

Experimental investigation of vortex ring state for multicopters

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ABSTRACT

The vortex ring state (VRS) regime for multicopters has been experimentally investigated within GARTEUR AG28: Vortex Ring State via thrust and power measurements as well as stereo particle image velocimetry, at descent angles between 40° and 90° , and normalised descent rates between -0.4 and -1.7, covering the whole area of flight susceptible to VRS. Two different configurations have been investigated: the isolated rotor, used as a reference test case, and the quadrotor configuration. The objective of the present study is to investigate the configurational effects on the quadrotor configuration in vortex ring state and the deviation of its performance from existing experimental data for conventional helicopters. Thrust data and stereo-PIV fields show that while both the single rotor and multicopter exhibit qualitative similarities to helicopters in their VRS behaviour, the quadrotor configuration proved more susceptible to VRS: based on mean thrust reduction, the quadrotor configuration showed a broader region of the flight envelope affected by VRS compared to the isolated rotor configuration. However, similar mean thrust reduction levels are obtained for the two configurations, with the highest reduction measured at roughly 30% for every configuration. The velocimetry measurements proved that, for the Quadrotor configuration, a single recirculation region wraps around the device when in vortex ring state, forming a unified vortex ring.

NOTATION

List of Acronyms

CoR	Centre of Rotation
DEHS	Di-Ethyl-Hexyl-Sebacate
DLR	German Aerospace Center
ERF	European Rotorcraft Forum
eROI	Extended Region Of Interest
eVTOL	Electric Vertical Take-Off and Landing

FOV	Field Of View
FPP	Flight Path Plane
SR	Isolated Single Rotor Configuration
RTG	Rotor Test Facility Göttingen
ROI	Region Of Interest
PSD	Power Spectral Density
QR	Quadrotor Configuration
S-PIV	Stereo Particle Image Velocimetry
UAM	Urban Air Mobility
UPM	Unsteady Panel Method
VRS	Vortex Ring State
TWS	Turbulent Wake State
X	Streamwise Direction
Y	Crosswise Direction
Z	Out-of-plane Direction

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List of Symbols

α	Angle of descent, $^\circ$
ρ	Air density, kg/m^3
ω	VRS frequency scaling, Hz
A	Rotor disk area, m^2
f	Frequency, Hz
P	Total power, kW
P_0	Profile power, kW
P_i	Induced power, kW
R	Rotor radius, m
T	Thrust, N
$T_{hov.}$	Thrust in hover, N
U	Streamwise velocity component, m/s
U_∞	Wind tunnel velocity, m/s
V	Crosswise velocity component, m/s
V_d	Descent velocity, m/s
V_h	Horizontal velocity, m/s
$V_{hov.}$	Velocity scale, m/s
V_{tip}	Tip speed, m/s
$V_{tot.}$	Translational velocity, m/s
W	Out-of-plane velocity component, m/s

INTRODUCTION

A rotorcraft performing aggressive descent manoeuvres may enter a potentially dangerous flight condition, termed vortex ring state (VRS). VRS arises when the descent rate of the vehicle becomes comparable to the induced velocity of the rotor at the rotor plane, as sketched in Figure 1b. Phenomenologically, VRS is characterised by reduced thrust, increased thrust fluctuations and vibrations, significant loss of control authority and insurgence of stability issues. VRS is also characterised by a notable modification in the flow topology: the rotor ingests its wake, and the flow field is dominated by an annular vortex positioned around the rotor plane. This change in flow topology renders low-order methods based on momentum theory inapplicable, as a well-defined slipstream through the rotor disk is no longer present. As a result, flight tests and wind tunnel experiments become essential for understanding and predicting the physics of VRS.

The pioneering works of Ref. 1–5 already touched upon the most critical issues of VRS. Most notably, Ref. 2 already discussed the progressive deterioration of a helicopter's attitude control as it enters various stages of VRS, measured via flight test. He reported on the insurgence of violent yawing/rolling coupled motions and sudden increase of the vehicle's descent rate. Ref. 3 focused on the non-dimensional characteristic curves, and proved the intrinsically unstable nature of flight in VRS, noting that the slope of the non-dimensional power plotted against non-dimensional descent velocity is negative.

