Analysis of Aeroelastic Stability of Wind Turbine Blades*, European Wind Energy Conference, London, United Kingdom, November 2004.

[5] D.P. Molenaar, *Cost effective design and operation of variable speed wind turbines*, PhD Thesis, Delft, 2003

[6] J. Ruh W. Schielen, *Various approaches for the Modelling of Flexible Robot Arms.* In Proceedings of the Euromech-Colloquium 219 on Refined Dynamical Theories of Beams, Plates, and Shells and their Applications, Kassel, 1986, Springer-Verlag, Berlin Heidelberg, pp. 420-429, 1987.

[7] J. Rauh, *Ein Beitrag zur Modellierung elastischer Balkensysteme.* Fortschritt-Berichte VDI, VDI-Verlag GmbH, Dusseldorf, Reihe 18: Mechanik/Bruchmechanik, Nr. 37, 126 p., 1989.

[8] W. Johnson, *Helicopter theory*, Dover Publications, Inc., 31 East 2nd Street, Mineola, N.Y. 11501

[9] P. Bongers, W. Bierbooms, S. Dijkstra, Th. van Holten, *An integrated dynamic model of a flexible wind turbine*, TU Delft, 1990

[10] D. M. Eggleston, F. S. Stoddard, "Wind Turbine Engineering Design ", *Kluwer Academic*, April 1, 1987, ISBN: 0442221959.