DYNAMIC INVERSION CONTROL FOR A COAXIAL HELICOPTER UAV

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Abstract

Helicopters offer unique flying capabilities beneficiary for UAVs operating in urban and obstacle rich environments. To exploit these flying characteristics, advanced control systems are needed as the flying characteristics come with natural instability and complex flight dynamics. Helicopters therefore present a formidable control problem, so that a lot of different control approaches have been and are still researched. One very promising method in this context, which is gaining wider use in the control theory for a wide array of problems, is Dynamic Inversion. The basic idea is to compensate the dynamics of the plant by incorporating the inverted plant dynamics into the controller. This enables full control of the system, where not only states can be commanded directly but also state transitions are under continuous control. For unstable systems as helicopters with changing dynamics, stabilizing and adapting elements have to be included to allow for inversion. A Dynamic Inversion control approach for a coaxial helicopter UAV is presented within this contribution.

Introduction

The Department of Flight Dynamics of the RWTH Aachen University has for several years been working on modelling, simulation and design of an unmanned coaxial helicopter's Flight Guidance and Control System (FGCS). In connection to a collaboration with EADS Innovation Works, Munich, a complete simulation environment and a Flight Guidance and Control System enabling autonomous operation for a coaxial helicopter UAV with a take-off mass of 200 kg has been set up, see Fig. 1.

Fig. 1: SHARC coaxial helicopter UAV [EADS].

The main focus of this contribution is the description of the employed simulation and its use for the investigation of the potential benefits of Dynamic Inversion control for flight guidance and control of the above mentioned SHARC helicopter. A main aspect is hereby the research of the dynamic properties und the associated consequences put upon the Flight Guidance and control system as well as the benefits of the coaxial helicopter's more symmetric design compared to the main and tail rotor configuration.

Dynamic Inversion

Dynamic Inversion is a control methodology by which the plant's dynamics are inverted by the controller and thus compensated, so that states and their associated trajectories can be directly commanded by a reference model. In case of a perfect inversion it is therefore possible to continuously and fully control the system and its states. Dynamic Inversion allows for full exploitation of the flight envelope and furthermore is the base for model following controller, where the plant's characteristic are replaced by model characteristics to be addressed.

Limitations to Dynamic Inversion

Although Dynamic Inversion offers a lot of benefits there are a number of shortcomings connected to its implementation. First of all, the perfectly accurate model of the to be inverted plant is not known, so that there are always discrepancies which can strongly influence the usability of Dynamic Inversion. Furthermore, under-actuated systems pose a problem as not all states can be directly addressed by the control inputs. As physical systems are strictly proper due to inertia, their inverse would be improper, which is not allowed as it would mean that the nominators order is higher than its denominators order, and thus the system would be able to change abruptly its states.

Theory of Dynamic Inversion

In Dynamic Inversion there is a linear approach and a non-linear approach. Non-linear Dynamic Inversion works by inversion of non-linear equations of motion compromising non-linear effects and geometric transformations [1]. Linear Dynamic Inversion is most conveniently performed as it is based on linear control theory which also nowadays forms the base for controllability and stability investigations. This is especially true in context to aerospace applications which are traditionally connected with higher conservatism due to safety issues. The following linear inversion scheme is described in [2] and fits in its algebraic form all problems given in state space form. It is the basis of the inversion controller used for the coaxial helicopter UAV in context of this contribution.

Starting from the linear state space formulation of a MIMO dynamic system, the state matrix can be expressed as:

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Cx + Du \quad (2)$$

Hereby *A* is the system matrix, *B* is the control matrix, *C* is the output matrix and *D* is the direct feed-through matrix. *x* represents the state vector and *u* the control vector of the system.

For a perfect inversion the serial connection of a transfer system *G* and its perfect inverse G^* generate a unity transfer system without any dynamics with $u=y^*$, see Fig. 2.

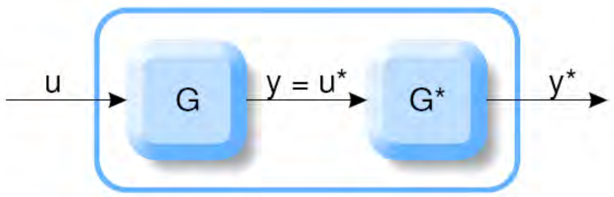

Fig. 2: Perfect Inversion [2].

