GROUND RESONANCE ANALYSIS METHOD FOR SKID LANDING GEAR HELICOPTER

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Abstract

Investigation of the stability on the ground single-rotor helicopter with skid landing gear, which has six degrees of freedom, blade has lagging and flapping hinges. Differential equations of the perturbed motion of the helicopter for this case are derived under the assumption that the blades is rigid, trust the rotor is zero, the angular velocity of the rotor rotation is constant. Assuming periodic motion parameters helicopter $x, y, z, \theta, \varphi, \vartheta$ *and blades* γ, β *, the differential equations are expanded in a Fourier series. When substituted in the motion equations of the helicopter stiffness matrix for different types of chassis defined unstable zones the helicopter, where continuous operation is possible structural failure.*

1. Introduction

This paper investigates the stability of the blades motion by mode of start rotating rotor on the ground has the name of ground resonance. This phenomenon appears when a helicopter rotor hub designs have begun to use the hinge connection of the blade that allows the blade to lead-lag in the rotation plane and in the flapping plane. In most cases, when the ground resonance vibrations could not stop, and they lead to the helicopter destruction. Only in rare cases, the vibration could stop a timely turning off the engine or helicopter takeoff. In the history of the helicopter were many cases when the helicopter was destroyed by the occurrence of oscillations of this type. These circumstances forced engineers to work on the development of the theory and reliable method of calculating the ground resonance phenomenon that would intelligently choose the characteristics of the structural elements that determine the stability margin of the helicopter on the ground at the design stage of the product.

A.Fayngolda work, R.Colmena, B.J.Zherebtsova, M.A.Lernera, A.I.Pozhalostina and other researchers who have examined the phenomenon of ground resonance and solved the problem of stability have contributed to the development of the theory of the helicopter stability and the choice of required damping to prevent ground resonance phenomenon.

1.1. Nomenclatures

m_F, m_l ,	–	Fuselage mass and mass of blade;
k	–	Number of blades;
I_x, I_y, I_z	–	Moment of inertia about the axes;
I_{xy}	–	Centrifugal moment of inertia;
S_v	–	Static moment of inertia relative to lagging hinge;
S_o	–	Static moment of inertia relative to axis of rotation;
I_o	–	Lag rotational inertia of blade relative axis of rotation;
I_1	–	Lag rotational inertia of blade around its center of mass;
I_v	–	Lag rotational inertia of blade relative lagging hinge;

l_2	–	Distance from axis rotation to lagging hinge;
l_0	–	Distance from the center hub to the plane of rotor.
$[C]$	–	Stiffness matrix of landing gear;
$\{W\}$	–	Column-vector displacement of helicopter
$\{F\}$	–	Forces of flex landing gear
ω	–	Rotational speed of the main rotor;
y_0	–	Distance from center of mass to axis rotation;
γ_j	–	Lead-lag angle of j[th] blade;
β_j	–	Flapping angle of j[th] blade;
$V=\sqrt{{V_{p_j}}^2+{V_{T_j}}^2}$	–	Total velocity of approach flow;
V_{p_j}	–	Component of the flow velocity in the plane of rotation of j[th] blade;
V_{T_j}	–	Component of the flow velocity perpendicular to V_{p_j};
V_{ind}	–	Induced velocity;
φ_j	–	Angle between the aerodynamic chord and the plane of rotation of j[th] blade;
α_p	–	Angle of attack of j[th] blade;
$\psi_j=\omega t+\frac{2\pi(j-1)}{k}$	–	Azimuth angle of j[th] blade.

1.2. The governing equation of motion

Using the procedure described in the writings of TsAGI [1], we obtain the motion equations of the helicopter:

$$(m_F+k\cdot m_l)\cdot\ddot{x}-k\cdot m_l\cdot e_0\cdot\ddot{\vartheta}-\left[\sum_{j=1}^{k}\begin{matrix}S_V\cdot\left((\ddot{\gamma}_j-\omega^2\cdot\gamma_j)\cdot\sin\psi_j+2\cdot\omega\cdot\dot{\gamma}_j\cdot\cos\psi_j\right)-\\-S_0\left(\left(\ddot{\phi}-\omega^2\cdot\phi\right)\cdot\sin\psi_j+\left(2\cdot\omega\cdot\dot{\phi}+1\right)\cdot\cos\psi_j\right)\end{matrix}\right]+$$
$$+K_{d,1}\cdot\dot{x}+C_{1,1}\cdot x+C_{1,2}\cdot y+C_{1,3}\cdot z+C_{1,4}\cdot\theta+C_{1,5}\cdot\varphi+C_{1,6}\cdot\vartheta=0;$$

