COMMAND FOLLOWING CONTROL LAW DESIGN BY LINEAR QUADRATIC OPTIMISATION

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Abstract

This paper presents a control law design for a linearised model of a Lynx-like helicopter in the 80 knots forward flight condition, with respect to an appropriate subset of the published handling qualities requirements. The synthesis method used was the Linear Quadratic Regulator (LQR) version of model following. The design is based principally on the rigid body quasi-steady state rotor model although a simplified representation of the actuator and blade flapping dynamics as pure delays is used to motivate the choice of the LQR input weighting matrix. The design assumes that all of the rigid body states can be measured. State estimation of actuator and rotor states is not used. The design is evaluated using a linear model which includes representations of the actuator and blade flapping dynamics.

Notation

A, B, C ,D	Linear dynamic system state space matrices
$\dot{h}$	height rate
I	identity matrix
J	Linear Quadratic Regulator 'cost' function
K	gain matrix in control law
M	parameter in solution of Lyapunov equation
p, q, r	roll rate, pitch rate, yaw rate
Q	Linear Quadratic Regulator State weighting matrix
q	pitch rate; also parametersation of $Q = C'_H \, q \, C_H$
R	Linear Quadratic regulator input weighting matrix
r_2	Coefficient of determination
T(s)	Loop transfer function
$\underline{u}$	input vector
u_B, v_B, w_B	body axis velocity components
$\underline{x}$	State vector
$\underline{y}$	output or measurement vector
α	minimum over frequency of smallest singular value of return difference matrix
β	sideslip angle
Δ	perturbation from trim value
ζ, ω_n	parameterisation of compensation
θ, φ, ψ	pitch, roll and heading angle
ρ	parametersation of $R = \rho I$
$\bar{\sigma}$, $\underline{\sigma}$	largest and smallest singular values
Φ	resolvent of system matrix $A : \Phi = (sI - A)^{-1}$

subscripts:

CG	command generator
H	helicopter
M	model

zero eg φ_0 trim value

Introduction

This paper describes the application of time-domain based linear quadratic optimisation to the design of a command following and stability augmentation control law for a single rotor helicopter. The application of linear quadratic methods to the helicopter problem is far from new [Ref 1, 2, 3]. However, it is appropriate to re-examine their applicability in the light of the new handling qualities requirements for military helicopters, [Ref 4, 5, 6] and this present work forms part of a broader study [Ref. 7] of multivariable control law synthesis methods. A limited design problem was specified as part of this study, in order to be able to make relatively quick comparisons of different control laws based on a linearised HELISTAB model of a Lynx-like helicopter for the 80 knot forward flight condition. (HELISTAB is a non linear analytical helicopter model developed at RAE Bedford [Ref.15].) Evaluation was to be performed with respect to an appropriate subset of the handling qualities requirement of [Ref 4]. This subset relates to the bandwidth and disturbance rejection qualities of the stability-loop, and also to pilot control of height rate, pitch rate, turn coordination and sideslip. The extent to which individual requirements are satisfied in detail will be discussed later on.

The following aspects will not be considered here although it is acknowledged that they cannot be neglected in the full design problem : non-linearity, robustness with respect to variations in trim condition and issues associated with the digital implementation of the control system.

Choice of Technique

The control law synthesis technique used in this paper is essentially the well known Linear Quadratic Regulator (LQR) method of optimal control [Ref 8] based on a 9-th order rigid body model, It is assumed that all of the rigid body states are available and the use of state estimators is not considered.

There are various factors that suggest this is a reasonable approach. The rigid body dynamics are accurately modelled whereas the actuator and rotor dynamics may not be. Furthermore while it appears to be technologically feasible to assume that all the rigid body states are available for feedback (possibly via a mixture of direct measurement and calculation using the geometrical relationships involving the Euler angles) the same cannot be said of the actuator and rotor states. In principle these states can be estimated using (for example) a Kalman Filter but the traditional version of this approach may not be robust [Ref 9] and the more recent LQG/LTR version has been criticised on other grounds [Ref 10]. Since we are in the fortunate position of having an adequate reduced order model, of which all the state variables are available for feedback, a full state feedback law is feasible and the LQR method becomes an option.

Robustness with respect to model error - and in particular with respect to dynamics not included in the model at all - is an important consideration. LQR control laws have good theoretical robustness properties [Ref 11] but while it is pleasant to have some guaranteed robustness, miracles cannot be expected. As has been emphasized by Tischler [Ref 12] the effects of these extra dynamics can by approximated by a pure time delay (we consider only the rotor flapping and actuator dynamics here but Tischler takes account of several other effects associated with the implementation of a digital control system). The 80 knot, forward flight linearised HELISTAB models with and without actuator and rotor dynamics were examined and representative figures of 75° phase lag at 10 rad/s were chosen. This corresponds to a time delay of 0.131 s. This is a large effect and it is clearly wise to try to take it into account in some way (as Tischler does). We shall return to this point later.

Finally, in addition to stability augmentation it is necessary to design the 'command interface' i.e. that portion of the control system which sits between the pilot and the stability loop and processes the pilot input to his inceptors in order to give good command following (Fig (1)). A model following version of LQR theory has long been available [Ref 13] which makes it possible to design the stability loop and command interface in an integrated way.

Basic LQR Theory

In this section we review the basic theory. Fuller details may be found in [Ref 8] and [Ref 13].

Let the linearised representation of the open loop helicopter be given by

$$\dot{\underline{x}}_H = A_H \underline{x}_H + B_H \underline{u}_H \qquad \underline{y}_H = C_H \underline{x}_H \tag{1}$$

(This representation is assumed to include cascade compensation). The desired behaviour of the system in response to pilot input $\underline{u}_M$ is given by a model:

$$\dot{\underline{x}}_M = A_M \underline{x}_M + B_M \underline{u}_M \qquad \underline{y}_M = C_M \underline{x}_M + D_M \underline{u}_M \tag{2}$$

The problem is to specify $\underline{u}_H$ in terms of $\underline{x}_H$, $\underline{x}_M$ and $\underline{u}_M$ so that the tracking error $(\underline{y}_H - \underline{y}_M)$ is small. For design purposes the pilot inputs $\underline{u}_M$ are taken to be step functions so that:

$$\dot{\underline{u}}_M = \underline{o} \tag{3}$$

With assumption (3) the dynamics of the model may be rewritten as:

$$\begin{bmatrix} \dot{\underline{x}}_M \\ \dot{\underline{u}}_M \end{bmatrix} = \begin{bmatrix} A_M & B_M \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_M \\ \underline{u}_M \end{bmatrix} \tag{4}$$

$$\underline{y}_M = [C_M \; D_M] \begin{bmatrix} \underline{x}_M \\ \underline{u}_M \end{bmatrix}$$