If the feed through matrix is regular, then the inversion law can be directly built by solving the second state equation for u and substituting it in the equation for x, which yields:

$$\begin{aligned} u &= D^{-1}(y - Cx) \\ &= -D^{-1}Cx + D^{-1}y \end{aligned} \quad (3)$$

With the inverse transfer system coupled to it afterwards, its respective equations become:

$$\begin{aligned} \dot{x} &= Ax + B(-D^{-1}Cx + D^{-1}y) \\ &= (A - BD^{-1}C)x + BD^{-1}y \end{aligned} \quad (4)$$

The relevant matrices can thus directly be found by a coefficient comparison as input *u* and output *y* of the system are exchanged, leading to:

$$\begin{aligned} y &= u^* \\ \dot{x}^* &= (A - BD^{-1}C)x^* + BD^{-1}u^* \\ y^* &= -D^{-1}Cx^* + D^{-1}u^* \end{aligned} \quad (5)$$

The relevant matrices are found by a comparison of coefficients:

$$\begin{aligned} D^* &= D^{-1} \\ C^* &= -D^{-1}C = -D^*C \\ B^* &= BD^{-1} = BD^* \\ A^* &= A - BD^{-1}C = A - B^*C = A + BC^* \end{aligned} \quad (6)$$

The calculation of these matrices can be easily performed as algebraic expressions by introducing the Laplace variable s as symbolic equation and solving it with MATLAB through the *inv* command.

Inversion of strictly proper systems

While before presented case is admittedly easy, a problem arises from real physics systems which are strictly proper. For this reason, the inversion cannot be performed directly as the inversion would be improper in consequence.

One possibility to overcome this problem is the introduction of "propering" filters causing the transfer function to be bi-proper and thus invertible. As these filters include additional zeros in the original transfer function, they should be kept small not to alter the system's behaviour. Although accordingly small values for the time constants T in within the filters are preferred, they can cause some problems during numerical simulation as they increase the stiffness of the system [2]. The magnitude should therefore be some orders smaller than that of the smallest pole.

Another possibility quoted in [2] is the use of a highly parameterized controller gain in an additional control loop connected to the plant's model which also generates a perfect inverse as implicit method. This approach is however not used in the following.

Through the inclusion of filters and the representation of the dynamics with the Laplace variable *s* the inversion can be calculated algebraically and then brought to numerical form for use in simulations.

Dynamic Inversion of unstable systems

Stable systems can be more easily dynamically inverted than unstable ones. Although it is directly possible to apply Dynamic Inversion to latter ones numerical deviation issues quickly lead to instability. This especially applies when the model fits the plant poorly.

One approach to allow for Dynamic Inversion of unstable systems is to include a stabilizing controller into the plant acting parallel to the Dynamic Inversion. As this controller just has to guarantee stability it is not necessary to operate with a high gain or to be designed optimally and a proportional controller is sufficient [2]. In this context the stabilizing controller will avoid too high deviations from the operation point while the Dynamic Inversion is mainly responsible for command compliance.

Imperfect model and adaptation

As it is impossible to perfectly match plant and plant's model used for linear inversion there are different approaches allowing for compensation of inversion errors accompanied with this incomplete knowledge of the plant's characteristic. Next to Robust Control aiming at ensuring stability and performance in the face of uncertainties one very important approach is the use of Adaptive Control. While there is not only the problem of changing characteristics which could be tackled in a classic controller by appropriate gain scheduling, but mainly the lack of information or even misinformation on the plant adaptive elements have to autonomously react on these deficiencies.

Neural Networks

Very important in the context of adaptation are Artificial Neural Networks (ANN) allowing for a representation of these effects and comprising a learning capability to adapt to changing scenarios. They consist of neurons made up by an activation function as basic elements which are connected by weights. The net learns by adjusting the weights. For this learning algorithms as for example back propagation are used. As ANN can theoretically according to their size and structure model any system they provide a very useable base for compensating unknown effects in context to Dynamic Inversion where they are used as online networks continuously updating their weightings[3,4].

With Neural Networks comes the problem that the adaptation can cause the control inputs to become so large that real effects as actuator rate limiting or deflection limits and limited control power prevent the system to execute the adaptations commanded by the neural net. One possibility to avoid this problem is Pseudo Control Hedging (PCH) [5]. Hereby the plant's characteristics that are not meant to be seen by the Neural Net, as they would cause an adaptation latter one would not be able to execute, are masked from the Neural Network. This ensures that the plant runs within its operational limits and can conduct the required commands and even adapt in the presence of input saturation.

