# Free Mesh Morphing Optimisation applied on Composite Stiffened Panels under Stability and Strength Constraints

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## ABSTRACT

*The paper presents a novel free mesh morphing technology based on Moving Least Square (MLS) applied to Structural Optimisation. The proposed approach moves from the field of surface reconstruction from 3D scattered data. From a more general standpoint, MLS methods seem promising methodologies to solve different morphing problems where existing meshes can be modified without specific needs to change their topology, i.e. their connectivity information. In this respect, MLS is a very effective and promising methodology for mesh morphing.*

*The proposed MLS morphing methodology has been applied to the optimisation of composite stiffened panels. The goal of the optimisation is to reduce the overall panel weight finding the best layups (thickness and percentages) for skin and stringers as well as the optimal shape for the stringers via MLS morphing, considering stability and strength constraints. The optimisation process acts on the shape of the stringers, via the proposed MLS approach, without requiring any remeshing towards the optimisation process. Nonlinear Finite Element analyses are used to predict the overall behaviour of the panels in terms of force vs. shortening curve up to final failure, discriminate between local skin instabilities and global ones, eventually leading to the overall failure of the structure. Strength criteria are additionally accounted by monitoring the maximum Tsai-Wu failure index overall the structure.*

## INTRODUCTION

Composite stiffened panels are nowadays widely used in primary lightweight structures: airplane wing covers, fuselage panels, helicopter tails and tailplanes are but few examples. Recent works underlined how the use of composite assemblies has been crucial in the structural design of modern tiltrotor structures requiring complex optimization processes since several constraints including those related manufacturing and dynamic stability have to be accounted for [1].

Optimisation of stiffened composite panels subjected to buckling and strength constraints has been dealt with several studies in the last decades. Among the first studies on this field a minimum weight design was performed by Butler and Williams [2] using VICONOPT; other minimum weight optimisations including buckling load constraints were proposed by Wiggenraad et al. [3]. Often, the presence of local optima and integer variables, like the number and the orientation of the layers, made the use of the genetic algorithms (GA) appealing in optimisation involving composite structures [3,4,5,6]. As an example, Kaletta and Wolf [7] applied a parallel computing GA, considering buckling and maximum strength constraints, to stiffened composite plate panels. The fitness evaluation was performed using directly eigenvalue finite element analyses.

However GA may require a higher number of function evaluations to converge to the optimum than other gradient-based or direct search algorithms. Attempt to reduce computational efforts have been made by using response surface techniques. As an example multi-objective GA were used together with response surfaces to optimise composite stiffened panels to working in post-buckling field [8].

The work herein presented moves from this literature review and tries to identify the optimal design of a stiffened composite panel by changing not only laminate thicknesses and percentages but also the geometry of the stringers, fully modelled in the FE models. Such approach will normally require to re-mesh the panel for each analysed configuration. Instead, the paper presents a non-linear morphing technology based on Moving Least Square approximations to morph the mesh of the stringers.

At first the Free Mesh Morphing technique is reviewed and applied to a typical stiffened panel to control the geometry of the stringers. Then a typical optimisation problem is presented and solved by coupling the MLS morphing technique with an effective Simplex (Nelder-Mead [9]) optimisation procedure enhanced using a constraint vs. objective ranking to account for non-linear constraints.

## FREE MESH MORPHING

Strictly speaking Free Mesh Morphing is a general purpose technique used to deform an existing mesh - or

better the nodes upon which mesh connectivity is defined - without requiring explicit information on the geometry initially used to create the mesh.
More in particular Free Mesh Morphing allows to modify the position of a set of target nodes in the space by defining a continuous and smooth deformation field to be added to the original position of the nodes. Morphing algorithms can thus be distinguished based on the way they define the deformation field to be applied.
In the structural field three main schemes have been used:

- **Linear scheme**: in this scheme the deformation field is defined by a linear combination of linear deformations spanned wise a set of controlling handles. This technique is known to be used by MSC and Altair in their size and shape tools. Generally speaking this technique is intuitive and easy to use as it produces very predictable deformation fields. It is also computationally effective but as main disadvantage it often leads to broken meshes;
- **non-linear scheme**: these have been more recently introduced in many research activities [10-14]: Radial Basis Function (RBF) and MLS approaches seem to be the more commons one. Despite being more computational expensive and sometimes less intuitive to use, they outperform the linear scheme with respect of smoothness and final quality of the deformed mesh. Both RBF and MLS can also be adapted to perform linear interpolation by properly down-selecting their inner interpolation functions.

More in particular this paper focuses on the use of MLS as available in Shaper [15] - a commercial software for general purpose size-and-shape optimisation.
From a general standpoint, MLS techniques belong to the field of data approximation form a scattered set of points. Applications of MLS can be found many different fields: computer graphics and visualization, image processing, regression models, supervised learning, mesh-free finite element methods are but few examples [10-14].
The following section briefly outlines the fundamentals of MLS approximation methods. More detailed information can be found in [14].