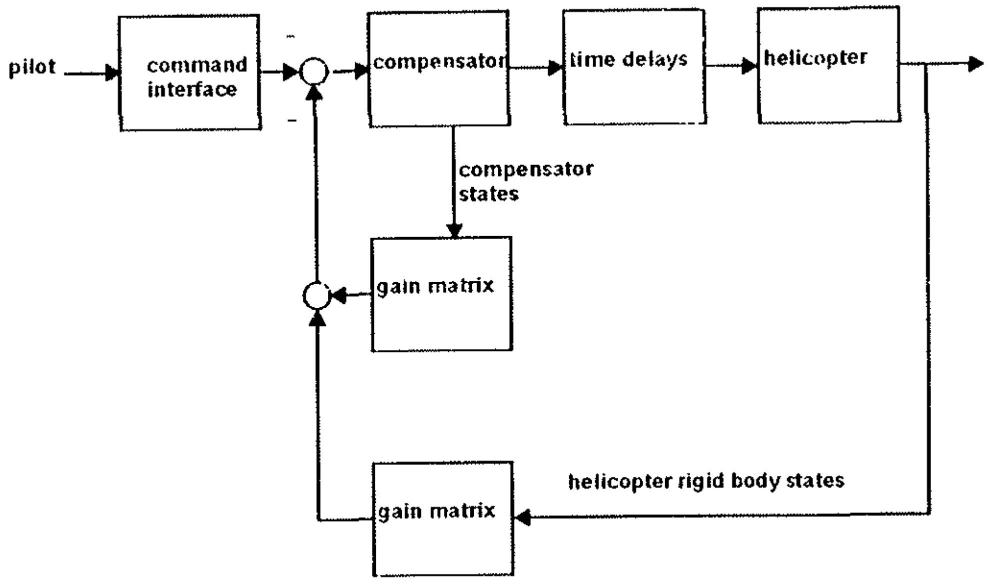

Fig 1: Control design block diagram including time delays used at design stage.

Equations (4) define the dynamics of a new system called the command generator with state vector $\underline{x}_{CG}$ such that:

$$\dot{\underline{x}}_{CG} = \begin{bmatrix} A_M & B_M \\ 0 & 0 \end{bmatrix} \underline{x}_{CG} = A_{CG}\,\underline{x}_{CG} \qquad (5)$$

$$\underline{y}_{CG} = [C_M \; D_M]\, \underline{x}_{CG} = C_{CG}\, \underline{x}_{CG}$$

The tracking problem may be expressed in terms of a cost function J which is to be minimised by an appropriate choice of $\underline{u}_H$:

$$J = \int_0^\infty (\underline{y}_m - \underline{y}_{CG})'q(\underline{y}_H - \underline{y}_{CG}) + \underline{u}_H' R \underline{u}_H dt \qquad (6)$$

Equation (6) is a version of the standard Linear Quadratic Regulator problem:

$$\dot{\underline{x}} = A\,\underline{x} + B\underline{u}$$

$$J = \int_0^\infty \underline{x}'\,Q\,\underline{x} + \underline{u}'\,R\,\underline{u}\,dt \qquad (7)$$

with $\underline{u} = \underline{u}_H$

$$\left.\begin{aligned} \underline{x} &= \begin{bmatrix} \underline{x}_H \\ \underline{x}_{CG} \end{bmatrix} \\ A &= \begin{bmatrix} A_H & 0 \\ 0 & A_{CG} \end{bmatrix} \\ B &= \begin{bmatrix} B_H \\ 0 \end{bmatrix} \\ Q &= \begin{bmatrix} C_H' q\, C_H & -C_H'\, q\, C_{CG} \\ -C'_{CG}\, q\, C_H & C'_{CG}\, q\, C_{CG} \end{bmatrix} \end{aligned}\right\} \qquad (8)$$

The (A,B) composite system is uncontrollable because $\underline{x}_{CG}$ is not influenced by $\underline{u}_H$. However, this does not prevent us from obtaining a solution to (7) since (as shown in [Ref 9]) the Riccati equation for (7) may be partitioned and the part associated with the uncontrollability can be ignored. The solution to (7) (ie. the control law that we seek) turns out to have the form:

$$\underline{u}_H = -\,[\,K_H\; K_{CG}\,] \begin{bmatrix} \underline{x}_H \\ \underline{x}_{CG} \end{bmatrix} \qquad (9)$$

where K_H is the full state feedback matrix that stabilises $\underline{x}_H$ when $\underline{y}_{CG} = \underline{o}$ and is obtained via the solution of

$$\dot{\underline{x}}_H = A_H\, \underline{x}_H + B_H\, \underline{u}_H$$

$$(10)$$

$$J_H = \int_0^\infty \underline{x}_H' C_H' q\, C_H\, \underline{x}_H + \underline{u}_H' R\, \underline{u}_H\, dt$$

and K_{CG} (which gives the command interface) is given by the solution of a matrix Lyapunov equation:

$$K_{CG} = R^{-1} B_H' M \tag{11}$$

$$M A_{CG} + (A_H - B_H K_H)' M - C_H' q C_{CG} = 0$$

Note that the control law eq.(9) contains contributions from the pilot inputs $\underline{u}_M$ as well as the model states $\underline{x}_M$. In the design process the components of $\underline{u}_M$ were assumed to be step functions but the same control law is assumed to be valid for the more general inputs which occur in practice.

Application of the Theory : Stability Loop Design

We now come to the application of the general theory given in the previous section to the specific problem at hand.

First of all, the handling qualities [Ref.4.] imposes a heading hold requirement in paragraph 3.2.6, which states that the heading shall return to within plus or minus 10% of its peak excursion within 10 seconds following a pulse input inserted directly into the control actuator. ([Ref 4] should be consulted for a detailed description of this and other handling qualities cited in the present paper) It is assumed that this requires the presence of the heading angle Ψ in the state vector. The linearised model of the helicopter is therefore a 9-th order, six degree of freedom system with state vector:

$$\begin{bmatrix} \Delta u_B \\ \Delta w_B \\ q \\ \Delta \theta \\ \Delta v_B \\ p \\ \Delta \varphi \\ r \\ \Delta \Psi \end{bmatrix} \tag{12}$$

where the 'Δ's indicate perturbations from trim values eg.

$$\Psi = \Psi_o + \Delta\Psi \tag{13}$$

Examination of equations (10) and (11) show that K_H needs to be known before K_{CG} can be found. This should not be taken to imply that the stability loop can be designed independently of the tracking problem. The state weighting matrix in eq.(10) $C_H' q C_H$ depends (via C_H) on the choice of variables that the system is to track. It was decided [Ref 7] that the choice of tracked variables most compatible with the limited design problem was:

$$\begin{bmatrix} \text{height rate} \\ \frac{d}{dt} \text{ pitch} \\ \text{turn rate} \\ \Delta \text{ sideslip angle} \end{bmatrix} = \begin{bmatrix} \dot{h} \\ \frac{d}{dt} \Delta \theta \\ \frac{d}{dt} \Delta \Psi \\ \Delta \beta \end{bmatrix} \tag{14}$$

