## PAPER $N^{\circ} 29$

A FREE WAKE ANALYSIS FOR HOVERING ROTORS AND ADVANCING PROPELLERS
by

## D. FAVIER, M. NSI MBA, C. BARBI, C. MARESCA

Institut de Mécanique des Fluides de Marseille U.A.-03 du C.N.R.S.

1, rue Honnorat, 13003 Marseille, France


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## ABSTRACT

From new experimental results obtained on either hovering rotors or propellers in axial flight, an attempt in improving and checking a free wake analysis model is made. In both flight cases the wake is divided into near and far regions which are empirically prescribed according to synthetized laws of contraction and convection obtained in each region. For hovering rotors, the free wake model is checked by comparisons with data obtained either on the circulation distribution measured along the blade or on the tip vortex path and induced velocity field measured in the wake. For the wake propeller in axial flight, a particular attention is given to the dependence of the prescribed wake geometry on the upstream conditions (operating parameter, collective pitch angle). As function of these parameters, the synthetized empirical laws are derived for the prescribed wake geometry in axial flight.

## NOMENCLATURE

| $A, B$ | $:$ Wake contraction coefficients, (Eq. 3) |
| :--- | :--- |
| $\alpha$ | $:$ Local section angle of attack, (deg) |
| $\alpha_{0}$ | $:$ Propeller blade pitch at $\xi=0.7,(\operatorname{deg})$ |
| $b$ | $:$ Number of blades |
| $C$ | $:$ Local blade chord, (m) |
| $C_{L}$ | $:$ Local lift coefficient |
| $C_{T}$ | $:$ Rotor thrust coefficient, $\left(C_{T}=T / 0(\Omega R)^{2} \pi R^{2}\right)$ |


| $Y$ | : Propeller operating parameter, ( $\left.\gamma=\mathrm{V}_{\infty} / \mathrm{nD}\right)$ |
| :---: | :---: |
| $\Gamma_{b}, \Gamma$ | : Circulation on the blade or in the wake, $\left(\mathrm{m}^{2} / \mathrm{s}\right)$ |
| D | : Diameter of rotor $\left(\mathrm{D}_{\mathrm{R}}=1.50 \mathrm{~m}\right)$ or propeller ( $\mathrm{D}_{H}=0.85 \mathrm{~m}$ ) |
| $\xi$ | : Reduced blade radius, $(\xi=r / R)$ |
| $K_{1}, K_{2}$ | : Tip vortex translation coefficients (Eq.4) |
| $\lambda$ | : Propeller advancing parameter, ( $\lambda=\gamma / \pi)$ |
| n | : Blade rotational frequency (r.p.s.) |
| OXYZ | : Fixed axis system coordinates, (Fig. 4) |
| $\psi$ | : Blade azimuthal angle, (deg) |
| $\psi_{b}$ | : Azimuthal periodicity, ( $\left.\psi_{\mathrm{b}}=360^{\circ} / \mathrm{b}\right)$ |
| $\psi_{s}$ | : Azimuthal instabilities occurence in the far wake, (Eq.5) |
| $r$ | : Radial distance from the axis of rotation, (m) |
| R | : Radius of rotor ( $R=0.75 \mathrm{~m}$ ) or propeller ( $\mathrm{R}=0.425 \mathrm{~m}$ ) |
| $X$ | : Propeller power coefficient, ( $X=P / \rho n^{3} D^{5}$ ) |
| T | : Rotor or propeller thrust, (N) |
| T | : Propeller thrust coefficient, $\left(\tau=T / \rho n^{2} D^{4}\right)$ |
| $\theta v$ | : Local blade twist angle, (deg) |
| ${ }^{\theta} 75$ | : Rotor blade pitch at $\xi=0.75$ (deg) |
| U,V,W | : Velocity components defined on Fig. 4 |
| $V_{\infty}$ | : Freestream axial velocity (m/s) |
| $\Omega$ | : Angular rotational speed, $\Omega=2 \pi n,(\mathrm{rd} / \mathrm{sec})$ |

Subscripts
"t" : Denotes quantity relative to the tip vortex
"0" : Denotes quantity nondimensionalized by $V_{\infty}$

## Superscript:s

```
"-": Denotes quantity evaluated at }\xi=0.
"~" : Denotes quantity nondimensionalized by \OmegaR
```


## 1. INTRODUCTION

The prediction of rotary wings performance in various flight conditions remains one of the more challenging of the unsolved problems of modern rotorcraft aerodynamics. Several methods have been proposed for airloads and wake geometry predictions, with or without advancing velocity
of the rotating plane (see refs. 1-10). These calculation procedures are generally developed on the basis of either prescribed $(1),(2)$ or free wake (3)-(6) models, with the use of lifting lines or lifting surfaces ${ }^{(7)}$, (8) for the blade loading distribution, and straight or curved $(9),(10)$ elements for the wake distorsion influence.