### *Moving Least Square:*

The Moving Least Square technique can be used to solve any function approximation problem. The problem can be formulated as follows:

Given N points located at positions $\mathbf{x}_i \in \Re^3$ where $i \in [1,\ldots,N]$, find a globally defined function $p(\mathbf{x})$ that approximates a set of given values $f(\mathbf{x}_i)$.

In order to have the maximum versatility, no specific structure is supposed in the data point distribution $\mathbf{x}_i$, which can be considered scattered in the space $\Re^3$.
While this problem seems very abstract, it may be easily applied to mesh morphing. For instance, by solving function approximation it is possible to find the function that represent the mesh nodes displacement field given the displacement of few points, the controlling handles.

A classical approach to solve this problem is represented by the Least Square that looks for the solution of the following minimization:

$$\min_{p \in \Pi_m} \sum_i \left(p(\mathbf{x}_i) - f(\mathbf{x}_i)\right)^2 \qquad (1)$$

The real problem here is to find the appropriate functional space for the function $p(\mathbf{x})$ able to be a good approximation of the real function $f(\mathbf{x})$ globally. Usually the space of polynomial of maximum degree *m* is taken.
To avoid the necessity to find a functional approximation space that is valid globally, Lancaster and Salkauskas proposed the MLS technique [14]. The idea is to perform individually a Weighted Least Square for each point $\mathbf{x}$ in the domain. So, the global approximation $p(\mathbf{x})$ is obtained from the union of a series of local functions i.e. $p(\mathbf{x}) = p_{\mathbf{x}}(\mathbf{x})$, where the functions $p_{\mathbf{x}}(\mathbf{x})$ are obtained as solution of the following minimization problem:

$$\min_{p_{\mathbf{x}} \in \Pi_m} \sum_i \left(p_{\mathbf{x}}(\mathbf{x}_i) - f(\mathbf{x}_i)\right)^2 \theta\left(\left\|\mathbf{x} - \mathbf{x}_i\right\|\right) \qquad (2)$$

The function $\theta$ is a non-negative weight function that depends only from the Euclidean distance between points in the solution space. The approximation is localized as much as possible if $\theta(r)$ is rapidly decreasing as $r \to \infty$. Extremely useful as weight functions are the compact support RBF functions suggested by Wendland [16].
Then, the approximation problem with MLS is localized in two ways:

- first, a local polynomial fit is evaluated continuously over the entire domain; each point $\mathbf{x}$ has is own local approximation.
- second, the number of points $\mathbf{x}_i$ that effectively have an influence on the local polynomial representation at $\mathbf{x}$ is kept limited by using RBF weight functions (better if with compact support).

By looking at Eq. (2) it is easy to see that if $\theta(0) \to \infty$ the MLS fit will be forced to interpolate the prescribed values at known points $f(\mathbf{x}_i)$.
Of course, the MLS approach is definitely more expensive than other approach because for every point $\mathbf{x}$ where the solution must be computed it is required to solve a Linear System of Equations to find the minimum

of Eq. (2). However, the size of these LSEs  can be kept small by using small support dimension for the weight functions.

In order to solve the MLS problem the user has to take a decision with respect to these parameters

1. The functional space for the local approximation functions $p_{\mathbf{x}}(\mathbf{x})$. Usually the functions are polynomial, so the user must decide the maximum polynomial order.
2. The type of weight functions. This choice may have an influence on the smoothness of the obtained approximation (see [13]).
3. The size of the local support of the weight function. Usually a good strategy is to fix the number of points that must belong to each support, and then adapt the local size accordingly. In this way a natural adaptation of the algorithm is realized, since where there are few spread points the support is large, while for more dense areas the support becomes smaller.

### *Samples:*

This section presents a first example on how MLS morphing techniques can be used to modify the shape of a very simple FE mesh: a cube made of bricks (8-nodes solid elements). The deformation field is defined by using 8 control points (so called free handles in Shaper) located on the 8 vertices of the cube.