The definition of sideslip angle β is taken from [Ref 8]:

$$\tan \beta = \frac{v_B}{u_B} \tag{15}$$

The output vector $\underline{y}$ was taken to be

$$\underline{y} = \begin{bmatrix} \dot{h} \\ \Delta \theta \\ \frac{d}{dt} \Delta \theta \\ \Delta \Psi \\ \frac{d}{dt} \Delta \Psi \\ \Delta \beta \end{bmatrix} \tag{16}$$

All of the components of $\underline{y}$ in eq.(16) are readily expressible in terms of the state variables and are important for requirements involving response to pilot input. There is another facet of the tracking problem which affects the design of the stability loop. According to section 3.2.2 of [Ref 4] the system is supposed to exhibit rate command characteristics so that in response to a step input on the appropriate inceptor the pitch attitude should (in the context of a linear treatment) increase indefinitely at a uniform rate - at least in the steady state. In classical SISO theory the vanishing of the steady state error requires compensation of the open loop system to ensure that it is type 2 (at least). The equivalent of this argument in the present situation is that if the pitch attitude is to increase indefinitely (in the idealised case of a linear model and a step input of infinite duration) *and* if we wish to allow for the possibility of coordinated activity on all four actuators to achieve this then the actuator weightings should reflect this and should be chosen to vanish at the appropriate rate in the low frequency limit: $s = j\omega \rightarrow 0$. In order to achieve this the four actuators were each preceeded by a compensator of the form

$$\frac{s^2 + 2\zeta\omega_n s + \omega^2_n}{s^2} \tag{17}$$

with $\zeta = 1/\sqrt{2}$, $\omega_n = 1$ rads^{-1}

the choice of $\omega_n = 1$ rad s^{-1} for the corner frequency of eq.(17) was made so that control activity above this frequency should not be unduly restricted, perhaps limiting the bandwidth (which is required to be above 3.5 rad/s for level 1). The open loop system (A_H B_H C_H D_H) is assumed to incorporate the compensator dynamics. A consequence of the use of a full state feedback law that should be noted is that the control law eq.(9) includes feedback from the compensator states and the effect of having this 'inner loop' closed around the compensator means that the closed loop compensator dynamics are not the same as the open loop compensator dynamics eq.(17). Although compensation has been discussed in terms only of the pitch attitude response the extra poles at the origin afforded by eq.(17) appear to suffice for all of the command tracking requirements. Indeed, there is probably an element of overkill since height rate needs only follow a step function and there is already one pole at the origin in the system due to the presence of the heading angle in the state vector. However, the same form of compensation on all of the actuators has been retained for conceptual simplicity.

Having specified C_H and the compensation (17) it remains to complete the design by specifying the weighting matrices R and $Q = C_H' \, q \, C_H$ in (10) and it is here that the problem of lack of visibility in the LQR method arises. We will present a rationale for the choice by Q and R that is adapted from an argument presented by Doyle and Stein [Ref 14].

The actuators are independent devices with similar ranges of variation and equal compensation so that it is appropriate to take

$$R = \rho I \tag{18}$$

(The choice of the scalar parameter ρ will be dictated below by robustness requirements). Similarly the components of $\underline{y}$ in (16) will be weighted independently ie q in $Q = C_H' q C_H$ will be chosen to be diagonal.

Doyle and Stein suggest in [Ref 14] that for good performance and robustness R and Q should be chosen so that the maximum and minimum singular values of the LQR loop transfer function

$$T_H(s) = K_H(sI-A_H)^{-1} \, B_H \tag{19}$$

be close together over the appropriate frequency range. This is a fairly labour intensive criterion to try to apply directly since it would be necessary to solve a Ricatti equation for each trial pair (Q,R). However (as is pointed out in [Ref 14]) an approximation can be made which removes the need for repeated solutions of the Riccati equation at this stage and enables Q to be chosen independently of R. The LQR loop transfer function $T_H(s)$ satisfies the following identity:

$$(I + T_H \, (-j\omega))'R \, (I + T_H \, (j\omega)) =$$

$$R + B_H' \, \Phi_H' \, (-j\omega) \, Q \, \Phi_H \, (j\omega) \, B_H \tag{20}$$

In the limit $\rho \to 0$ (corresponding to a high bandwidth of the nominal system) we get the approximation

$$T_H' \, (-j\omega) \; T_H \, (j\omega) \sim \frac{1}{\rho} \, B_H' \; \Phi_H' \; (-j\omega) \; Q \; \Phi_H(j\omega) \; B_H \quad \rho \tag{21}$$

Hence the <u>ratio</u> of the maximum and minimum singular values of T_H is (approximately) independent of ρ so that Q can be chosen to get this ratio as close as possible to unity. A numerical optimisation routine was used to choose the diagonal elements of q in $Q = C_H' \, q \, C_H$ so as to minimise the mean value of log $(\bar{\sigma} \; (T_H)/\underline{\sigma} \; (T_H))$ over the frequency range 1–6 rads^{-1}.

With Q found, the value of ρ is chosen on the basis of robustness. If ρ is made smaller then the bandwidth of the (nominal) system will increase but the system will become less robust. Therefore we try to make ρ as small as possible subject to retaining an adequate margin of stability. As a measure of stability we use the smallest singular value of the return difference matrix minimised over frequency ie for the <u>nominal</u> system we would consider the quantity

$$\alpha = \inf_{\omega} \underline{\sigma}(\; I + T_H(j\omega)) \tag{22}$$

The smaller this quantity is, the less robust is the system.

It is well known [Ref 11] that consideration of eq.(22) in conjunction with the return difference identity eq.(10) leads to the 'guaranteed' stability margins of the Linear Quadratic Regulator. However it has already been noted that the effects of the rotors and actuators are far from negligible and it would be prudent to try to take these effects into account, preferably in a way that does not rely on a detailed accurate knowledge of these higher order dynamics. A way of doing this is to evaluate eq.(22) with time delays inserted into each of the four actuators between the compensator and the helicopter (Fig 1). The values of the four time delays were all taken to be 0.131 s.

What is a suitable smallest acceptable value for α in eq.(22)? The selection of a suitable value was motivated by consideration of the single input case where α is simply the distance of closest approach to the critical point on the Nyquist diagram. If the gain and phase margins in this case are constrained to be 12 dB and 45° then this supplies two points on the Nyquist diagram and the corresponding value of α can be estimated by joining these two points with a straight line and is approximately 0.6.