Although the hovering case appears to be the simplest flight configuration, it has been one of the most extensively studied in recent years. In most hovering flight investigations, the free wake analysis is based on an iterative calculation of the circulation distribution on the blade, while the codes validations are generally made by comparison with data obtained on wake geometry, induced velocity field and aerodynamics airloads measurements. Although very few experimental results (11), (12), (13) are presently available on the distribution circulation along the blade radius, the need exists for a direct comparison and a better codes validation of this quantity.

Moreover, the comparisons between calculations and available experiments have shown good agreement in some flight configurations, and increasing deviations in other ones. Specially for hovering rotors with non linear twisted blade, or sharp evolutive tip shape. Among others, one major reason of these discrepancies seems to lie in a still inadequate modelling of important parameters of the downwash wake. In addition to an adequate wake geometry generalized ${ }^{(14)}$ to both hovering and advancing flights a few examples of these parameters are : the complete modelling of the far wake region where spatial and azimuthal vortex instabilities take place ; the evolution of tip vortex core size and strength along the path, as well as their correlation with the circulation distribution established near the blade tip.

In the present work an attempt is made to adress some of the points previously raised. For both hovering and axial flight configurations, the paper presents some new experimental results obtained on rotors and propellers wakes. For the hovering rotors, the validation of a free wake analysis model is checked by direct comparison with circulation distribution measured along the blade radius by laser velocimeter. For propeliers in axial flight, a particular attention is given to the complete description of the near and far wake associated with different advancing parameters. Specially it is
shown that synthetized laws of contraction and transtation for the tip vortex can be expressed in a form quite similar to those already established on rotor in hover (1), (2).

## 2. EXPERIMENTAL FACILITIES AND TEST PROCEDURES

2.1. Models and test conditions

Model-scales of rotor and propeller were set-up in the open test chamber (elliptical test section : $3.3 \times 2.2 \mathrm{~m}^{2}$ ) of the I.M.F.M.-S1 subsonic wind-tunne1 (freestream velocity $0 \leqslant V_{\infty} \leqslant 50 \mathrm{~m} / \mathrm{s}$ ). Photographs showing each model mounted on its supporting mast are presented in Figs. 1 and 2.

Table 1 : Test conditions :

|  | ROTOR : | PROPELLER : |
| :---: | :---: | :---: |
| : Diameter | 1.50 m : | 0.85 m |
| : Root cut out | $0.22 \mathrm{R} \quad$ : | 0.176 R |
| : Angular rotational frequency | $143 \mathrm{rd} / \mathrm{s}$ : | 140-280 rd/s |
| : Rotational tip speed | $107 \mathrm{~m} / \mathrm{s} \quad:$ | $60-120 \mathrm{~m} / \mathrm{s}$ |
| : Coning angle | $2.5^{\circ} \quad:$ | $0^{\circ}$ |
| : Number of blades | 2-6 : | 4 |
| : Airfoil section | $\begin{array}{lll} \hline \text { OA } 209 \\ \text { BV } 23010 & \vdots \\ \hline \end{array}$ | NACA 64 A 408 |
| : Blade chord | 0.05 : | see Table 2 |
| Blade twist | -8.3 $\left.{ }^{\circ} \mathrm{L}\right)$ : | see Table 2 (N.L.) : |

Table 2 : Blade propeller thickness, chord and twist distributions


The model rotor consists of a fully articulated rotor hub which can be equipped with interchangeable sets of blade. Rotor geometry and hovering test conditions are summarized on Table 1 . The different sets of rotor configuration tested (see Fig. 3) are numbered from 1 to 7 , and correspond to various combinations of blade twist, airfoil section and tip shape. All these different tip geometries are calculated so that the radii of the corresponding blade remain constant and equal to $R=0.750 \mathrm{~m}$.