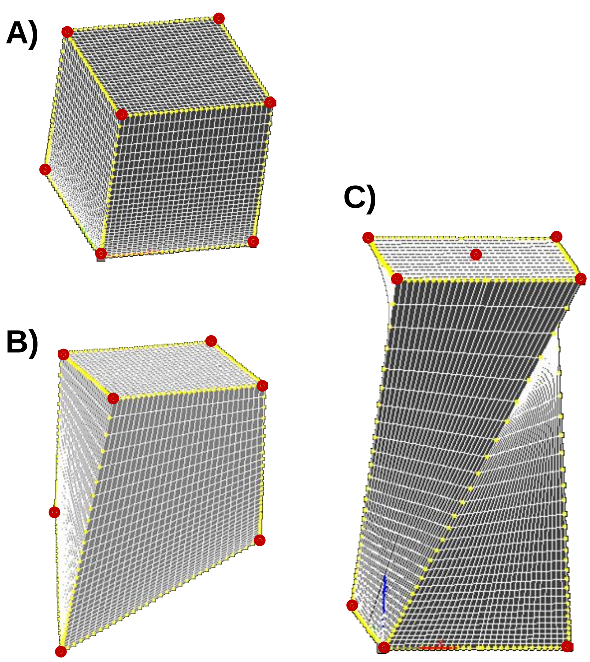


*Fig. 1: a first example of MLS morphing: a) original mesh; b) single handle movement; c) combined handle movements.*

Figure 1.A) presents the original FE mesh of the cube together with 8 controlling handles located at the vertices. In Fig.1.B) a handle on the bottom-face is translated and the cube deformed accordingly; finally in Fig. 1.C) a new handle is added on the upper face of the cube and it is linked (by means of a rigid constraint) to the other 4 top-face handles. The handle is then translated vertically and rotated of 90 degrees, the mesh of cube following the deformation imposed by the handles. The same configuration of handles would as an example allow to inflate and to contract the cube as well as to deform it to become a more general parallelogram.

It is worth noticing that:

- a relative small number of handles has been required to apply a deformation field overall the model;
- the deformation field works by deforming elements the more they are closer to a moving handles. This is exactly what we expect from a morphing algorithm assuring the mesh to remain almost untouched in those regions where no handles are moved;

handles can be hierarchical combined to assure rigid transformations and, if required, more morphing operators can be superimposed using different controlling handles.

## STIFFENED PANEL CONFIGURATION:

The Free Mesh Morphing technique described in the previous section is applied to the optimisation of a composite stiffened panel assembly representing a typical wing cover section.

Clearly, the scope of the work is not the actual design of a real-world composite wing cover - requiring to consider many different design criteria - but mainly an attempt to explore the advantages of free mesh morphing and optimisation techniques in the preliminary design phases. In this respect, a simplified design approach has been followed attempting to minimise the weight of the structure against buckling and strength criteria.

As reported in Fig. 2, the considered structure consists of a low-curvature stiffened panel 2000*mm* wide and 1750*mm* long, with a curvature radius of 15700*mm*. Five stringers with back-to-back C cross section are equally spaced with a pitch of 250*mm*.

A central strip of the panel skin - corresponding to the rib landings - has been constrained not to translate vertically as well as the lateral edges of the structure corresponding to the spar landings. Reinforcement pad-ups have been added to these areas by increasing the nominal thickness of the panel skin.

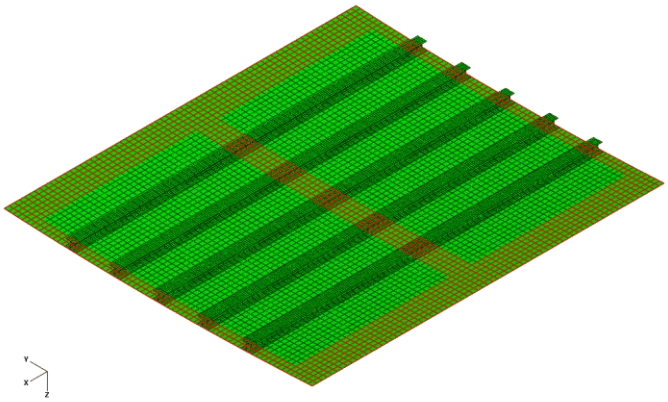

*Fig. 2: considered structure assembly: a two-rib composite stiffened panel.*

The panel is made of a unidirectional CRFP material whose nominal mechanical properties are reported in Tab. 1 below.

| Description | Value |
|---|---|
| Elastic modulus $E_{11}$ | 13500*[N/mm²]* |
| Elastic modulus $E_{22}$ | 8500 *[N/mm²]* |
| Poisson coefficient $\nu_{12}$ | 0.35 |
| Shear modulus $G_{12}=G_{13}$ | 4200 *[N/mm²]* |
| Shear modulus $G_{23}$ | 3150*[N/mm²]* |
| Compression strength X | 1280*[N/mm²]* |
| Compression strength Y | 250*[N/mm²]* |
| Tension strength X | 2210*[N/mm²]* |
| Tension strength Y | 75*[N/mm²]* |
| In plane shear strength | 95*[N/mm²]* |
| Nominal ply thickness | 0.30*[mm]* |
| Density | 1600*[Kg/m³]* |

*Tab. 1: mechanical properties of the stiffened panel's material.*

The rationale is to optimise not only the thicknesses of panels and stringers but also the cross section of the latter. After a brief discussion on the adopted FE model, two different morphing options are presented: the first one uses a reduced number of control handles to shape the stringer cross section to remain constant span-wise; the second one exploits the non-linear behaviour of the MLS approximations to shape the foot and head flanges in a smooth way.
Whilst the second morphing approach has been used to underline the capabilities of MLS, the first has been selected to perform the optimisation described in the following. This choice is motivated not only by the need to contain as much as possible the number of controlling handles and - as a consequence - of design variables during the optimisation search, but also by other manufacturing and general cost related considerations.