Interpretation of Time and Frequency Responses

It should be noted with regard to the numerical results presented in the remainder of this paper that:

a) All simulations are based on linear models and are therefore only strictly accurate for small signals.
b) While the design process is based principally on a 9-th order rigid body model, in order to gain some appreciation of the robustness of the design the helicopter model used for evaluation includes actuator dynamics (modelled as first order lags) and a 6-th order rotor model. The actuator time constants used were 25 rad/s for the pedals and 12.6 rad/s for the others.

Stability Loop : Results

Attitude hold criteria:

According to requirement 3.2.6 of the handling qualities specification document [Ref 4], following an impulse to the appropriate actuator the attitudes and heading should return to within ± 10% of the peak excursion within 10s. Simulated pitch, roll and heading impulse responses are presented in Fig 2. It is clear that all three responses satisfy this requirement. Of the three the roll response is the least well damped and this is presumably because the roll angle is not directly weighted with our particular choice of Q. We shall return to this point again in our discussion of the turn manoeuvre.

Stability Loop Bandwidth

Requirement 3.4.10 of [Ref 4] demands that heading responses to disturbances applied directly at the actuators shall meet the criteria set in paragraphs 3.4.1.1 (pitch) 3.4.5.1 (roll) and 3.4.7.1 (yaw).

Examination of the time history of step responses makes it clear that these responses are of the attitude command type and their bandwidths are calculated accordingly.

The values obtained were (Table 1):

Table 1.

Channel	Bandwidth (rad/s)	Time Delay (s)
Pitch	4.1	0.12
Roll	5.9	0.09
Heading	5.1	0.04

The gain limited bandwidth is undefined for the roll response, but pilot induced oscillations are not a problem at this stage since as yet there is no pilot input.

In addition to the bandwidth and phase delay criteria there is a requirement that all modes should have damping ratios exceeding 0.35 for level 1 and 0.25 for level 2. For the evaluation system which includes models of the rotor and actuator dynamics all but one of the damping ratios exceed the level 1 boundary. The exception is a rotor mode with a damping ratio of approximately 0.22 which is only lightly coupled to the rigid body states and in addition is almost identical in the open and closed loop systems.

Response to Pilot Inputs

Having now completed the design and evaluation of the stability loop we turn to the design of the 'command interface'. The four pilot inceptors will each be designed to give a distinct response corresponding to a particular manoeuvre. Because we are working with linearised models it is both valid and conceptually simple to consider

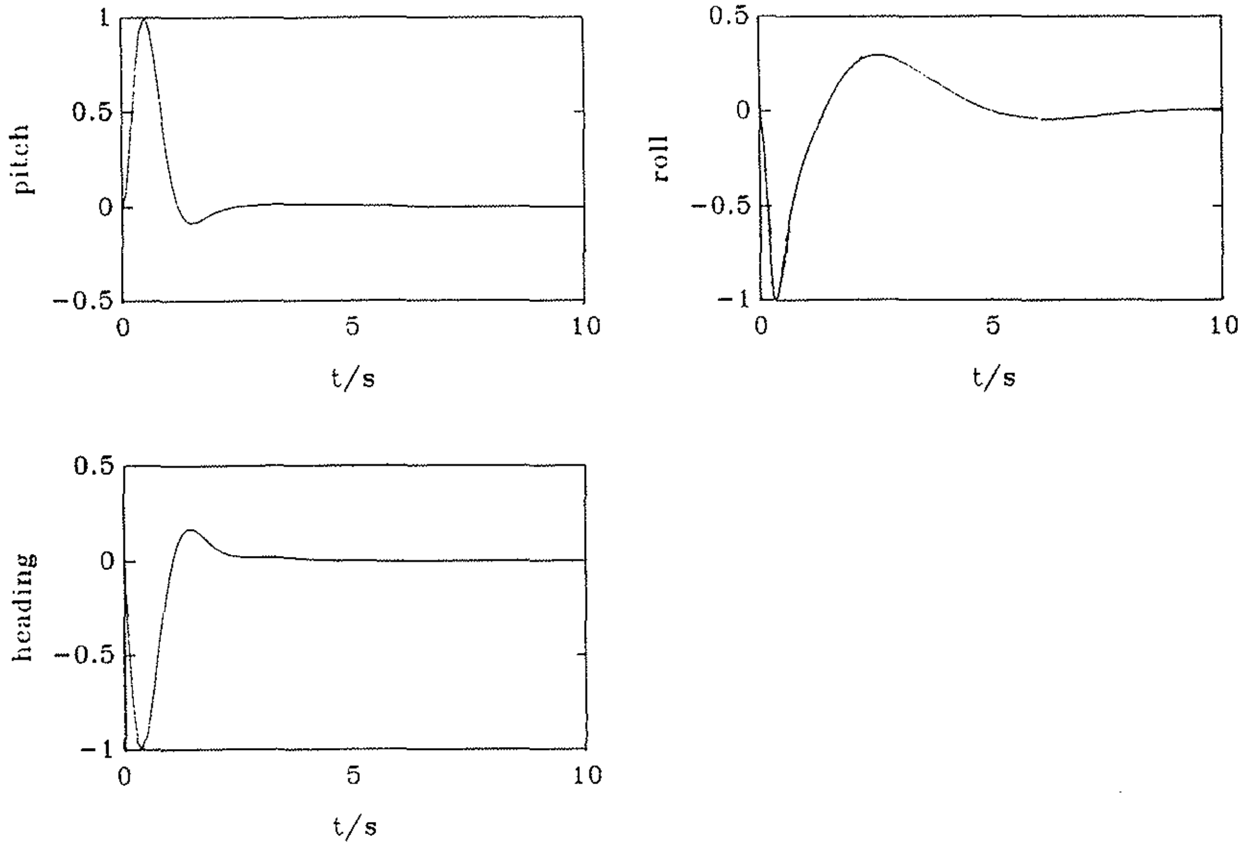

Fig 2: Simulated attitude and heading responses to impulses input directly to actuators. (Normalised so peak excursion is one).

each inceptor and the requirements that relate to it in turn. In each case we will specify a model of the 'ideal' response to the inceptor which then gives the command interface via eq.(5), eq.(9) and eq.(11).

First Inceptor – Height Rate Control

The model chosen was

$$A_M = [\ -1/2.5\ ]$$

$$B_M = [\ 1/2.5\ ]$$

$$C_M = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (23)$$

$$D_M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The interpretation of this model is particularly simple : in response to a unit step on the first inceptor the pitch, heading and sideslip angles would remain unchanged and the height rate should exhibit a first order response with a time constant of 2.5 s.