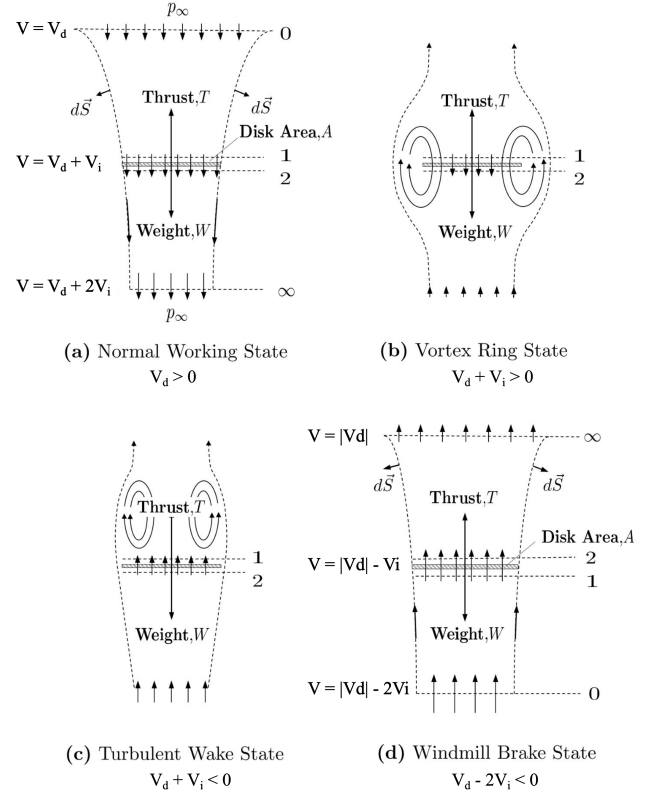


Figure 1. Sketches of different flow topologies around a rotor for varying descent rates, adapted from (Ref. 6).

Indeed, for a given rotor pitch, an increase in descent rate causes an increase in induced velocity, or, equivalently, an increase in required power, leading to a further increase in the descent rate. Refs. 7 and 8 focused on the unsteady aerodynamics of VRS by investigating the time histories of thrust, torque and angular velocity of a model helicopter via wind-tunnel tests. The experimental campaign identified significant thrust fluctuations during the onset of VRS, with variations reaching approximately 30% of the mean thrust values. Moreover, a power spectral density analysis revealed that in VRS, significant periodic components in thrust fluctuations are observable. Beginning in the late 1980s, the emergence and growing interest in tiltrotor air vehicles sparked a renewed focus on the phenomenon of VRS. Scientists and engineers investigated whether existing experimental correlations, computed for conventional helicopters, would extend to tiltrotor configurations, following a series of incidents involving tiltrotor platforms. Notably, a high-profile accident involving the Bell-Boeing V-22 Osprey was attributed, at least in part, to the onset of VRS during a critical phase of flight. For instance, Ref. 9 focused specifically on the Bell-Boeing V-22 Osprey by experimentally investigating the VRS phenomenon on a scaled version of the rotor-wing configuration and found that, in addition to the expected reduction in thrust, VRS could cause severe low-frequency roll oscillations coupled with thrust fluctuation with the same frequency content. Remarkable work was done by Ref. 10, which extensively surveyed the existing literature to develop a VRS model which

would extend across different aerial platforms and provide a low-order model for predictions and real-time simulations. The developed model proved successful in predicting the tilt-off behaviour of tiltrotor configurations, despite the significant difference in the insurgence VRS in conventional helicopters. On the topic of VRS models, Ref. 11 utilised numerical simulations to evaluate various criteria for predicting VRS boundaries and obtained consistent results with flight test data.

Throughout the last decade, research on a new kind of aerial platform intensified: electric vertical take-off and landing (eVTOL) vehicles have been investigated, to optimise the design and deployment of air vehicles in the scope of electric urban air mobility (UAM) and public services such as early wildfire fighting, humanitarian crisis and last-mile delivery, (Ref. 12). Multiple lines of research have been developed, spanning from preliminary aircraft design (Ref. 13) and passenger acceptance (Ref. 14), to vertiport design and integration in urban environments (Ref. 15). Concerns arose in the scientific community regarding the behaviour of such novel air vehicle designs in VRS: Ref. 16 found that eVTOL platforms exhibit VRS at higher forward flight speeds compared to conventional helicopters, due to the higher disk loading and resulting higher induced velocity. Additionally, eVTOLs are often designed to operate in urban environments, where aggressive descent manoeuvres may represent a critical flight condition.

Refs. 6, 17 performed an experimental investigation on small-scale rotors for multicopters in axial descent, focusing on the interactional effects of rotors in side-by-side configuration at multiple separation distances and for various geometrical parameters. Interestingly, it was found that, in VRS, a singular vortex ring structure formed around the dual-rotor configuration at small rotor separation distances. However, the dual rotor configuration showed very similar performance to the reference isolated rotor configuration. It is to be noted, however, that the experimental investigation only assessed the axial descent condition without taking into consideration different descent angles.