### *Finite Element Model:*

Finite Element Analyses have been performed to predict the behavior of the structure assembly using Abaqus [17].
All parts of the structure have been modeled using laminated shell elements with four nodes (S4R) with six degrees of freedom at each node and a single integration points throughout the thickness for each generalized composite ply. The elements used to model the skin have characteristic dimension of about 25*mm*, resulting in a total number of 4900 elements. Each stinger is modeled using 1260 shell elements: 4 elements on each side of the foot flanges 3 elements on each side of the upper ones and 4 elements along the web. Linear material models are used.
The connection between the skin and the stringer feet has been modeled using tie contacts so to allow the nodes of the foot flange to change their position, as a result of the morphing, without needs to modify the mesh of the skin.
Boundary conditions have been defined to account for spars and ribs: vertical translations of nodes corresponding to spars and ribs landings have been constrained not to translate perpendicular to their plane.
One end in the span-wise direction has been constrained against longitudinal translation; the other is loaded by applying a uniform displacement.

Two different analyses are performed on each configuration undergoing the optimisation process:

- **eigenvalue**: eigenvalue FE analyses are performed to obtain the first bucking load of the structure. Eigenvalue analyses require an average CPU time of about 3 minutes;
- **non-linear implicit**: displacement controlled implicit FE analyses have been performed to obtain the load vs. shortening curve as well as structure strength via the Tsai-Wu failure criteria or global instabilities leading to the premature collapse of the structure. Non-linear analyses require an average CPU time of about 30 minutes.

It may be argued why performing an eigenvalue analysis followed by a non-linear one, being the latter capable to predict first buckling load as well. The reason is mainly related to the need of performing optimisation searches. In this respect, the difficulties of estimating the first buckling load through the non-linear analysis are overcome simply looking at the obtained eigenvalues.
Figure 3 shows a typical force vs. displacement curve up to the geometrical collapse with the corresponding first buckling load as obtained by the eigenvalue analysis. Tsai-Wu strength criteria is reported as well.
Fig. 4 and Fig.5 show related local and global buckling patterns.

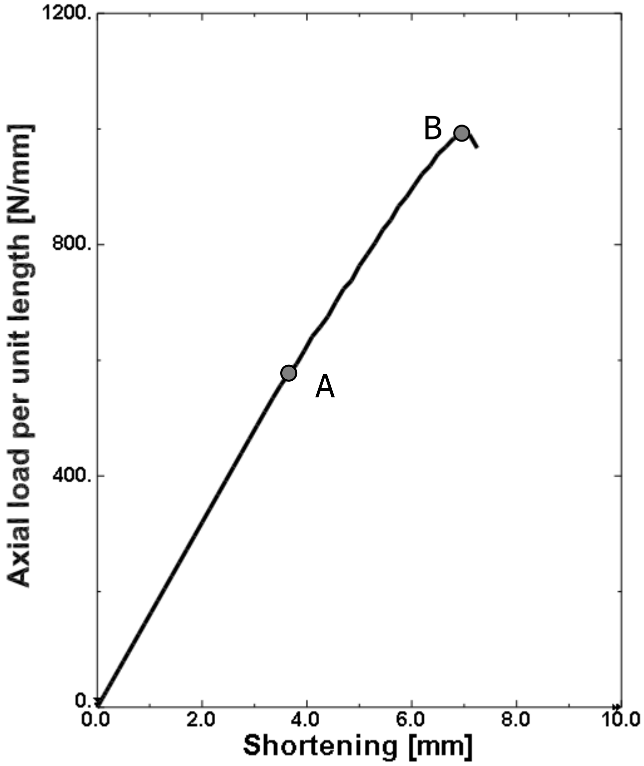


*Fig. 3:* *typical force vs. shortening curve with first buckling load from related eigenvalue analysis marked as A).*