$$(m_F+k\cdot m_l)\cdot\ddot{y}+S_0\cdot\sum_{j=1}^{k}\begin{pmatrix}\ddot{\beta}_j+\ddot{\vartheta}\cdot\cos\psi_j+\ddot{\theta}\cdot\sin\psi_j-2\cdot\omega\left(\dot{\vartheta}\cdot\cos\psi_j-\dot{\theta}\cdot\sin\psi_j\right)-\\-\omega^2\left(\vartheta\cdot\cos\psi_j-\theta\cdot\sin\psi_j\right)\end{pmatrix}+$$
$$+\sum_{j=1}^{k}B\cdot\omega\left(\dot{\vartheta}\cdot\cos\psi_j+\dot{\theta}\cdot\sin\psi_j+\dot{\beta}_j\right)+C\cdot\omega\cdot k\cdot\dot{y}+$$
$$+C_{2,1}\cdot x+C_{2,2}\cdot y+C_{2,3}\cdot z+C_{2,4}\cdot\theta+C_{2,5}\cdot\varphi+C_{2,6}\cdot\vartheta=0;$$

$$(m_F+k\cdot m_l)\cdot\ddot{z}-k\cdot m_l\cdot e_0\cdot\ddot{\theta}-\left[\sum_{j=1}^{k}\begin{matrix}S_v\cdot\left((\ddot{\gamma}_j-\omega^2\cdot\gamma_j)\cdot\cos\psi_j+2\cdot\omega\cdot\dot{\gamma}_j\cdot\sin\psi_j\right)-\\-S_o\left(\left(\ddot{\phi}-\omega^2\cdot\phi\right)\cdot\cos\psi_j+\left(2\cdot\omega\cdot\dot{\phi}+1\right)\cdot\sin\psi_j\right)\end{matrix}\right]+$$
$$+K_{d,2}\cdot\dot{x}+C_{1,1}\cdot x+C_{1,2}\cdot y+C_{1,3}\cdot z+C_{1,4}\cdot\theta+C_{1,5}\cdot\varphi+C_{1,6}\cdot\vartheta=0;$$

$$(I_x+k\cdot m_l\cdot y_0^2+\frac{k}{2}\cdot I_o)\ddot{\theta}-\ddot{\phi}\cdot I_{xy}+k\cdot m_l\cdot y_0\cdot\ddot{z}-I_o\cdot\omega\cdot k\cdot\dot{\vartheta}-\frac{k}{2}\cdot\omega^2\cdot\theta\cdot I_o+$$
$$+\sum_{j=1}^{k}\begin{pmatrix}\frac{1}{2}I_o\left(\ddot{\vartheta}\cdot\sin2\psi_j-\ddot{\theta}\cdot\cos2\psi_j\right)-S_o\left(y_0\cdot\ddot{\phi}\cdot\cos\psi_j+\ddot{y}\cdot\sin\psi_j\right)+I_o\left(\ddot{\beta}\cdot\sin\psi_j\right)+\\+2\omega\left(\frac{1}{2}I_o\left(\dot{\vartheta}\cdot\cos2\psi_j-\dot{\theta}\cdot\sin2\psi_j\right)+S_o\left(y_0\cdot\dot{\phi}\cdot\sin\psi_j\right)+S_v\left(y_0\cdot\dot{\gamma}_j\cdot\sin\psi_j\right)\right)+\\\omega^2\left(\frac{1}{2}I_o\left(-\vartheta\cdot\sin2\psi_j+\theta\cdot\cos2\psi_j\right)+S_o\left(y_0\cdot\phi\cdot\cos\psi_j+\sin\psi_j\right)+S_v\left(y_0\cdot\gamma_j\cdot\sin\psi_j\right)\right)+\end{pmatrix}+;$$
$$+C_{4,1}\cdot x+C_{4,2}\cdot y+C_{4,3}\cdot z+C_{4,4}\cdot\theta+C_{4,5}\cdot\varphi+C_{4,6}\cdot\vartheta=0$$