The simulated height rate response (for the evaluation model which contains representations of actuator and rotor dynamics) is shown in the top part of fig 3. Requirement 3.4.3 of [Ref 4] demands that the response closely approximates a delayed first order response with specified limits on the effective time constant, time delay and coefficient of determination (r^2). The values obtained are (Table 2):

Table 2

	Level 1 Boundaries	Actual
Time constant	<5.0	3.1
Time Delay	<0.2	0.11
r^2	$0.97 < r^2 < 1.03$	0.9991

The results summarised in the table showing that the direct height rate response is

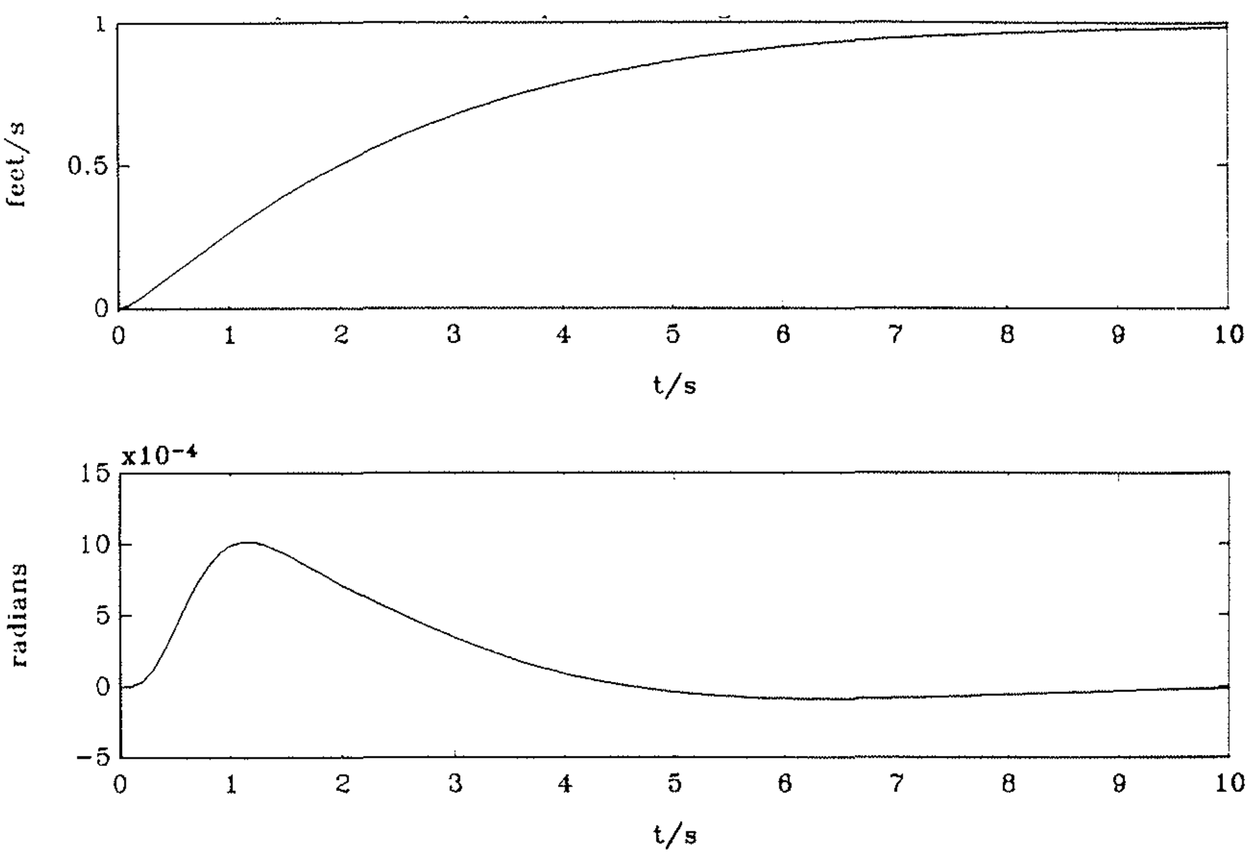

Fig 3: Height rate and pitch attitude response to unit step on first inceptor.

Level 1. In addition to the direct response there is a coupling requirement (3.4.4.1.1 of [Ref 4]) that the ratio

$$\left| \frac{\text{peak pitch}}{\text{peak normal acceleration}} \right|$$

should not exceed 1.0 deg/(ft/s^2). In this case

peak pitch $\hat{=}$ 0.0010 radian

peak normal acceleration $\hat{=}$ 1/3.1373 ft/s^2

giving a coupling of approximately 0.18 deg/(ft/s^2) which is well within the prescribed limit.

Second Inceptor - Pitch Rate Control

The second inceptor is designed to control pitch with rate command characteristics. The model chosen was

$$A_M = [\ 0\ ]$$

$$B_M = [\ 1\ ]$$

$$C_M = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \varphi_o \end{bmatrix} \qquad (24)$$

$$D_M = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The response of this model to a step may be described as follows. The height rate and heading angle do not change. The pitch attitude tracks the integral of the inceptor and the rate of change of pitch attitude tracks the inceptor directly. The ideal behaviour of the sideslip angle demands some explanation. If during the pitching manoeuvre the aircraft follows a straight and level flight path with zero heading angle then

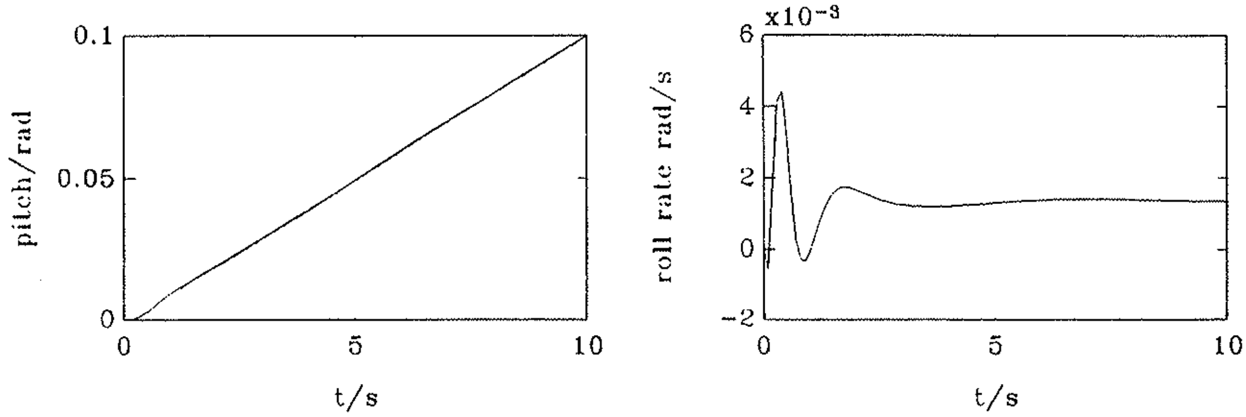

Fig 4: Pitch attitude and roll rate response to a step input on the second inceptor.