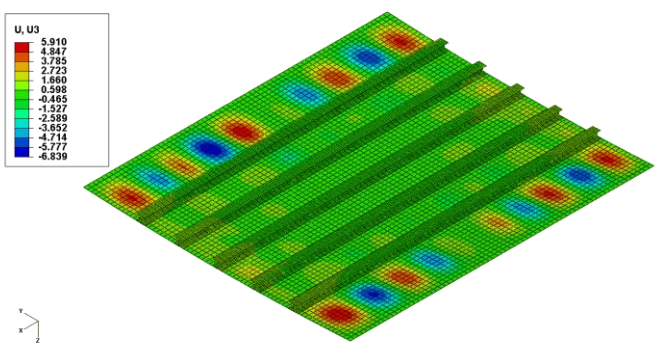

*Fig. 4:* *first buckling load corresponding to point A) in Fig 3 above.*

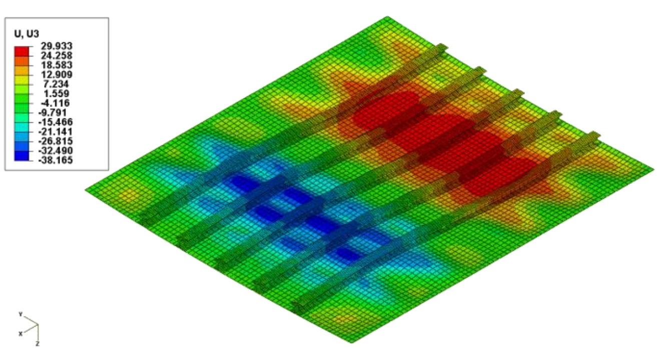

*Fig. 5:* *structural collapse due to global geometrical instability corresponding to point B) in Fig 3 above.*

### *Mesh Morphing:*

This section presents how the proposed MLS morphing technology can be used to control the shape of the stringers.
Two different examples are presented: the first aims at having constant stringer sections span-wise with straight flanges; the second one is an extension of the previous and attempts to alter the foot flange width in a continuous way along the ribs. The first example will be used as morphing scheme in the optimisation searches whilst the second should be considered an example of the advanced morphing capabilities made available by the proposed MLS approach.

***Constant span-wise cross section:*** within this scenario, the shape of a stringer should be changed so to have constant cross section. More in particular foot and head flanges should be inflated and deflated. This is achieved by defining 14 handles per stringer: 7 at each end as shown in Fig. 6 below. The handles are moved in the stringer cross-section plane, the handles on the rear section follow the movement of the front ones so to assure the stringer section to remain constant span-wise.

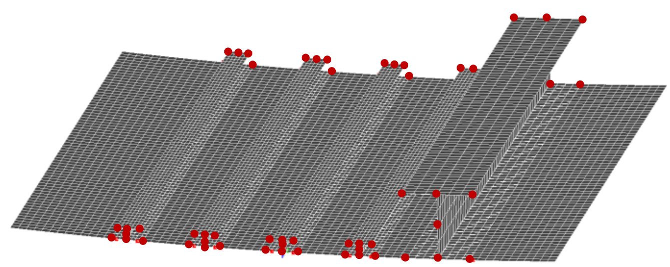

*Fig. 6:* *stringer morphing: constant span-wise cross sections.*

As shown in Fix. 6, each stringer can be morphed independently from the others. However the same stringer cross-section will be enforced during the optimisation process.

***Variable span-wise cross section:*** this second example should be considered as an extension of the previous one. The idea is to free-up the stringer shape to have a non-constant and smooth (i.e. non-linear) cross section span-wise. This is achieved by adding more controlling handles.
Figure 7shows an example of how the stringer foot can be "inflated" in the rib-bay assuring a constant and given width on the rib-landing areas. The stringer top flange is morphed in a similar way in Fig. 8.

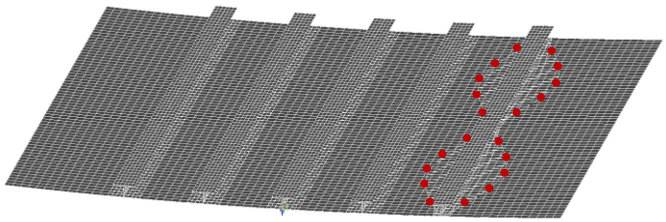

*Fig. 7: stringer morphing: smooth span-wise cross sections - a first configuration.*

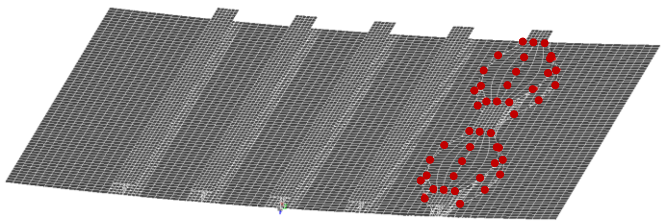

*Fig. 8: stringer morphing: smooth span-wise cross sections - a second configuration.*

It is worth noticing that in both cases a smooth morphing has been achieved preserving the quality of the mesh. The relatively large number of handle used to achieve the morphing may be seen as a drawback, however it is recalled that - at least within Shaper - hierarchies of handles can be defined so to possibly reduce the number of parameters to be controlled by during an optimisation process.