$$(I_y+k\cdot I)\cdot\ddot{\phi}-I_{xy}\cdot\ddot{\theta}-\sum_{j=1}^{k}\left(I_1\left(\ddot{\gamma}_j+\omega^2\cdot\gamma_j\right)-S_o\begin{pmatrix}y_0\cdot\ddot{\theta}\cdot\cos\psi_j-y_0\cdot\ddot{\vartheta}\cdot\sin\psi_j+\\+\ddot{x}\cdot\sin\psi_j+\ddot{z}\cdot\cos\psi_j\end{pmatrix}\right)+$$
$$+2\omega\cdot\left(k\cdot\dot{\phi}\cdot D-\frac{1}{2}k\cdot\omega\cdot D-\sum_{j=1}^{k}\begin{pmatrix}\dot{\vartheta}\cdot\sin(\psi_j)\cdot e_O\cdot E-\dot{x}\cdot\sin(\psi_j)\cdot E+\dot{\theta}\cdot\cos(\psi_j)\cdot e_O\cdot E-\\-\dot{z}\cdot\cos(\psi_j)\cdot E+\dot{\gamma}\cdot D\end{pmatrix}\right)+$$
$$+C_{5,1}\cdot x+C_{5,2}\cdot y+C_{5,3}\cdot z+C_{5,4}\cdot\theta+C_{5,5}\cdot\varphi+C_{5,6}\cdot\vartheta=0;$$

$$(I_z+k\cdot m_l\cdot y_0^2+\frac{k}{2}\cdot I_o)\cdot\ddot{\vartheta}-I_o\cdot\omega\cdot k\cdot\dot{\theta}-\frac{k}{2}\cdot\omega^2\cdot\vartheta\cdot I_o+$$
$$+\sum_{j=1}^{k}\begin{pmatrix}\frac{1}{2}I_o\left(\ddot{\vartheta}\cdot\cos2\psi_j+\ddot{\theta}\cdot\sin2\psi_j\right)-S_o\left(y_0\cdot\ddot{\phi}\cdot\sin\psi_j+\ddot{y}\cdot\cos\psi_j\right)+I_o\left(\ddot{\beta}_j\cdot\cos\psi_j\right)+\\+2\omega\left(\frac{1}{2}I_o\left(\dot{\vartheta}\cdot\sin2\psi_j-\dot{\theta}\cdot\cos2\psi_j\right)-S_o\left(y_0\cdot\dot{\phi}\cdot\cos\psi_j\right)-S_v\left(y_0\cdot\dot{\gamma}_j\cdot\sin\psi_j\right)\right)+\\\omega^2\begin{pmatrix}\frac{1}{2}I_o\left(\vartheta\cdot\cos2\psi_j+\theta\cdot\sin2\psi_j\right)+S_o\left(y_0\cdot\phi\cdot\sin\psi_j-y_0\cdot\cos\psi_j\right)+\\+S_v\left(y_0\cdot\gamma_j\cdot\sin\psi_j\right)\end{pmatrix}\end{pmatrix}+$$
$$+C_{4,1}\cdot x+C_{4,2}\cdot y+C_{4,3}\cdot z+C_{4,4}\cdot\theta+C_{4,5}\cdot\varphi+C_{4,6}\cdot\vartheta=0;$$

$$I_v\cdot\ddot{\gamma}_j+I_1\cdot\ddot{\varphi}+\omega^2\cdot(-\phi\cdot I_1-I_v\cdot\gamma_j)-S_v\cdot\begin{pmatrix}(y_0\cdot\ddot{\theta}+\ddot{z})\cdot\cos\psi_j+\\+\left(y_0\cdot\ddot{\vartheta}+\ddot{x}\right)\cdot\sin\psi_j\end{pmatrix}+$$
$$+k_2\cdot\dot{\gamma}_j+C_b\cdot\gamma_j+2\omega\cdot\begin{pmatrix}\dot{\theta}\cdot\cos(\psi_j)\cdot y_0\cdot E+\dot{\vartheta}\cdot\sin(\psi_j)\cdot y_0\cdot E-\\-\dot{x}\cdot\sin(\psi_j)\cdot E-\dot{z}\cdot\cos(\psi_j)\cdot E+\\+\dot{\phi}\cdot D+\dot{\gamma}_j\cdot D+\frac{1}{2}\omega\cdot D\end{pmatrix}=0;$$

$$I_o\cdot\ddot{\beta}_j+I_o\cdot\begin{pmatrix}\ddot{\vartheta}\cdot\cos\psi_j+\ddot{\theta}\cdot\sin\psi_j-2\cdot\omega\left(\dot{\vartheta}\cdot\sin\psi_j-\dot{\theta}\cdot\cos\psi_j\right)-\\-\omega^2\left(\vartheta\cdot\cos\psi_j+\theta\cdot\sin\psi_j\right)\end{pmatrix}+$$
$$+\omega\left(-\dot{\theta}\cdot\sin(\psi_j)\cdot A-\dot{y}\cdot B-\dot{\vartheta}\cdot\cos(\psi_j)\cdot A-\dot{\beta}_j\cdot A\right)=0,$$