with the definition eq.(15) of sideslip angle we have

$$\tan \beta = \sin \varphi \tan \theta \qquad (25)$$

which when linearised with sufficient accuracy for small trim angles gives.

$$\Delta\beta = \varphi_0 \, \Delta\theta + \theta_0 \, \Delta \varphi \qquad (26)$$

Now φ and θ_0, while both small, are of roughly the same magnitude so that constraining $\Delta\beta = 0$ would mean there would have to be excursions on the roll angle of similar size to those on the pitch angle which conflicts with the requirement to minimise the coupling from pitch to roll angle (3.4.4.2 of [Ref 4]). Therefore in this case the model asks for a small excursion in sideslip angle

$$\Delta\beta = \varphi_0 \, \Delta\theta \qquad (27)$$

during the pitching manoeuvre. There is still coupling to the roll angle but it is less that that obtained with C_M (6,1) set equal to zero.

Fig. 4 shows the time domain responses of pitch attitude and roll rate for a unit step on the second inceptor. The left graph clearly shows that it is a rate command system as was intended. The right graph shows that the pitch-to-roll coupling is approximately 0.5 which falls within the Level 2 region as defined by requirement 3.4.4.2 of [Ref 4].

Fig. 5 shows the frequency response from the second inceptor to the pitch attitude. Assessed as a rate command system this gives a bandwidth of 2.8 rad/s and a phase delay of 0.125 seconds. This is level 2 for combat and target tracking, level 1 for all other mission task elements.

Third Inceptor - Turn Control

The derivation of the model for turn is slightly more complicated than in the previous two cases. First of all, in the 'ideal turn' we shall suppose that the height rate,pitch attitude and sideslip angle should remain undisturbed. In a steady turn we expect the heading angle to change uniformly. The complication arises because the roll angle plays an important part in turn coordination, and is supposed to have a rate-command characteristic (requirement 3.2.2 of [Ref 4]). Unfortunately the tracking matrix C_H does not pick out the roll angle, so we need to get at it indirectly.
Within the linear approximation, in a steady turn, we expect that

$$\frac{d}{dt} \Delta\Psi \; \alpha \; \Delta\varphi \qquad (28)$$

for an idealised roll rate response we would expect

$$\frac{\Delta\varphi(s)}{U_m(s)} = \frac{1}{s} \qquad (29)$$

Combining these last two equations gives us the model for heading angle expressed in transfer function form

$$\frac{\Delta\Psi(s)}{U_m(s)} = \frac{1}{s^2} \qquad (30)$$

All of the idealised responses are thus incorporated in the following state space

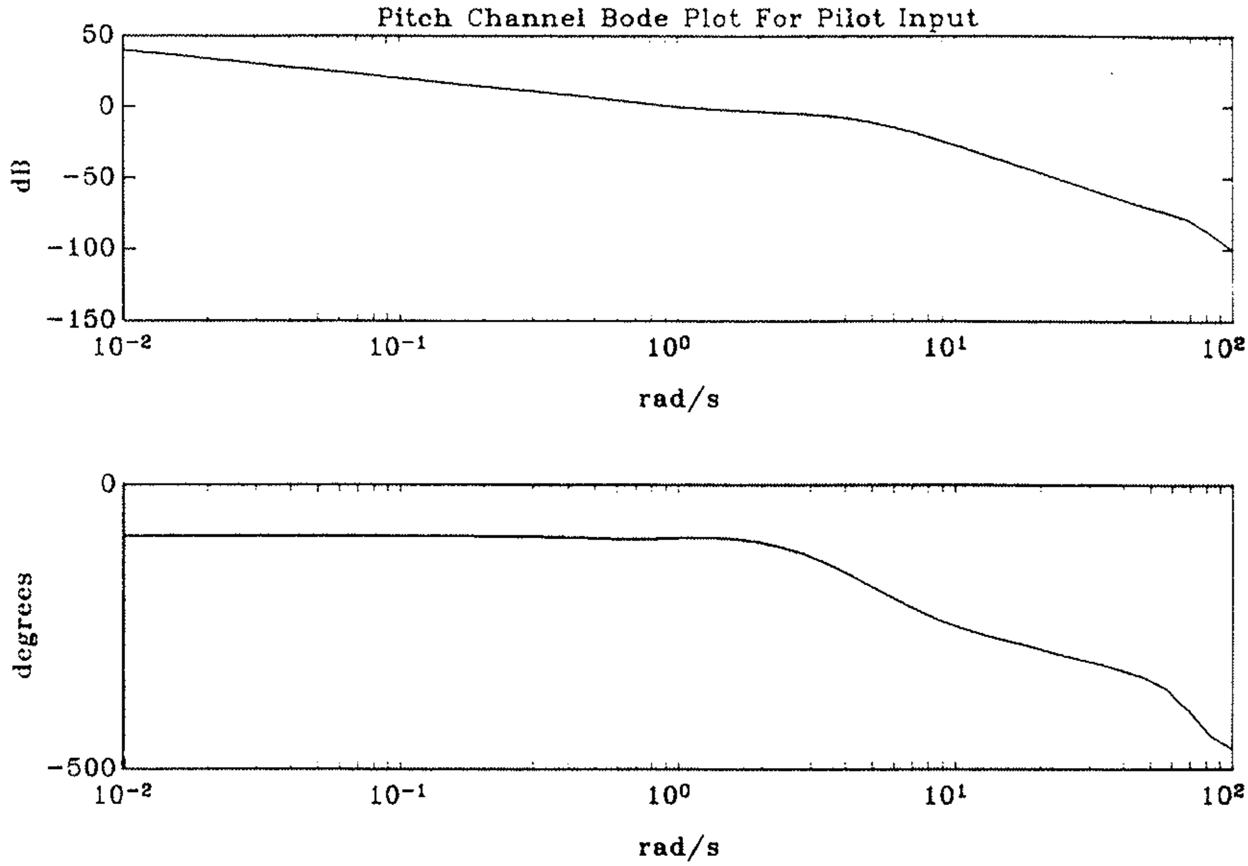

Fig 5: Bode plot for frequency response of pitch attitude from second inceptor.

model

$$A_M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B_M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad (31)$$

$$D_M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Examination of the roll angle response to a step on the third inceptor (not reproduced here) shows that it is indeed of the rate command type.Evaluated as a rate command system the bandwidth and phase delay are 4.3 rad/s and 0.084s respectively which is well within Level 1 for combat and target tracking.