## STRUCTURAL OPTIMISATION:

The goal of the proposed optimisation is to reduce the overall panel weight finding the best layup(thickness and percentages) forskin and stringers as well as the optimal shape for the stringers via MLS morphing considering stability and strength constraints. In this respect, the optimisation process acts both on the shape of the stringers via the proposed MLS approach and on the thickness and the lay-ups of the stringers and the panel skin.
Despite this kind of optimisation problems is not new in literature [1-8] and it is often considered in the day-to-day industrial design, no works have been found at the best knowledge of the authors using MLS methods coupled with an optimisation engine in the attempt to simultaneously optimise size and shape together with laminate layups. Optimisations have been performed using Nexus [18]

### *OptimisationProblem:*

The optimisation problem has been formulated to minimise the panel weight assuring the panel not to buckle before the limit load is reached and not to fail (Tsai-Wu criteria or global instability) before the ultimate load.
From a mathematical standpoint the optimisation problem follows below:

$$\begin{cases} \min\limits_{\vec{x}\in D}\left(W\left(\vec{x}\right)\right) \\ F_{CR}\left(\vec{x}\right)\geq F_{Limit} \\ F_{TsaiWu=1}\left(\vec{x}\right)\geq F_{Ultimate} \end{cases} \tag{3}$$

Even if it is recognized that the proposed approach is a simplified one, it is also believed to represent typical aerospace design conditions.

***Design variables:*** design variables can be grouped in two main sets: variables that control the shape of the stringers, i.e. the position of the handle in the morphing process, and variables that control the thicknesses and the lay-up of panel skin and stringers.
As far as the stringer shape in concerned, Tab. 2 reports the range of variation of the flanges and of the stringer height being the shape controlled via 3 displacements of the controlling handles: one to stretch/shrink the upper flanges, one to stretch/shrink the foot flanges and the last one to stretch/shrink the height of the web.

| Description | Min | Max |
|---|---|---|
| Stringer height *[mm]* | 23.5 | 82.5 |
| Stringer half-foot width *[mm]* | 12.5 | 62.5 |
| Stringer half-head width | 12.5 | 62.5 |

*Tab. 2: optimisation variables: stringer cross-section domain of interest.*

The optimisation of the composite laminates forming the panel skin and the stringer sections has been performed considering overall equivalent thicknesses orientation-by-orientation. Symmetric and balanced laminates are ensured.
Table 3 reports the considered variables and their domains of interest.

| Description | Min | Max |
|---|---|---|
| Stringer 0 ply thickness *[mm]* | 0.75 | 5.00 |
| Stringer 90 ply thickness *[mm]* | 0.75 | 5.00 |
| Stringer +/-45 ply thickness*[mm]* | 0.75 | 5.00 |
| Skin 0 ply thickness *[mm]* | 0.50 | 5.00 |
| Skin 90 ply thickness *[mm]* | 0.50 | 5.00 |
| Skin +/-$\alpha$ ply thickness*[mm]* | 0.50 | 5.00 |
| Skin $\alpha$ ply orientation *[degrees]* | 20.0 | 80.0 |

*Tab. 3: optimisation variables: composite laminates.*

It is stressed that an additional degree of freedom is introduced in the design of the skin laminate: layers at an arbitrary orientation of $\alpha$ can be introduced in laminate. When $\alpha$ is set to 45 degrees classical [0/+45/-45/90] staking sequences are obtained.

***Geometrical constraints:*** few geometrical constraints have been added among the design variables to restrict the feasibility region of the optimisation domain to more realistic solutions:

- the width of the head flanges should be less than the foot flange;
- the overall panel thickness should range between 3.0*mm* and 7.0*mm*.
- the overall thickness of the stringer flanges should range between 2.5*mm* and 6.5*mm*, leading to a total web thickness ranging between 5 and 13*mm*.

***Strength requirements:*** as aforementioned the panel should be designed to assure minimum bucking load and minimum failure one.
This is achieved within the optimisation procedure by adding three additional constraints:

- the first buckling load (obtained via an eigenvalue analysis) should be greater than a threshold value representing the limit load;
- the load that makes Tsai-Wu criteria to be 1.0 either in the stringers or in the skin should not be less than the ultimate load;
- collapse due to non-linear instabilities should not be less that the ultimate load.

## *Optimisation Results:*

The optimisation process described in the previous section is continuous and therefore solvable via gradient-based optimisation algorithms, however the need of using a finite-difference scheme to evaluate derivatives of objective and constraint functions with respect of the 10 design variables and the possibility to remain trapped in local minima make the use of gradient-based algorithm less appealing.
On the other side, the optimisation problem as formulated above appears to be continuous and smooth at least with respect of the design variables even if the response of the structure may turn out to be non-linear and non-smooth due to instabilities and relevant stress concentrations.
Overall, the previous considerations (relatively small number of continuous design variables but accurate derivatives and possibly non-smooth constraint responses) make the Nelder-Mead method an attractive alternative to tackle this optimisation problem. The main drawback of the Nelder-Mead [9] method is that in its original formulation it is unable to deal with constraints. However the modified version of the method available in Nexus [18] overcomes this limitation.