For the model propeller the operating conditions are defined on Table 1, and the thickness, chord and non linear twist distributions along the blade radius $\xi$ are given on Table 2. A large scale of propeller operating parameters $(0.2<\gamma<1.1)$ has been investigated in order to cover several flight conditions varying from zero thrust to maximum thrust.

### 2.2. Measurement procedures

Several measuring techniques suited for surveying the flow around the blades and the flow in the near and far wake regions have been used, including $X$-wires anemometry, 2D-1aser velocimeter, and flow visualisations. Figure 4 gives the fixed system coordinates (OXYZ) used for the wake survey and sketches the measuring techniques implemented for the propeller tests. More detailed informations concerning these techniques can be found in Refs. (11), (15), (16).

- Aerodynamic forces measurements (averaged thrust and torque) are made by means of strain-gauge cells mounted on the supporting mast of the models.
- The 3D wake velocity field is measured by two complementary techniques : a $X$-hot wires probe displaced behind the rotating plane along radial and axial coordinates $(r, Z)$ by means of teledriven gears (see Figs. 1,4) ; and a $2 D$ Laser Doppler velocimetry technique (backscattering mode, Bragg cells for reversed flow regions on the hovering rotor). Figure 5 presents the L.V. system and its automated supporting device which allows displacement of the focal measurement volume all along the three orthogonal axes and around the blade isee Ref. 15).
- Tip vortex paths $(r, Z, \psi)$ are measured on both models by means of a hot-wire technique ${ }^{(15),(16)}$ which allows additionally the determina-
tion of the far wake position where the vortex instability starts appearing and develops. However, in order to determine the tip vortex path for the wake region very close to the blades, a visualization flow technique has also been used. The technique consists of emitting white smoke filaments either upstream the rotating plane (propeller) or in the reversed flow region (rotor in hover). Then, a stroboscopic flash synchronized on the blade rotation lights up the emission lines which are filmed by a video camera and recorded on a magnetoscope tape.
- The measurement of the circulation distribution along the rotor blades is carried out by means of a 2D Laser method suited for the hovering flight case. This technique allows to perform circulation measurement bv only considering the variation, as function of azimut ( $0 \leqslant \psi \leqslant 2 \pi / b$ ), the velocity tangential component on the upper and lower side of the rotating plane (see Ref. 11).

All the azimuthal-dependent data are stored and analyzed through an acquisition and reduction data system (interface and IIP 9845B computer). The initiation and synchronization of acquisitions are realised by means of a Photo cell and an angular counter. For the L.V. acquisition data histograms are realised at each phase angle on 200 samples of validated instantaneous velocities (see Refs. 11,15). Moreover, each velocity component U.V,W has been represented by 10th order Fourier series at any given measuring point.

## 3. FREE WAKE ANALYSIS

The resolution technique and the basic equations used for the code are described in details in references (4) and (15). Only summarized here are the main operations mode and the new far wake modelling introduced.

The blade is considered as a lifting line spanning the quarter chord sections, and the bound vorticity is continuously distributed along this line. The wake is formed by a finite number of discrete vortex lines shed from each blade. The operating mode consists of dividing this wake in a strong rolled-up tip vortex filament as a result of grouping some tip vortices, and in several weaker trailing vortices lines representing the inboard vortex sheet. From this wake representation the tip vortex line can be computed and adaptated from its starting point ( $\psi=0$ ) on the em
ting blade until the first encounter of the following blade which corresponds to the azimuth $\psi=2 \pi / b$.

For the axial flight the initial prescribed wake geometry will be discussed later on in section 5 .

For the hovering case the initial geometries of the tip vortex line and inboard sheet are assumed to be described according to the empirical laws synthetized in Refs. (1), (2). The tip vortex contraction and convection are then given by the following expressions:

```
Radial coordinate :
```

$$
\begin{equation*}
r_{t} / R=A^{\prime}+\left(1-A^{\prime}\right) e^{-B^{\prime} \psi} \tag{1}
\end{equation*}
$$

## Axial coordinate :

$$
Z_{t} / R= \begin{cases}K_{1}^{\prime} \psi & \text { for } \psi \leqslant 2 \pi / b  \tag{2}\\ K_{1}^{\prime}(2 \pi / b)+K_{2}^{\prime}(\psi-2 \pi / b) & \text { for } \psi \geqslant 2 \pi / b\end{cases}
$$

Where the coefficients $A^{\prime}, B^{\prime}, K_{1}^{\prime}, K^{\prime}{ }_{2}$ are only dependent on thrust coefficient, solidity, and collective pitch(see Ref. 1,2). The vortex sheet geometry is also expressed in a similar form.