Dynamic Inversion for Helicopters

In context to non-linear inversion, very often the rotational dynamics are separated from the translational dynamics, and the relevant transformation matrices are used to describe the nonlinear interaction between them. Especially in the case of helicopters, where there is a strong interconnection between rotational and translational dynamics, this way seems useful. In fact, rotational dynamics can be perceived as a kind of actuator for translational dynamics[1, 7]. This becomes clear as the main rotor, which is a dynamic system for itself though with a higher magnitude of bandwidth compared to the rigid body dynamics, causes the thrust, which is scaled by the collective pitch and directed in pitch and roll attitude by the respective cyclic controls. Thus, through the tilting of the thrust vector, acceleration in the horizontal can be caused.

In many publications Dynamic Inversion is limited to this rotational dynamics [1, 4] as here most of the helicopters dynamics are inherent and the vertical and yaw motion are relatively easy to control as they possess first order dynamics. The outer control loops are accordingly put up by classic controller synthesis and the overall cascade structure. As the outer control loops are used for translational dynamics and thus velocity and position control, it can be useful to extend the dynamic version to this part of the controller structure. In [6] this is pursued to ensure execution of a defined trajectory with high bandwidth and agility. Hereby the classic bandwidth separation between inner and outer loop can be decreased allowing better manoeuvrability in the translational regime. The cascade structure is maintained but for each control loop Dynamic Inversion is used. For this reason the tracking of commands and disturbance rejection can be improved compared to a classical design.

The approach to use a simple linear model for inversion and combine it with a neural network is pursued in [8] for an unmanned RMAX helicopter. It is therefore similar to the here employed one.

Modelling

The basis for the investigation is a dedicated simulation model of the helicopter consisting of rotor and fuselage aerodynamics.

The coaxial helicopter is modelled with a 6-DOF simulation expanded by the flapping motion of the individual blades as primary rotor dynamic and thus input to the rigid body motion of the helicopter. The rotor aerodynamics is calculated by the use of the Blade Element Theory, whereby experimental data for the induced velocities in the rotor area for coaxial helicopters is available from former research programs on this topic at EADS. In addition, the results are validated by a second modelling approach utilizing a vortex method as “Free-Wake Time Marching” simulation, which allows the calculation of transient rotor aerodynamics and the influence of control and disturbance inputs to the rotor, see Fig. 2. The rotor is seen as a dynamic system causing forces and moments on the helicopter. The fuselage aerodynamics are simplified by the use of drag surfaces, which is a valid approach, especially for the near hover flight, see also [9].

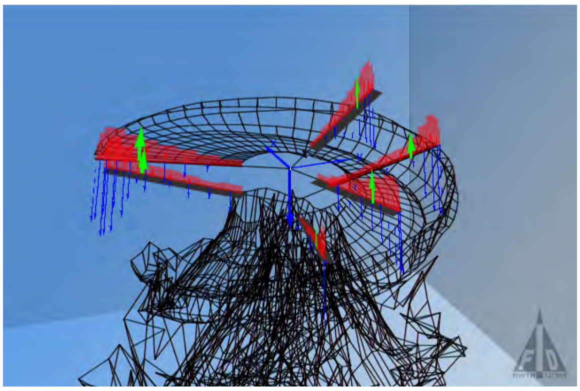

Fig. 3: Coaxial rotor with wake (shown only for one upper blade) in hover flight. Circulation in red, blade thrust in green and induced velocities in blue.

Control System

The Flight Guidance and Control System possesses a classic cascade structure with stacked control loops reaching form the Basic Controller (inner loop) to the Mission Controller (outer loop) commanding the Flight Guidance. Since the prevailing dynamics of the helicopter are dealt with by the Basic Controller with controlled states of pitch and roll attitude as well as vertical velocity and yaw rate, different control strategies have been investigated. These include classic analytical Pole Zero Placement, which is especially suitable for the near hover flight as well as Robust Control by the H-infinity method allowing a robust response to uncertainties and changes in flight dynamic properties which are typical for helicopters throughout the flight envelope. The Mission Control by the Department of Flight Dynamic is a tailored test-system for the evaluation of the control system. It allows the controllers to be commanded and configured (controller gains, limiters etc.) and enables the automatic performance of predefined flight manoeuvres as for example defined in ADS-33E-PRF. It is set up as state machine under Stateflow, a MATLAB/Simulink toolbox. Details can be found in [9].