***Modified Nelder-Mead Method:*** the Nelder-Mead method attempts to minimize a nonlinear function of n real variables using only function values, without any derivative information. The Nelder-Mead method thus falls in the general class of direct search methods. The method works maintaining at each iteration a (possibly) non degenerate simplex, i.e. a geometric figure in n dimensions of nonzero volume that is the convex hull of n + 1 vertices.
In the original formulation, each iteration of the method begins with a simplex and its associated objective function values. One or more test points are computed, along with their function values, in the attempt to improve the simplex with respect of the objective function.
In the original implementation, a direct comparison between objective values is used to sort and modify the points forming the simplex.
The implementation herein proposed compares the points forming the simplex using a rank based approach: feasible solutions are always preferred to unfeasible ones; among unfeasible solutions, the less violated are preferred; among feasible solutions, the ones with minimum objective are preferred. The proposed approach directly allows to consider non-linear constraints and intrinsically favors the algorithm in the search of feasible solutions over performing more classical approaches based on penalty.

***Results:*** A first optimisation run has been performed searching for the minimum weight panel with a buckling load per unit length above 450*N/mm* and a failure load per unit length above 675*N/mm* (being this the minimum between Tsai-Wu and global instability loads).
The rationale is to design a panel working under the buckling load in limit conditions and capable to sustain ultimate loads without failures. Despite first-ply-failure approach is recognized to be conservative in many practical applications, this is believed to represent a good benchmark for the proposed optimisation procedure. It can be easily extended to account for more realistic and possibly less conservative strength criteria (including as an example damage tolerance and reparability criteria).
Limiting to this test case, the optimisation was started with an initial random simplex of 11 points and required 192 iterations, corresponding to a total number of 280 function evaluations to converge to the final solution.
Figure 9 reports the convergence history of the algorithm. It is worth noticing that despite the relative high number of iterations, the number of function evaluations has been contained.
A classical gradient based optimisation would have required at least 11 evaluations per iteration (as derivatives would have computed via finite differences at each iteration) leading to an equivalent number of iterations of about 25 - which seems a realistic forecast considering the number of variables. In this respect, it seems that the Simplex method turned out to be as

effective as a gradient based algorithm in terms of number of evaluations.

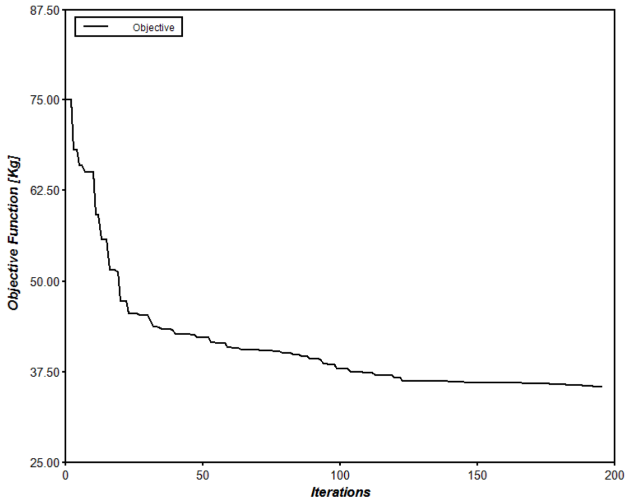


*Fig. 9: optimisation history: objective function vs. iterations.*

The characteristics of the identified design, in terms of design variables and requirements, are briefly reported in Tab. 4.

| Description | | Value |
|---|---|---|
| Stringer height | *[mm]* | 23.71 |
| Stringer half-foot width | *[mm]* | 29.89 |
| Stringer half-head width | *[mm]* | 15.40 |
| Stringer 0 ply thickness | *[mm]* | 2.74 |
| Stringer 90 ply thickness | *[mm]* | 1.95 |
| Stringer +/-45 ply thickness | *[mm]* | 0.75 |
| Skin 0 ply thickness | *[mm]* | 0.50 |
| Skin 90 ply thickness | *mm]* | 1.35 |
| Skin +/-$\alpha$ ply thickness | *[mm]* | 1.01 |
| Skin $\alpha$ ply orientation | *[degs.]* | 44.75 |
| Overall Weight | *[Kg]* | 35.67 |
| First Buckling Load | *[N/mm]* | 451.05 |
| Failure load (Tsai-Wu=1) | *[N/mm]* | 697.12 |
| Collapse Load | *[N/mm]* | 723.22 |

*Tab. 4: optimisation results: design variables, objective and constraints values.*

The obtained results show that the value of the first buckling load is at the imposed constraint value of 450 *N/mm*, being the driving constraint. A residual margin of around 7% has been achieved on the Tsai-Wu failure load which results to happen slightly before the structural collapse due to global instability.
The values of the geometrical variables describing the stringer geometry are far enough from the imposed geometrical limitations and no further margins seem to exist for further weigh reductions.
Considering the strength requirements it can be seen that the Tsai-Wu criterion has been exceeded just before the global instability of the structure near the foot of the stringers due to high global bending.