The current work aims to experimentally investigate the VRS for multiple descent angles for 1-rotor and 4-rotor multicopters using thrust and power measurements, as well as planar stereoscopic particle image velocimetry (S-PIV).

EXPERIMENTAL SETUP

The experiments were conducted in the Rotor Test Facility Göttingen (RTG) of the German Aerospace Center (DLR). A photograph of the experimental setup taken during the S-PIV campaign is shown in Figure 2. In the scope of the current VRS experiments, the RTG provides a recirculating wind tunnel with an open test section (cross-section 1.6 m x 1.6 m) used to simulate the descent velocity component. To simulate this flight condition, the rotor axes have been oriented horizontally, with their downwash facing the incoming jet of the wind tunnel. The support structure hosting the motors is placed on a tiltable platform, which allows for accurate control of the relative angle, α , between the rotor plane and the freestream velocity, U_∞ . Indeed, by changing α and U_∞ , one can simulate different descent angles and descent velocities.

A sketch of the simulated flight condition is displayed in Figure 3, where different views of the quadrotor configuration are presented. The red, blue and black arrows represent the descent and horizontal velocity vectors, V_d and V_h , respectively, and their sum V_{tot} . Additionally, an axonometric view of the four rotors is also reported for illustrative purposes, showing the quadrotor configuration in descent flight.

Rotor rig

The quadrotor setup was similar to that used in Refs. 18–20, but using slightly larger propellers, since those were discontinued by the manufacturer. The experiment uses a 2-bladed, fixed-pitch multicopter rotor of type KDE 15.4x4.3" with a 0.196 m radius. Up to four 180 W brushless DC motors with a rotation rate of 4800 RPM (80 Hz) were used for all test cases, as in Figure 2. The device is positioned on a tiltable platform that allows adjustment of the relative angle between the device and the wind tunnel, α . Two different configurations have been tested: the Single Rotor (SR), which will be used as a reference configuration, and the Quadrotor (QR). The QR configuration is obtained by operating all four motors simultaneously and is set to operate in the tip-to-gap configuration, *i.e.* with a constant phase shift of 90° between neighbouring rotors, as shown in the top view of Figure 3. During the experiments, the rotor phase and rotation rate were monitored, remaining within 1° of the expected position at all times. Additional relevant flow parameters, such as wind tunnel exit velocity, ambient air pressure, and density, have been simultaneously acquired via a multi-channel data logger from Dewetron.

S-PIV setup

A stereoscopic particle image velocimetry setup has been developed to investigate the flow field around the device for multiple descent angles, descent velocities and configurations. The S-PIV setup features two 2560x1600 pixels Phantom VEO 640L sCMOS cameras for image pairs acquisition, positioned with the opening angle between both camera axes set at 100° . The cameras were positioned on the rotating platform to maintain the same field of view (FOV) irrespective of the tested descent angle. The cameras are equipped with two 24 mm f#2.5 lenses, motorised Scheimpflug adapters and are positioned at approximately 500 mm from the laser sheet plane, yielding a magnification factor of ≈ 3.5 pixel/mm. The cameras are operated at 15 Hz, and the proprietary software from LaVision, DaVis 11, is used for image acquisition. Stroboscopic illumination is provided by a double-pulsed Nd:YAG laser by Evergreen, capable of outputting 200 mJ at 15 Hz. The laser beam is shaped into a laser sheet about 5 mm thick. The laser sheet is shaped outside the wind tunnel, where the laser sheet optics are placed, and then it is projected in the measurement region by means of two 45° mirrors. The last mirror is placed inside the wind tunnel and is fixed in place using a micrometric rail for fine adjustments. The flow is seeded using DEHS fed into a CTS 1000 particle