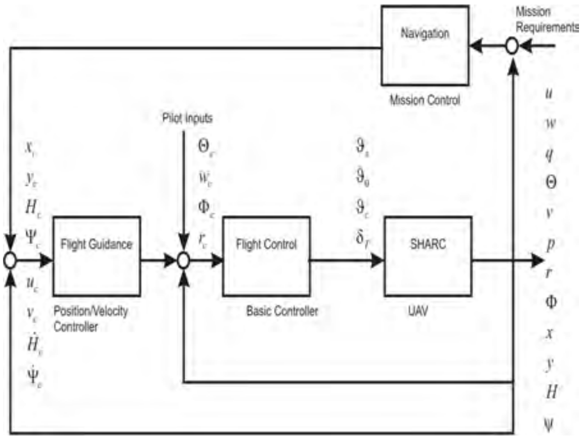

Fig. 4: Overall structure of the FCGS.

Application of Dynamic Inversion to the coaxial helicopter UAV

For the coaxial helicopter a linear Dynamic Inversion as described in [2] was chosen. Controlled variables include vertical and yaw rate and in contrast to latter one not the angular rates for pitch and role but attitude angles. Therefore, the structure of the control system remains unchanged with the Basic Controller with analytic pole placement replaced by a Dynamic Inversion controller. To account for model deficiencies, a neural network was implemented. The structure of the Basic Controller with Dynamic Inversion is shown in Fig. 5.

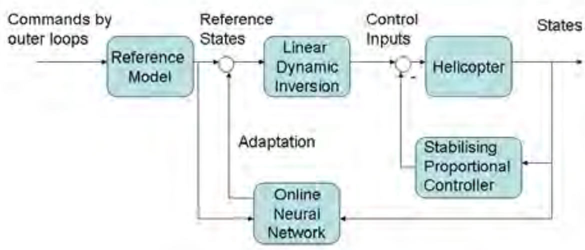

Fig. 5: Scheme for Basic Controller with stabilizing proportional controller and adaptive Neural Network for Dynamic Inversion.

For the design of the Dynamic Inversion, a state space system comprising the translational velocities u , v, w, the rotational rates p, q, r and the attitude angles Φ and Θ were used. The corresponding input and state vector **u** and **x** (**x**=**y**)are:

$$\mathbf{x} = [\mathrm{u\ v\ w\ p\ q\ r}\ \Phi\ \Theta\ ]^T,\ \mathbf{u} = [\vartheta_{coll}\ \vartheta_s\ \vartheta_c\ \delta_T]^T.$$

As command reference first order plants with 0.1 seconds time constant were used for yawing and vertical motion, while second order models with the same time constant were used for pitch and roll angle.

Due to the reduced couplings in control inputs for the more symmetrical coaxial configuration it was decided to reduce the control matrix *B* to main control effects, leaving the four primary control Inputs. Additionally, the couplings between longitudinal and lateral dynamics were neglected, see also following signal flow diagrams, Fig. 6-7.

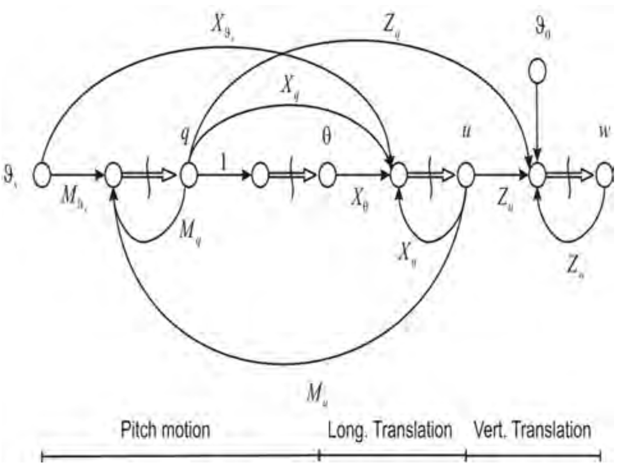

Fig. 6: Signal flow diagram for Longitudinal Motion as used in linear Dynamic Inversion.