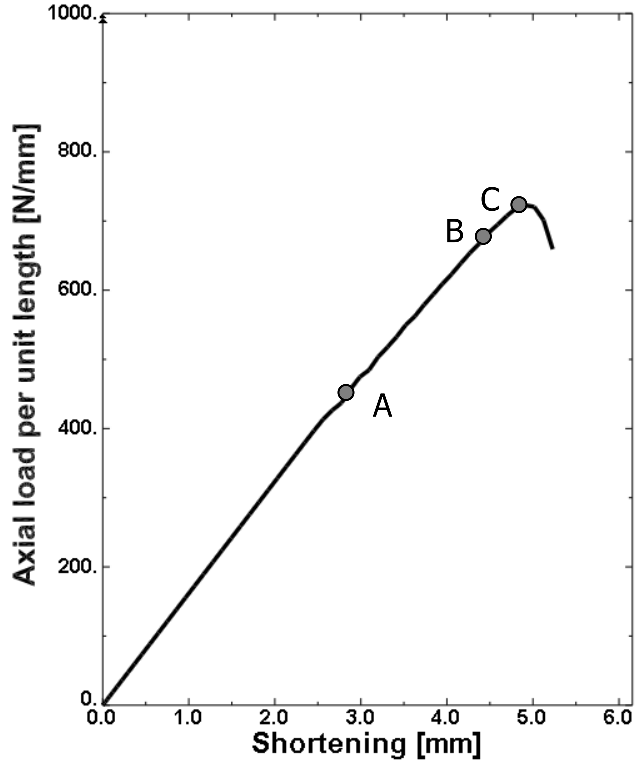


*Fig. 10: force vs. shortening curve of the optimised panel configuration: A) first buckling load; B) Tsai-Wu at 1; C) collapse.*

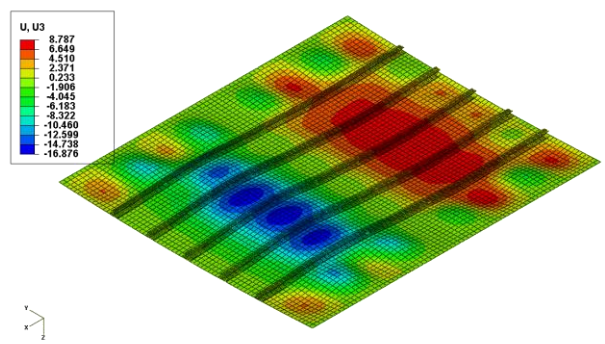


*Fig. 11: structural collapse corresponding to point C) in Fig 10 above.*

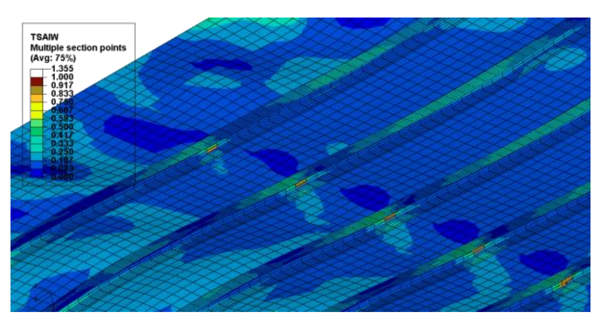


*Fig. 12: contour of Tsai-Wu failure criteria corresponding to point B) in Fig 10 above.*

## CONCLUSIVE REMARKS:

The paper focuses on the use of mesh free morphing applied to structural optimisation problems. A practical example considering a composite stiffened panel working in post-buckling has been presented.
The proposed morphing procedure seems an appealing alternative to more classical (linear) approaches and allows- in this specific application - to modify size and shape of the stringers.
The resulting optimisation process uses 10 design variables to control both shape variables (that define the geometry of the stringers) and lay-up variables controlling the thicknesses of each orientation of the composite laminates.
A modified version of the Nelder-Mead Simplex method is used in the search of the minimum weight panel subjected to first buckling and strength constraints. The method has the advantage of not requiring derivative information. Limiting to this application, the optimisation procedure seems to over-perform Genetic Algorithm and to rank almost equally to other well established gradient based methods with respect of the number of overall function evaluations being the final solution bounded by the constraints – suggesting no further improvements in the overall weight are possible.

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