(1.2.1)

where

$$A=\frac{c_y^\alpha\cdot\rho}{2}\int_{l_0}^{R}r^3bdr;\ B=\frac{c_y^\alpha\cdot\rho}{2}\int_{l_0}^{R}r^2bdr;\ C=\frac{c_y^\alpha\cdot\rho}{2}\int_{l_0}^{R}rbdr;$$
$$D=\frac{\rho}{2}\int_{l_0}^{R}c_{xp}r^3bdr;\ E=\frac{\rho}{2}\int_{l_0}^{R}c_{xp}r^2bdr.$$

The system (1.2.1) is the complete system motion equations of the helicopter on an elastic foundation with rigid blade connected to the hub flapping and lagging hinge, with the influence of the aerodynamic forces acting on the rotor blades.

The system (1.2.1) has been obtained on the basis of the hypotheses used in [1], but with these equations we get a closed system of equations, since they were obtained in our view unjustified exclusion of some terms of the equations in the derivation of the basic equations for the blades.

1.3. Stability analysis of helicopter

To analyze the stability of the helicopter during the promotion of the rotor, there are several methods. A common method is the R.Coleman and B.Ya.Zherebtsova [2], which is widely used for the analysis of simplified systems.

We obtained for the system (1.2.1) is not possible to use the method R.Coleman and B.Ya.Zherebtsova, so to analyze the stability of the method of expansion of differential equations in a Fourier series. The solution of the system (1.2.1) reduces to the solution of algebraic equations with constant coefficients. This can be done by expanding the parameters $x, y, z, \theta, \varphi, \vartheta, \gamma_j, \beta_j$ of the Fourier series, assuming their periodic:

(1.3.1) $$F_i = \frac{1}{2}a_{i,0} + \sum_{n=1}^{nq}(a_{i,n}\cos(n\cdot\omega\cdot t) + b_{i,n}\sin(n\cdot\omega\cdot t)); F_i = x,..,\beta_j;$$

Substituting (1.2.2) into the helicopter equation of motion, we get the vector equations with unknown coefficients $a_{i,n}$ and $b_{i,n}$ in the form of:

(1.3.2) $$\{F_i(a_{i,n}, b_{i,n})\} = 0, i = \overline{1,k}, n = \overline{1,N}\ .$$

Each i^{th} equation as a real periodic function with period T can be expanded in a Fourier series:

(1.3.3) $$F_i(t) = \frac{1}{2}zz_{i,0} + \sum_{n=1}^{nq}(zc_{i,n}\cos(n\cdot\omega\cdot t) + zs_{i,n}\sin(n\cdot\omega\cdot t)) = 0\cdot$$

Where

$$zz_{i,0} = \frac{\omega}{\pi}\cdot\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} F_i dt = 0;\quad zc_{i,n} = \frac{\omega}{\pi}\cdot\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} F_i\cdot\cos(\omega\cdot t)dt = 0;$$

$$zs_{i,n} = \frac{\omega}{\pi}\cdot\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} F_i\cdot\sin(\omega\cdot t)dt = 0.$$

The system of equations of motion can be written in the general form:

(1.3.4) $$\{z_i\} = \left[z_{i,0}, zc_{i,1}, zs_{i,1}, ..., zc_{i,n}, zs_{i,n}\right]^T.$$

Expressing a system of equations (4.22) for the coefficients of the Fourier series can be written in matrix form:

(1.3.5) $$[\Phi]\begin{Bmatrix} a_{i,n} \\ b_{i,n} \end{Bmatrix} = Q,$$

where $[\Phi]$- the characteristic matrix, which in this case describes the properties of the oscillations of the form "Ground" resonance.

The motion equations were obtained using the software package Maple, it is a written program for the formation of the matrix. In the future, for more convenient and fast work with the matrix and output the results to have been written in the program package Matlab. Calculation on a modern computer is no more than two minutes.

1.4. Results of calculation

The initial data for program testing and calculations were chosen skid landing gear, used in the design of the helicopter ANSAT. Was selected three possible the landing gear: composite, high steel and staffing steel operated by helicopter ANSAT.