Moreover for the far wake region, the previous code version (
assumed that, for $\psi \geqslant 8 \pi / b$, the whole wake could be replaced ${ }^{(2)}$ by a strong vortex ring of radius 1.2 R and arbitrary intensity ( $4 \Gamma$, where $\Gamma$ is the initial tip vortex strenght at its emitting point).

From the experimental rotor wake surveys a new far wake modelling has been introduced as depicted in Fig. 6. In the near region, the free wake analysis is realised until $\psi=2 \pi / b$, and the prescribed wake Equations (1), (2) are applied until $\psi=5 \pi / b$. Beyond the azimuth $\psi=5 \pi / b$, experiments indicate that some vortex instabilities begin to start while unsignificant wake contraction is observed. Consequently, as $5 \pi / b \leqslant \psi \leqslant 10 \pi / b$ the wake geometry is described by an helicoildal path with constant pitch (Eq. (2) and $r_{t} / R=$ constant). For $\psi \geqslant 10 \pi / b$, the remainder
of the wake exhibits strong azimuthal and spatial vortices instabilities and a semi-infinite cylinder model with constant vorticity distribution is used for this last region.

From this wake representation the calculation process consists in the three-steps iterating procedure presented in Fig. 7. The required input data are : the geometrical configuration (number of blades, solidity, collective pitch, blade twist, ...), the $2 D$ aerodynamic airloads and moments coefficients, and the total thrust coefficient. As function of these input data the initial wake geometry is deduced from the empirical modelling (step 0 on Fig, 7). The steps 1 and 2 are repeated until the tip vortex line is tangent to the induced velocity field. Then the proc from steps 1 through 3 is iterated until the bound circulation of step 3 converges to the blade circulation distribution of step 1.

## 4. RESULTS ON HOVERING ROTORS

Measurements of the tips vortex path, the induced velocity components, and the circulation distribution along the blades, have been carried out for the different hovering rotor configurations of Fig. 3 . These three kinds of data have been compared with corresponding results of the free wake calculation model.

In order to check the far wake modelling previously described in section 3 , the comparisons have been made on the one hand with the calculation model based on the vortex ring as far wake (denoted model 1), and on the other hand with the calculation model using the semi-infinite cylinder as far wake geometry (model 2). The examples of comparisons between experiment and calculations presented in this section concern rotors 5 and 7 in four-bladed configuration, and traduce the influence of tip shape and collective pitchangle.

The results of Fig. 8, 9, 10 concern the rotor 7 (b $=4$; $\theta .75=10^{\circ}$ ), and generally show that a better agreement between experiment and calculation is obtained when considering the results from model 2 using the circular cylinder as far wake geometry.

For the tip vortex path (Fig. 8) a better agreement is obtained with model 2 , specially for the radial contraction of the near wake region $\left(0^{\circ} \leqslant \psi \leqslant 90^{\circ}\right)$. The influence of the far wake modelling and the deviation between both calculation models is quite clear on the radial circulation distribution presented in Fig.9. The results indicate that the maximum circulation value, its radial location, as well as the spanwise distribution of lift coefficient are better predicted from model 2.

Concerning the axial velocity, Fig. 10 gives for nine different rotation phases $\left(\psi=0^{\circ}, 5^{\circ}, 10^{\circ}, 20^{\circ}, 35^{\circ}, 45^{\circ}, 55^{\circ}, 70^{\circ}, 85^{\circ}\right)$ the radial evolution of the $W$-component measured and calculated at $Z / R=0.177$ in the wake. In addition to the plots calculated from model 2, the Figure also presents the results deduced either from model $1^{*}$ (using the ring vortex only during the free wake steps 1 and 2 of Fig. 7), or from model 3 (using the ring vortex through all steps of Fig. 7). The comparison clearly indicates that the prediction of the velocity field is quite improved when using mode 12.