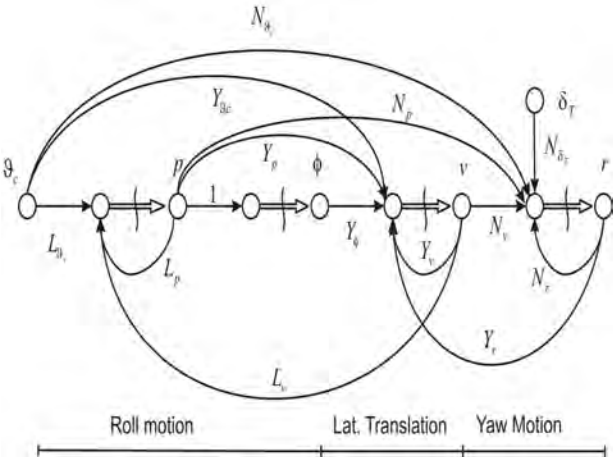

Fig. 7: Signal flow diagram for Lateral Motion as used in linear Dynamic Inversion.

As the neural network identifies the plant, its output can be used as additional input to the Dynamic Inversion reducing the inversion error. The dynamics of the systems are therefore incorporated in the inversion, while the parameters are maintained by the Neural Network. In the easiest form, the Neural Network has an input layer, one hidden layer and output layer. The hidden layer is built by a sigmoid activation functions as these have the advantage of upper and lower bound and they are continuously differentiable. The output layer is given as linear function. The activation function within the hidden layer for a single neuron is:

$$\sigma(z) = \frac{1}{1+e^{-z}} \tag{7}$$

The structure of the net can be seen in the following figure. For the learning of the net the back-propagation algorithm is used [3]. Due to the MIMO system the network can be described by matrices simplifying its calculation online during simulation. Aspects of bounds, stability and PCH are not to be addressed here. The relevant theory can be found in the literature [3-5].

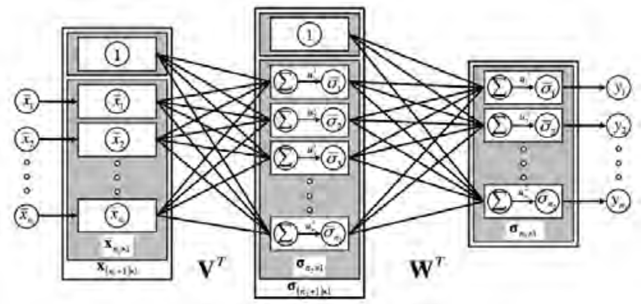

Fig. 8: Neural Network with input, hidden and output layer and relevant weighting matrices V^T and W^T [10].

The output y of the net can be calculated by using the relevant inputs x in conjunction with the weighting matrices and the activation functions resulting in:

$$y = \sigma_2(W^T \sigma_1(V^T x)) \, . \tag{8}$$

If the output layer is linear then the expression simplifies to the term within brackets. For simulation a sigmoid hidden layer and a linear output layer is used.

Simulation

In the following a Dynamic Inversion controller in combination with a stabilizing proportional controller and an adaptive online Neural Network was investigated in linear and non-linear Simulation. For latter on the blade element method was used for the rotor aerodynamics.

Linear Simulation

To setup the controller, the linear Dynamic Inversion was first conducted with a linear plant model. As stabilizing element for the plant a simple proportional controller with not optimized parameters was used. To account for model deficiencies a Neural Network was setup and the learning rate adapted within the simulation. The Neural Network was fed with commanded states, its derivatives and the error between commanded and measured states as well as latter absolute values. The learning rate was adjusted empirically within the simulation.

The following Fig. 9-12 show the results for this inversion simulation for different elements of the control setup active. It can be seen that due to numerical reasons the simulation becomes unstable when no stabilizing controller is used. The controller therefore allows for nominal performance, when the plant's model used for the Dynamic Inversion design equals the plant. The commands are then perfectly matched. In the presence of deficiencies, the inversion cannot longer be perfectly accomplished and deviations appear. By inclusion of a Neural Network acting on the Dynamic Inversion, these deviations can be compensated through an online adaptation.