Mathematical model of the landing gear in the equations represented by the matrix compliance, which defines its stiffness characteristics as the characteristics of the elastic foundation. To construct the matrix compliance skid landing gear using a software package ANSYS, in which a finite element model. Loading sequence design matrix is formed compliance.

The results are compared with the results of the method R.Coleman and B.Ya.Zherebtsova [2], where the problem of the plane motion stability of a mechanical system, mounted on an elastic foundation, which is the hub of heavy blades fixed on her with lagging hinge with the dampers.

Comparison of the results shows satisfactory convergence of the results, as shown in Figures 1.4.1 – 1.4.3. In the figures, there are areas that are by R.Coleman and B.Ya.Zherebtsova not covered due to lack of flapping blades.

Considered a composite landing gear (see Figure 1.4.1), which has three areas of instability: 13-20 rad/sec, 34-42 rad/sec, 47-59 rad/sec. Operating speed of the main rotor fall into the second area, which is illegal and can lead to the destruction of the helicopter. The helicopter on the high steel landing gear (Figure 1.4.2), there are two areas of instability: 14-22 rad/sec, 33-47 rad/sec. In this case, the area also gets into the zone operating speed of the main rotor. On staffed landing gear (Figure 1.4.3) there are two areas of instability: 22-33 rad/sec, 56 and above rad/sec. Operating speed of the main rotor are located away from areas of instability. Ground resonance at preset rpm, the first region of instability is at a high speed rotor, which does not lead to significant problems in the promotion of the main rotor.

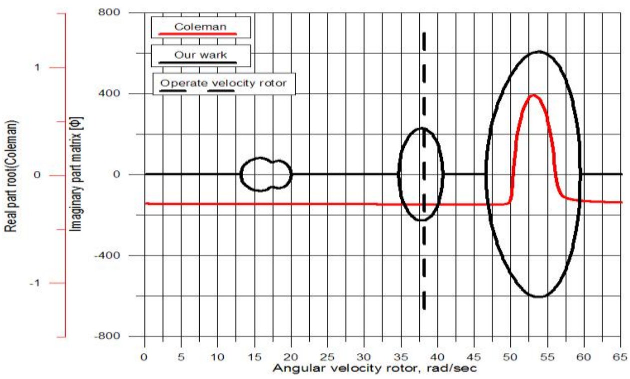

Fig.1.4.1. Stability analysis of helicopter R.Coleman method and a method for composite landing gear

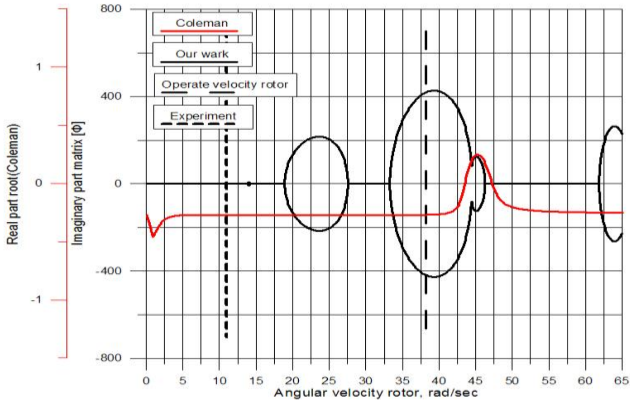

Fig.1.4.2. Stability analysis of helicopter R.Coleman method and a method for high-steel landing gear

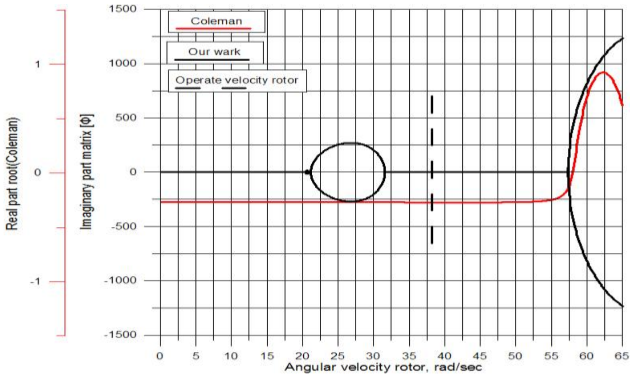

Fig.1.4.3. Stability analysis of helicopter R.Coleman method and a method for staffing landing gear

2. Modeling start rotating helicopter rotor in a software MD Adams

The paper deals with modeling of start rotating rotor on elastic foundation (Fig. 2.2) using the software MD.Adams. We compare the results of Adams with the results obtained by the simplified mathematical model calculation, as described in [2].