Fig. 11 concerns the hovering rotor 5 and presents the comparison of calculated and measured distributions of circulation and lift coefficient for $b=4 ; \theta_{.75}=8^{\circ}$. The plots from models 1 and 2 are additionally compared to the results from a free wake analysis code ${ }^{(8)}$ based on a lifting surface method. Both results deduced from this last model and from the present model 2 are shown to agree together and with experiment all along the blade radius, and until the maximum circulation peak is reached ( $\xi=0.95$ ). For this radial section both methods underestimate the maximum circulation value as obtained from L.V. measurements.

All the tests conducted for the determination of circulation distributions along the blades have shown an important effect of the tip geometry on the radial location and amplitude of the maximum circulation peak. As an example, Fig. 12 compares the blade circulation measured on rotor 5 (tapered tips) and on rotor 7 (rectangular tips). Although the number of blades $(b=4)$, the collective pitch $\left(\theta .75=8^{\circ}\right)$, the linear blade twist $\left(\theta_{v}=-8.3^{\circ}\right)$, and all other rotor operating conditions were the same for both rotors, quite different spanwise distributions and location of the peak value can be observed in Fig. 12.

## 5. RESULTS ON ADVANCING PROPELLERS

As previously mentionned, the empirical modelling of the trailing vortices geometry mainly concerns the hovering flight case, and several prescribed wake geometry models are presently available at least for linear twists and conventional tip shapes. Only a few investigations have been concentrated on axial/forward flight (see refs. 14, 15,17). An example of empirical modelling of the propeller wake, sufficiently complete to serve as an input data for a free wake analysis code, is given in this section.

As shown in Fig. 13, the propeller wake has been investigated over a large range of thrust coefficient and operating conditions $\left(23^{\circ} \leqslant \alpha_{0} \leqslant 32.5^{\circ}\right.$, and $\left.0.2 \leqslant \gamma \leqslant 1.1\right)$. As function of these parameters the tip vortex contraction and convection have been synthetized according to the following expressions :

Radial coordinate :
(3) $\quad r_{t} / R=A+(1-A) e^{-\psi / B}$

$$
\text { for } 0 \leqslant \psi \leqslant \psi_{S}
$$

with : $A\left(\alpha_{0}, \gamma\right)=P_{A}(\gamma)+\alpha_{0} Q_{A}(\gamma) ; B\left(\alpha_{0}, \gamma\right)=P_{B}(\gamma)+\alpha_{0} Q_{B}(\gamma)$
where $\left(P_{A}, Q_{A}\right) ;\left(P_{B}, Q_{A}\right)$ can be expressed as 2nd order polynomial expressions of $\gamma^{2}$ (see Ref. (16)). Figs. 14A and $14 B$ give the evolution of $A, B$ contraction coefficients as function of $\alpha_{0}$ and $\gamma$.

Axial coordinate :

$$
Z_{t} / R= \begin{cases}k_{1}\left(\psi / \psi_{b}\right) & \text { for } 0 \leqslant \psi \leqslant \psi_{b}  \tag{4}\\ k_{1}+k_{2}\left(\psi / \psi_{b}-1\right) & \text { for } \psi_{b} \leqslant \psi \leqslant \psi_{s}\end{cases}
$$

with : $k_{1}=z_{0}+Z_{1} x+z_{3} x^{3} ; k_{2}=z^{\prime}{ }_{0}+Z^{\prime}{ }_{1} x+z^{\prime}{ }_{2} x^{2}+z^{\prime}{ }_{3} x^{3}$ where coefficients $\left(Z_{i}\right)_{0 \leqslant i \leqslant 3}$ and $\left(Z_{i}\right)_{0 \leqslant i \leqslant 3}$ can be expressed ${ }^{(16)}$ as polynomial expressions of 2nd order in $\alpha_{0}$, and 1st order in $\gamma$. The $X$-parameter is defined as $X=\lambda / \lambda_{T}=\gamma / \gamma_{T}$, where $\lambda_{T}$ is the advancing parameter corresponding to the zero thrust coefficient, which is given by $\lambda_{T}=0.7 \mathrm{tg}\left(\alpha_{0.7}\right)$ as shown on Fig. 13. The variations of $K_{1}$ and $K_{2}$ as function of $\alpha_{0}$ and $\gamma$ are given in Figs. 15A and 15B.