Requirements 3.4.6.1 and 3.4.6.2 of [Ref 4] relate to bank angle oscillations and sideslip excursions during a turn. With a rate command system for roll angle a turn is initiated by an impulse-like pilot input rather than a step like input. Fig (6) is the handling qualities plot for the bank angle oscillation requirement 3.4.6.1, which is seen to be at Level 1. Fig (7) is the handling qualities plot for turn coordination - again Level 1.

Finally 3.4.4.2 (roll-to-pitch coupling) requires that the ratio

$$\left| \frac{\text{maximum pitch rate}}{\text{desired roll rate}} \right|$$

Should not exceed 0.25 following a step input for at least 5 seconds in order to be classed as Level 1, which is found to be satisfied.

Fourth Inceptor - Sideslip Angle

The model chosen was

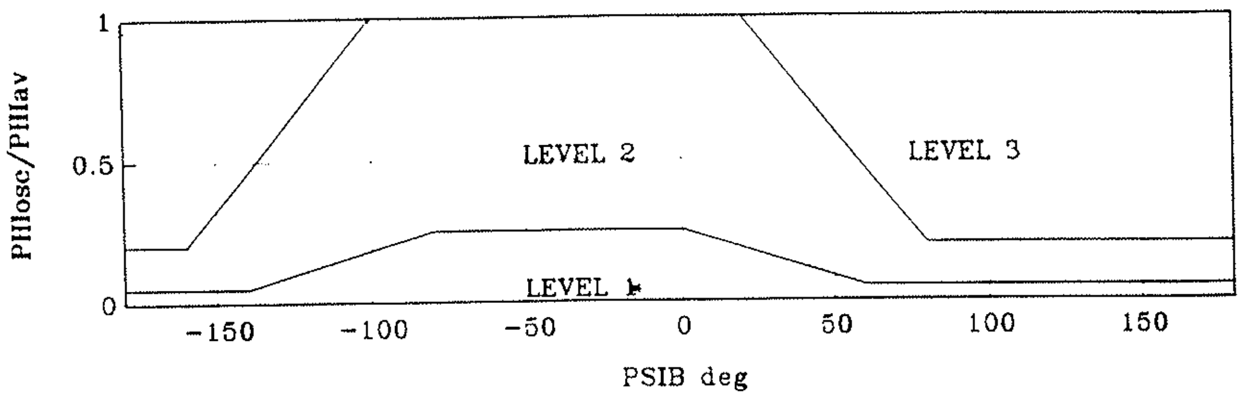

Fig 6: Handling qualities plot for bank angle oscillations requirement.

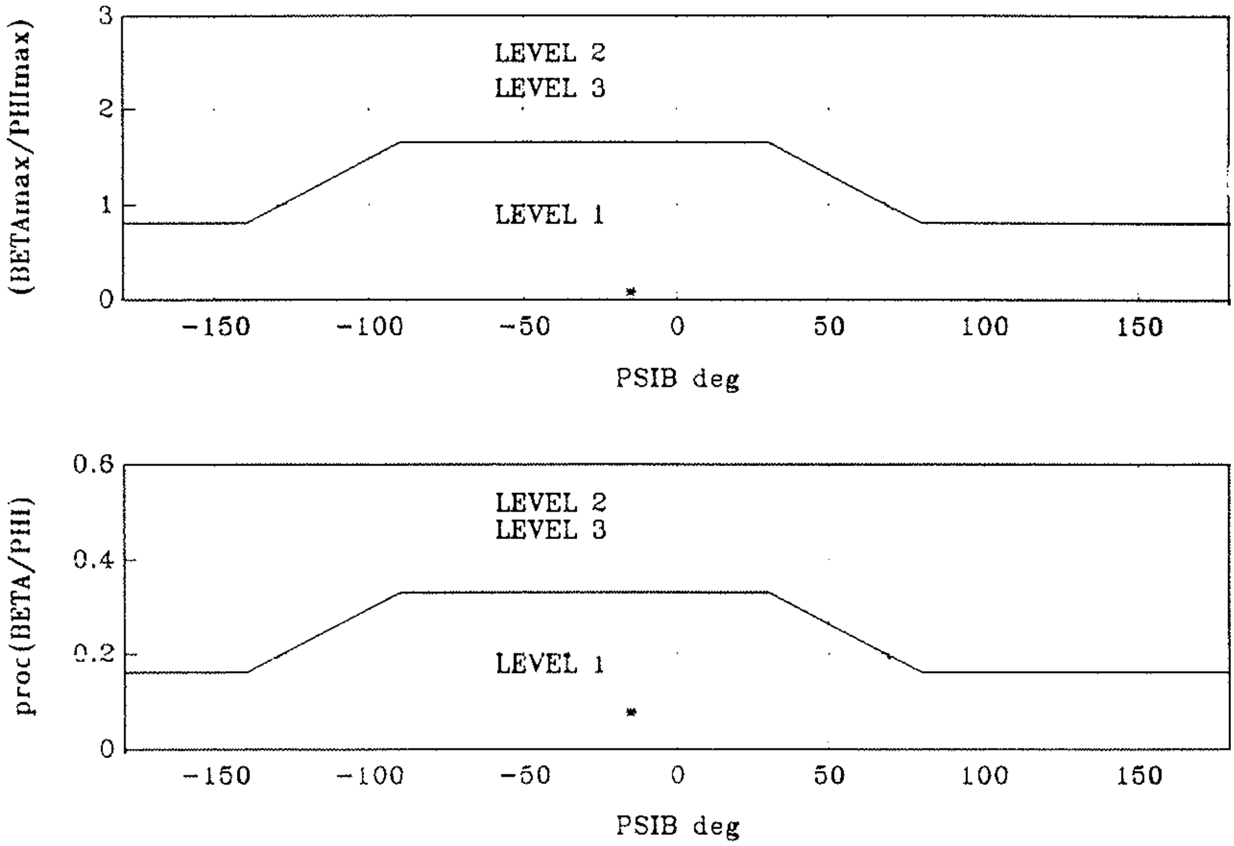

Fig 7: Handling qualities plot for turn coordination.