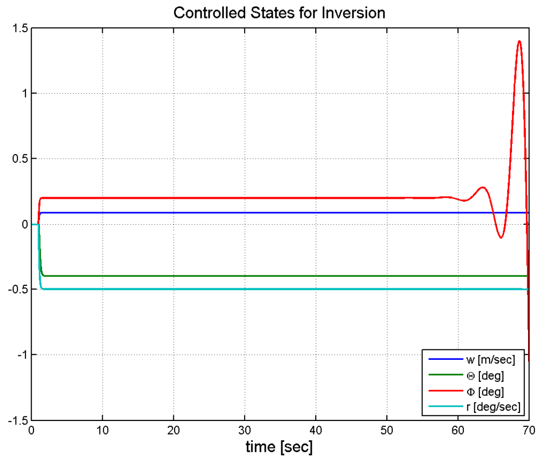

Fig. 9: Step inputs to controlled system with inversion only and following instability due to numerics.

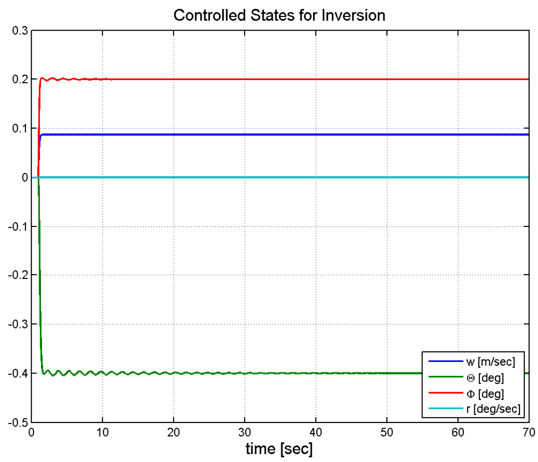

Fig. 10: Step inputs to controlled system with inversion and stabilizing proportional controller.

In the last case with adaption the plant's dynamic and control input matrix elements were changed randomly by +/- 100 %. The results are shown for stabilizing controller only and stabilizing controller with Neural Network adaptation in Fig. 11-12.

The Neural Network is able to compensate the oscillation and to maintain the states close to the commanded values whereas in the case with proportional controller only the system response clearly deteriorates.

The results showed good compliance due to the adaptive Neural Network also for from the model differing plants underlining the justification of this approach in combining Dynamic Inversion control with a stabilizing controller and a Neural Network for adaptations.

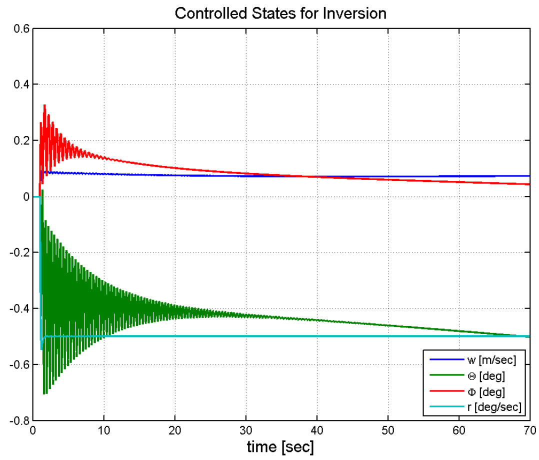

Fig. 11: Step inputs to controlled system with stabilizing proportional controller in presence of model deviation.

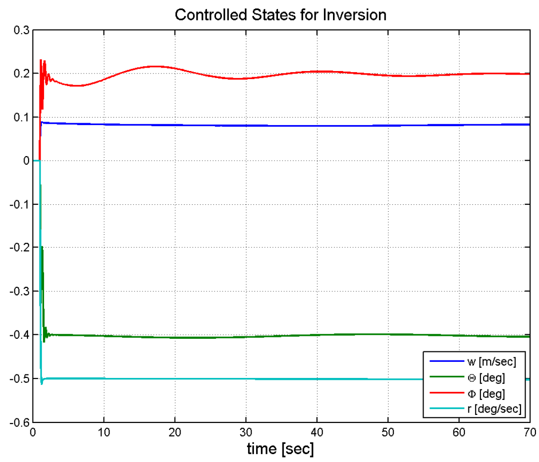

Fig. 12: Step inputs to controlled system with stabilising proportional controller and adapting online Neural Network in presence of model deviation.

Non-linear Simulation

The control system evaluated as Basic Controller in the linear simulation was transferred in the non-linear simulation and incorporated within the overall FCGS.