MD.Adams is an integral part of the software product family MD (MD.Nastran) and is intended for complex modeling of complex mechanical system, create virtual prototypes for virtual testing.

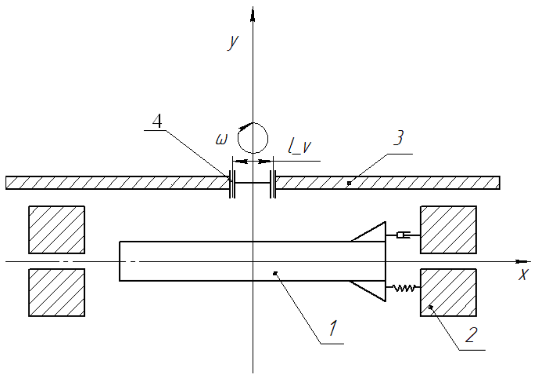

Fig. 2.1. Simplified model of promotion rotor on elastic foundation

(1 - fuselage, 2 - support, 3 - blade, 4 - lagging hinge)

In MD.Adams built a solid model, where the elastic base is a spring (Fig. 2.2). The cylinder is the main rotor shaft and is the axis of rotation. The box is a hub with a width equal to resound lagging hinge.

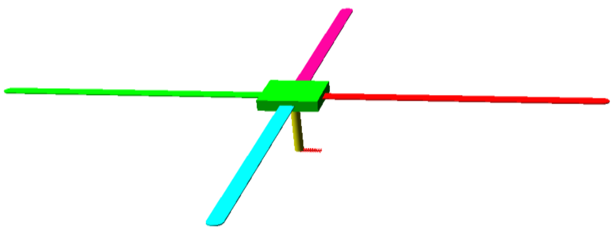

Fig. 2.2. Scheme of representing a simplified model helicopter in MD.Adams

In the joint hub and blades fitted lagging hinge and springs in torsion. The center of mass is put a damper spring with the longitudinal axis, and set limits on the movement in all directions except the cross.

The initial data is the mass-inertial characteristics of the body and blades of the helicopter, as well as the stiffness characteristics of the elastic foundation and lagging hinge, which are derived from the calculation of natural frequencies of helicopter on the landing gear. According to their own frequencies, determined the most dangerous frequency, affecting the stability. These values are chosen frequency of the spring in the center of mass of the helicopter used in the model as input.

2.1. Results of calculation

The initial data for the calculations are the mass-inertia and stiffness characteristics of the helicopter Ansat.

On the stage of the calculation is required to determine the number of steps per unit of time sufficient to produce a reliable result. Screw spun constant acceleration to the angular velocity of 65 rad/sec for 50 seconds, while making 50 000 time steps. Research has proven that the minimum number of points for the shortest period of oscillation should be 20-50, which will provide enough reliable solution. Otherwise, the solution can disperse.

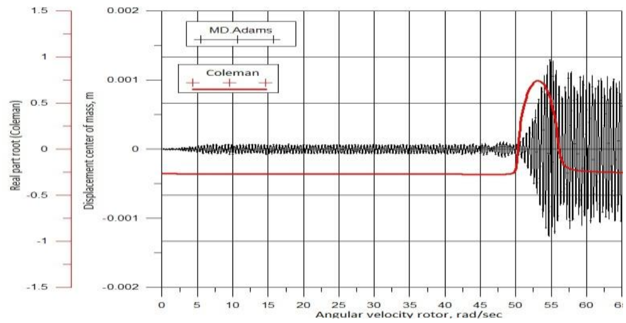

Fig. 2.3. Instability zones by the method [2] and MD.Adams

2.2. Conclusions

To analyze the stability start rotating rotor of the helicopter was considered a complete system of motion equations of the helicopter. A distinctive feature was the consideration of the motion of the main rotor blades in two planes, accounting for all six degrees of freedom of the helicopter fuselage, and the study of various options skid landing gear.

Method to analyze the stability of the helicopter by expansion differential equations, Fourier series shows the instability of oscillations of the earth's resonance.

The calculation results in a package MD.Adams with the previously used method shows satisfactory agreement, which can be seen from Fig. 2.3. This means that MD.Adams reveals vibrations such as ground resonance.

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