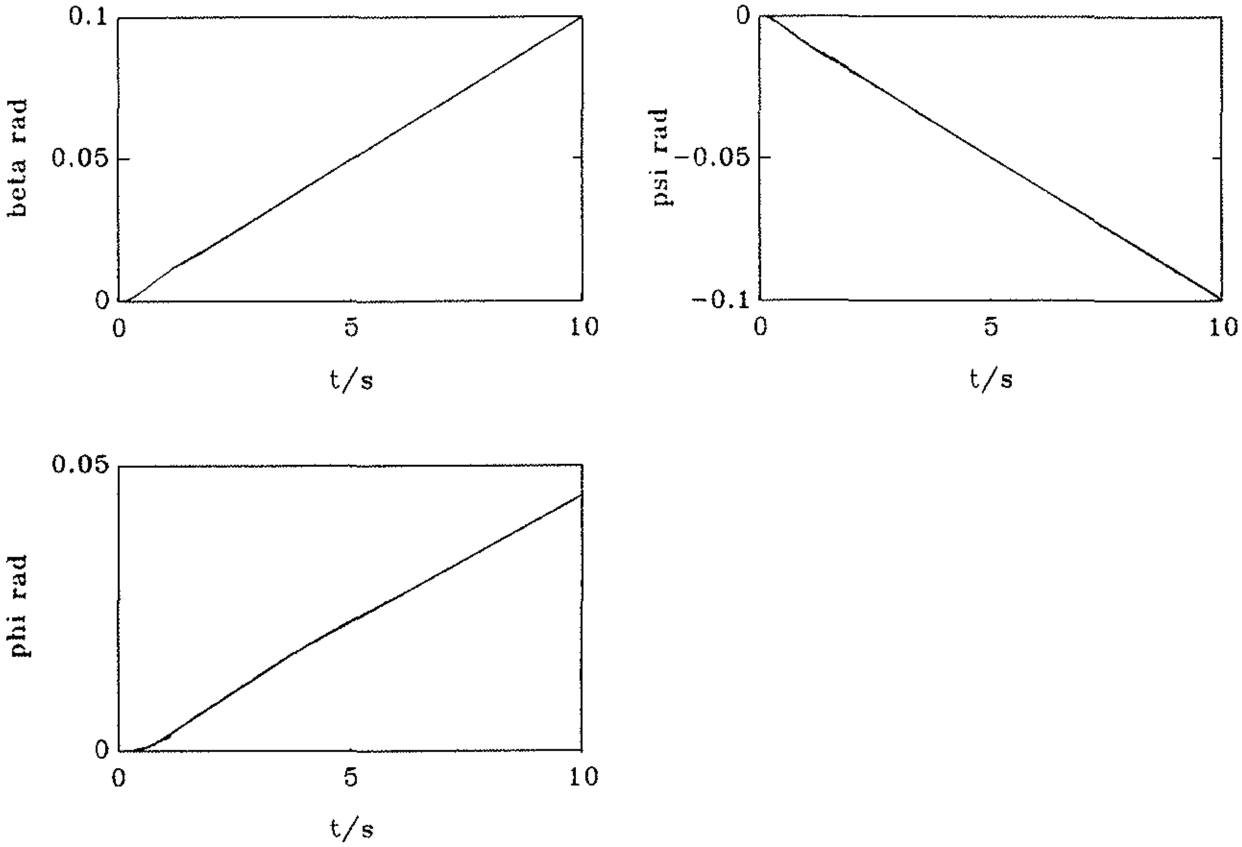

Fig 8: Time response to step input on 4th inceptor.

$$A_M = [\ 0\ ]$$

$$B_M = [\ -1\ ]$$

$$C_M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \qquad (32)$$

$$D_M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

The interpretation of this model is as follows. In response to a step input the height rate and pitch attitude should not change. The (perturbation in the) sideslip angle should track the integral of the step, and the heading angle should be the negative of the sideslip angle. in order to rationalise these last two specifications we not that if the aircraft is in straight and level flight then the steady state relationship between the perturbations is

$$\Delta\beta = \theta_o \, \Delta\varphi - \Delta\psi \qquad (33)$$

Thus the sideslip $\Delta\beta$ is a function of both $\Delta\varphi$ and $\Delta\Psi$ or put another way, specifying $\Delta\beta$ does not fix $\Delta\Psi$ uniquely. However since the trim pitch angle θ_o is very small we can choose

$$\Delta\Psi = -\Delta\beta$$
$$\frac{d}{dt}\Delta\Psi = \frac{-d}{dt}\Delta\beta \qquad (34)$$

for the model ie we neglect the contribution of the perturbation in the roll angle to the perturbation in the sideslip angle.

Fig (8) show the (simulated) step response of the actual system. It can be seen that equation (24) is obeyed to high accuracy. (The discrepancy between the two sides of eq.(33) due to the excursion of the roll angle is about 1%). The sideslip and roll both have the same sign which appears to satisfy 3.4.9.2 of [Ref 4]. Insofar as 3.4.9.1 and 3.4.9.3 apply to a linearised model they would seem to be automatically satisfied.

Fig (8) shows that the heading angle exhibits a rate command response. The bandwidth is 3.6 rad/s and the phase delay is 0.032s which is a level 1 response.

Discussion

The paper has presented results which illustrate the application of Linear Quadratic Regulator theory to the design of a stabilization and command augmentation system for a linearised model of a single rotor helicopter. A subset of the handling qualities from [Ref 4] were examined for the case of forward flight. In most cases a Level 1 performance was achieved, the notable exceptions being the pitch-to-roll coupling and pitch channel bandwidth in the case of pilot input. The evaluation model of the helicopter included representations of the rotor flapping and actuator dynamics. The results are fairly encouraging but it must be stressed that there are many other effects which should be included in a full treatment of even the linearised problem [Ref 12] and these effects would inevitably cause a deterioration in performance.

Several issues can be raised concerning the use of the LQR model following technique used in this paper. The command following problem imposes constraints on the design of the control law in a general way via the structure of the stability loop state weighting $C_H' \, q \, C_H$ and in specific ways via the specific choice of C_H and cascade compensation. Although it is aesthetically pleasing to have a method which can synthesize the stabilization and command following in a unified way it is recognised that the designer's freedom is restricted in some undesirable ways. For example, direct feed forward from the pilot's inceptors to the helicopter actuators does not seem to fit into this framework.

The evaluation model of the helicopter - as mentioned above - included detailed representations of actuator and rotor flapping dynamics which were believed to be reasonably accurate for the flight regime under consideration. The design model also included a representation of these extra dynamics but in a much simpler form (ie. modelled as pure time delays at the actuators) which was used to aid selection of the actuator weighting parameter ρ. It should also be noticed that once ρ has been chosen the Riccati and Lypunov equations that have to be solved to get the control law (9) make no further reference to the rotor and actuator dynamics.

The control law presented here has a rather complicated structure (eight integrators for the cascade compensation, plus several more for the various command models). This is probably more complicated than necessary. It may also be possible to reduce the number of integrators by using a finite end time version of LQR theory. (for example

One traditional objection to the LQR methodology is the lack of visibility ie the difficulty in choosing the state and actuator weighting matrices - particularly the former - to achieve the desired characteristics of the closed loop system. It is felt that the version of the Doyle and Stein method [Ref 14] used here is a reasonably 'visible' approach since the three main areas in which the designer needs to make choices:

(i) Choice of C_H
(ii) Choice of frequency range for optimisation of $C_H' \, q \, C_H$
(iii) Choice of ρ

relate fairly clearly to the problem specification and to the dynamics of the unaugmented helicopter.

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