By incorporating the controller into this more general simulation non-linearity, non-modelled dynamics (blade flapping) resulting in a compared to the plant reduced order model used for inversion, as well as real effects in form of actuator dynamics are introduced. These cause errors in the control system and lead to deviations from the commanded reference states.

As test a sinus command was given for the Θ angle with an amplitude of +/-10 deg and a frequency of ω=3 rad/sec starting at zero seconds. Due to the discrepancies between model and plant the Dynamic Inversion with proportional controller only was not able to generate the commanded oscillation and shows unstable behaviour despite the proportional controller, see Fig. 13.

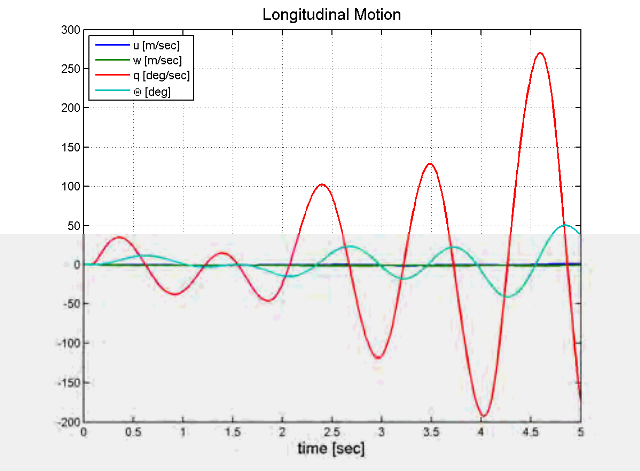

Fig. 13: Non-linear simulation of longitudinal motion for pitch oscillation with inversion and stabilizing controller only leading to divergence.

In the second case a Neural Network with V being a 8x13 and W being a 4x9 matrix was used for adaptation. Although not perfectly matching the amplitude, the curves adapt quickly to the command and the Neural Net even improves stability, see Fig. 14.

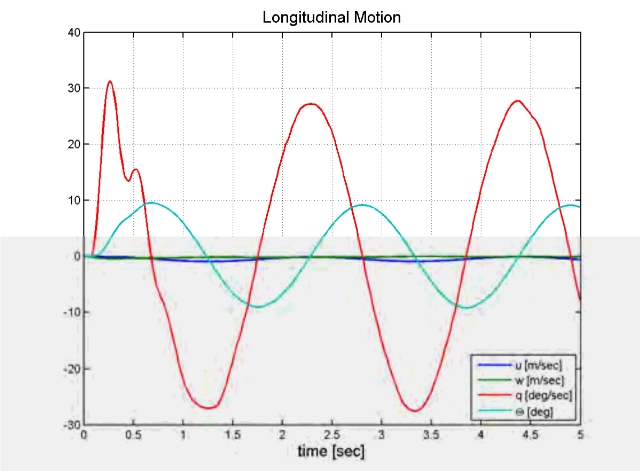

Fig. 14: Non-linear simulation of longitudinal motion for pitch oscillation with inversion, stabilizing controller and online Neural Network.

Due to the stabilizing controller and the adaptivity of the Neural Network the controller remains robust and clearly adapts to the changing plant characteristics.

However, the dynamic structure is important for the accuracy of the Dynamic Inversion. The inversion of a system with a reduced order model has it limitations also a simple Neural Network without incorporated dynamics as used in this context cannot overcome. Nevertheless, even with these limitations it is possible to build a basic adaptive inversion controller with by simple elements.

Conclusion and Outlook

Together with the identified dynamic properties of the helicopter the necessary steps for the implementation of a Dynamic Inversion control for operation of an unmanned helicopter were presented. It is shown that the use of insight into the dynamic properties and their evolution throughout manoeuvres Dynamic Inversion allows a beneficiary control of the helicopter with the possibility to generate explicitly specified state trajectories by reference models.

The problems of stability and model discrepancy were addressed by the inclusion of a proportional controller and a Neural Network. The results in the linear and non-linear simulation show the benefits of this approach.

For highest accuracy the model order should comply with the system's dynamic order or posses adequate adaptation capabilities to account for the model discrepancy. The inversion possesses the order of the nominal system while the neural net allows for adaptation of its parameters. However, a simple net without dynamics acting on the Dynamic Inversion can therefore not compensate missing dynamics within the inversion model.