In the previous expressions (where azimuths are expressed in degrees), $\psi_{\mathrm{b}}$ only represents an azimuthal periodicity ( $\psi_{\mathrm{b}}=36^{\circ} / \mathrm{b}$ ), while $\psi_{S}$ gives the azimuthal position of the far wake region where vortex insta.bility appears. This azimuthal far wake position has been also synthetized by the following empirical law :

$$
\left(\psi_{\mathrm{s}}-\psi_{\mathrm{b}}\right) / \mathrm{b} \psi_{\mathrm{b}}=1 / 4 \quad\left\{8.5-\alpha_{o} / 10-\gamma(2+\gamma)\right\}
$$

It can be noticed that similar form of the empirical laws are obtained for either the hovering flight (Eqs. 1-2) or the axial flight (Eqs. 3-4). However, it should be pointed out that coefficients (A,B) and $\left(K_{1}, K_{2}\right)$ are dependent on the upstream conditions $\left(\alpha_{0}, \gamma\right)$ for the propeller wake.

In order to illustrate this fact, Fig. 16 presents the tip vortex paths obtained in the wake of the propeller when operating at three different couples of parameters $\left(\alpha_{0}, \gamma\right)$, selected so that the thrust coefficient remains constant at $\tau=0.16$ (see Fig. 13). Axial and radial tip vortex coordinates $\left(r_{t} / R, Z_{t} / R\right)$ are plotted on the figure as function of the blade rotation $\psi$. The polynomials fitting from Eqs. (3),(4) are represented by full lines, while the $\psi_{s}$ far wake position is indicated by arrows as deduced from Eq. (5).

Although the thrust coefficient is constant for the three couples of ( $\alpha_{0}, \gamma$ ) investigated, quite different wake geometries are shown on Fig.16. It can be seen that the wake contraction increases with decreasing $\alpha_{0}$. Moreover, the azimuthal vortex instability is occuring along the $r_{t} /$ R-curve at an azimuth $\psi_{s}$ which increases with increasing operating parameters $\gamma$.

After the tip vortexpaths have been determined, the 30 velocity field of the propeller wake has been measured in five different downstream planes $Z / R=$ constant. In each of these planes the $U, V, W$ components have been measured, as function of $\psi$, along 15 blade radius sections ( $0.2<\xi<1$ ). This detailed wake survey, realised for the three couples of parameters ( $\alpha_{0}, \gamma$ ) previously defined in Fig. 16, allows a complete description of the prescribed wake (axial and radial behaviours, inboard vortex sheet) at each azimuth of the blade rotation.

An example given in Fig. $17 \mathrm{~A}, \mathrm{~B}$ concerns the radial velocity field as deduced from $U(\psi)$ and $V(\psi)$ components measured at two given $Z / R$ downstream the rotating plane. For $Z / R=0.231$ and $Z / R=0.776$, Figs. 17A, $B$ respectively present for $\alpha_{0}=27^{\circ}$, a detailed radial flowfield which can be considered as steady when the four blades are fixed at azimuths $\psi=0^{\circ}$, $90^{\circ}, 180^{\circ}, 270^{\circ}$. From such representations at different $Z / R$ and by means of closed contours of increasing size around the trailing vortex core, the total circulation evolution is evaluated along the downstream wake. Thus, checking the validity of models relating the spanwise circulation distribution to the fully rolled-up vortex structure (as the BETZ model, see Refs. 18,19), becomes possible.

Moreover, concerning the inboard vortex sheet, its evolution rimbeen deduced from the wake survey as follows :

For a given couple of parameters $\left(\alpha_{0}, \gamma\right)$, and a given radius $r_{b} / R$ on the blade, the corresponding axial sheet position and velocity have been expressed by polynomials fitting of the forms :

$$
\begin{equation*}
Z / R=C_{0}+\sum_{i=1}^{7} C_{i}\left(r_{w} / R\right)^{i} ; \quad W=C_{0}^{1}+\sum_{i=1}^{7} C_{i}^{1} w_{i} \tag{6}
\end{equation*}
$$

where coefficients $\left(C_{0}, C_{i}\right)$ and $\left(C_{0}^{1}, C_{i}^{1}\right)$ are expressed as 2nd order polynomials expressions of $\psi$ :

$$
c_{i}=B_{0}^{i}+B_{1}^{i} \psi+B_{2}^{i} \psi^{2} \quad \text { for } \quad i=0,1,2, \ldots, 7
$$