Future work will focus on extension of this approach by non-linear Dynamic Inversion and improved Neural Networks accounting for dynamics. Furthermore, with increased bandwidth the PCH will be included in the design.

Nomenclature

A	Dynamic matrix
ANN	Artificial Neural Network
B	Input matrix
C	Output matrix
D	Feed-through matrix
G	Plant
DOF	Degree of Freedom
FCGS	Flight Control and Guidance System
H	Height above ground
L_i	Roll body-fixed Moment-Derivative with respective index i

M_i	Pich body-fixed Moment-Derivative with respective index i
MIMO	Multiple Input Multiple Output
N_i	Yawing body-fixed Moment-Derivative with respective index i
PCH	Pseudo Control Hedging
p	Angular rate along body-fixed x-axis [deg/s]
q	Angular rate along body-fixed y-axis [deg/s]
r	Angular rate along body-fixed z-axis [deg/s]
t	Time [s]
T	Time constant
u	Translational velocity in body-fixed x-axis [m/s]
u	Input vector
UAV	Unmanned Aerial Vehicle
v	Translational velocity in body-fixed y-axis [m/s]
V	Weighting matrix (input to hidden layer)
w	Translational velocity in body-fixed z-axis [m/s]
W	Weighting matrix (hidden to output layer)
x	North position relative to origin [m]
x	State vector
X_i	Longitudinal body-fixed Moment-Derivative with respective index i
y	East position relative to origin [m]
y	Output vector
Y_i	Lateral body-fixed Moment-Derivative with respective index i
z	Vertical position relative to origin [m]
Z_i	Vertical body-fixed Moment-Derivative with respective index i
δ_T	Differential pitch [deg]
Ω	Angular velocity of blade rotation [deg/s]
Φ	Bank attitude [deg]
ϑ_0	Collective blade pitch [deg]
ϑ_c	Cyclic lateral pitch [deg]
ϑ_s	Cyclic longitudinal pitch [deg]
Θ	Pitch attitude [deg]
Ψ	Heading [deg]
β	Blade flapping angle [deg]
σ	Activation function
θ	Blade element pitch [deg]
ω_0	Controller Design Bandwidth [1/s]

Indices

*	True Inverse
c	"commanded"

References

[1] Enns, D. and Keviczky, T., Dynamic Inversion Based Flight Control for Autonomous RMAX Helicopter, Proceedings of the 2006 American Control Conference, Minneapolis, Minnesota, USA, June 14-16, 2006

[2] Buchholz, J. J., v. Grünhagen, W., Inversion Impossible?, Research Paper, Bremen/Braunschweig, 2007, ISBN 978-3-638-90059-1

[3] Lewis, F. L., Jagannatha, S. and Yesildirek, A., Neural Network control of robot manipulators and nonlinear systems, Francis & Taylor Inc. Philadelphia, PA, 1999

[4] Calise, A. J. and Rysdyk, R. T., Nonlinear Adaptive Flight Control using Neural Networks, IEEE Control Systems Magazine, 1998

[5] Johnson and Anthony J. Calise, PSEUDO-CONTROL HEDGING: A NEW METHOD FOR ADAPTIVE CONTROL, Advances in Navigation Guidance and Control Technology Workshop, November 1-2, 2000,Redstone Arsenal, Alabama

[6] Johnson, E. N. and Kannan, S. K., "Adaptive Trajectory Control for Autonomous Helicopters," Journal of Guidance, Control, and Dynamics, Vol. 28, No. 3, 2005, pp. 524-538

[7] PADFIELD, G.D., "Helicopter Flight Dynamics", Blackwell Publishing, Oxford, 2007

[8] Corban JE, Calise AJ, Prasad JVR, Hur J, Kim N. Flight evaluation of adaptive high bandwidth control methods for unmanned helicopters. AIAA Guidance, Navigation and Control Conference and Exhibit, Monterey, California, 2002

[9] Loechelt, S., Linke, F., Simulation of a Coaxial-Helicopter UAV's Flight Test Program based on ADS-33E-PRF for Control System Performance Evaluation, AHS 66th Annual Forum, Phoenix, AZ, May 11-13, 2010

[10] Holzapfel, F., Nichtlineare adaptive Regelung eines unbemannten Fluggerätes, Dissertation, 2004