In the previous expression (6), the radial position in the wake $\left(r_{w} / R\right)$, is deduced from $\left(r_{b} / R\right)$ at each given $\psi$ by :

$$
\begin{equation*}
\left(r_{w} / R\right)=\left(r_{t} / R\right) *\left(r_{b} / R\right) \tag{7}
\end{equation*}
$$

Fig. 18 gives a $r-Z$ plane representation of the axial field as obtained from Eqs (6)-(7) for $\alpha_{0}=23^{\circ}$. The paths of solid symbols " 0 " shed from the blade at different radius $r_{b} / R$, can be followed along the vortex sheet at different phases $\psi$ of the blade rotation ( $60^{\circ}<\psi<540^{\circ}$ ).

When plotting the inboard vortex sheet evolution in a $3 D$ represen-
tation, as shown in Fig. 19A,B,C for $\alpha_{0}=23^{\circ} ; 27^{\circ} ; 32.5^{\circ}$, the completed and detailed wake distorsion is obtained from polynomials fitting of Eqs. (6)-(7). Specially the accelerated wake flow near the tip, as well as the decelerated one near the hub are clearly visualized on the Figures. Between these two regions the presence of a neutral zone progressing downstream with a rather constant axial velocity is also shown on these figures.

The whole empirical wake model so obtained in this section is presently used as an initial prescribed wake geometry for the adaptation of the free wake analysis code to the axial flight case.

## 6. CONCLUDING REMARKS

An attempt in improving and checking the validity of a free wake analysis for hovering rotors or propellers in axial flight has been made in this study.

In the hovering flight case an improvement of the calculation results has been obtained by the introduction of a far wake model geometry as a semi-infinite cylinder. Azimuthal and axial positions of the cylinder have been suggested by data deduced from the experimental $X$-wires wake survey.

The validity of the free wake code, including this far wake model, has been checked by direct comparisons either with the tip vortex path and induced velocity field measured in the wake, or with the circulation distribution measured along the blade radius by a L.V. technique. This technique has appeared to be a powerful tool for surveying the flow around the blade and determining the spanwise circulation distribution. Due to the fact that free wake analysis methods are generally based on an iterative process of the biade circulation distribution, this quantity constitutes one of the most valuable to be determined when comparisonswith calculation results from free wake analysis are concerned.

In the axial flight case, a complete prescribed wake geometry has been obtained for the wake associated to a propeller operating in a large range of advancing parameters. From the empirical laws synthetizing the wake geometry, it has been shown that contraction and convection of the tip vortex
can be expressed in a form quite similar to those already established for rotors in hover. However, the ( $A, B$ ) contraction coefficients and the ( $K_{1}, K_{2}$ ) translation coefficients have been expressed as function of the propeller operating parameters. The empirical wake modelling so obtained constitutes an initial prescribed geometry sufficiently complete to be used as an input for the free wake analysis.

In order to derive a generalized wake geometry valid for several flight conditions, the present study also indicates the need for further detailed empirical wake modelling and corresponding prescribed wake definition.

## ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support provided by the "Direction des Recherches Etudes et Techniques" under Grant D.R.E.T. No 84/00

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## CAPTION OF FIGURES

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Fig. 19 : 3D representation of the vortex sheet for $\tau=0.16$ and $\alpha_{0}=23^{\circ}(\bar{A})$;

$$
\alpha_{0}=27^{\circ} \quad \text { (B) ; } \alpha_{0}=32.5^{\circ} \quad \text { (C) }
$$



Fig. 1

Fig. 2



Fig. 3









MODEL $1^{\text {w }}$ (ring vortex $\}-\cdots$
MODEL 2 (cylinder) ———


Fig. 10


Fig. 11


Fig. 12

Fig. 13



Fig. 14
29-20




Fig. 15
(A) $Z / R=0,231 ; a_{0}=27^{\circ} ; \Delta \psi=4,5^{\circ} ; \ldots V_{\infty} / 4$


Fig. 16


Fig. 17A
(B) $Z / R=0.776 ; \alpha_{0}=27^{\circ} ; \Delta \psi=4.5^{\circ} ; \infty V_{\infty} / 4$


Fig. 17 B


Fig. 18


Fig. 19