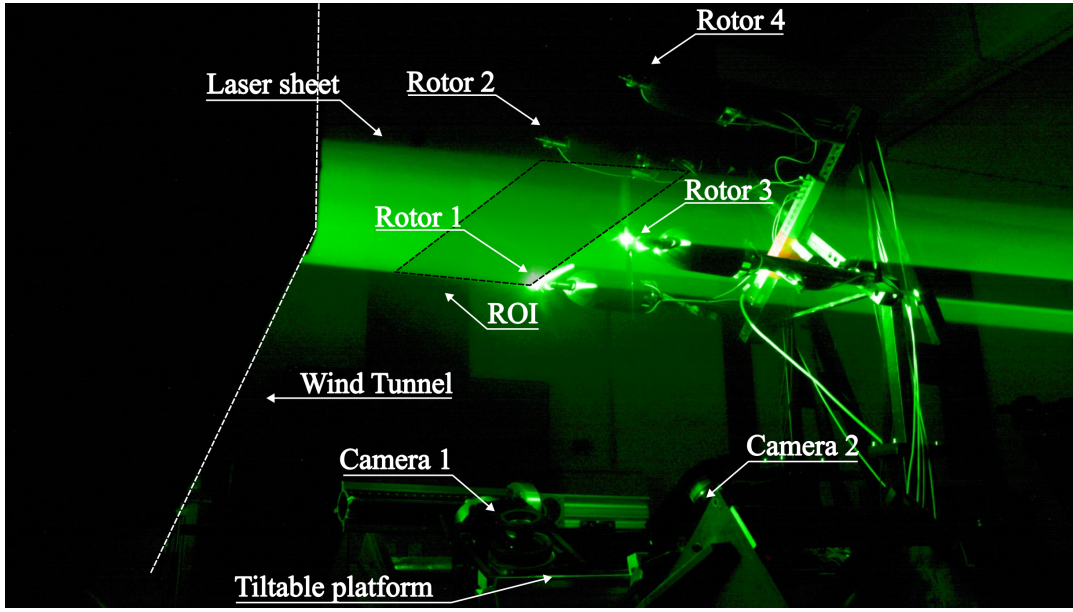


Figure 2. A photograph of the experimental setup during S-PIV recordings.

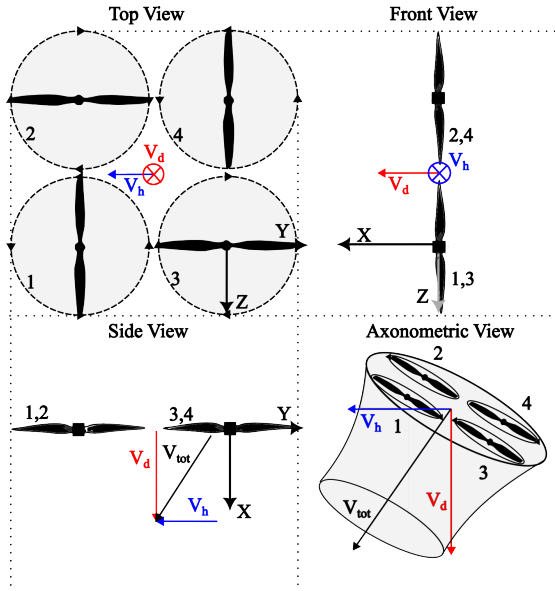


Figure 3. Schematic representation of the quadrotor configuration in multiple views.

generator by Seika, producing particles of about $2 \mu\text{m}$ in size. The deployment of the experimental setup is as follows: the laser sheet is aligned with rotors 1 and 3, which are the bottom rotors of Figure 2, passing through both rotors' hubs. The adopted reference system is shown in Figure 3 and it is centred on the hub of rotor 3, with X indicating the streamwise direction, Y the crosswise direction and Z the out-of-plane direction. Flow fields will be presented as in the side view shown in Figure 3. Stereoscopic calibration has been carried out using the LaVision Type 31 two-plane calibration target and the Pinhole camera model, yielding a stereo calibration error smaller than 0.1 pixel. A total of 700 images were acquired for each time average S-PIV flow field. The region of

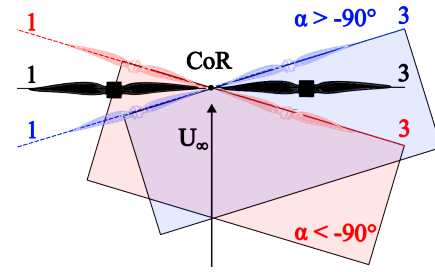


Figure 4. Alternative ways of measuring the flow field around the quadrotor at $\alpha < -90^\circ$ (—) and at $\alpha > -90^\circ$ (—).

interest (ROI) roughly measures $2.6R \times 4.3R$ in the stream-wise and crosswise directions, respectively, which proves adequate for the SR configuration, where only one rotor is visible in the FOV. For the QR configuration, however, the device extends for over four radii in the Y direction, making the current FOV too narrow to capture the flow structures extending past the front and back rotors. Hence, the following procedure has been developed to achieve an extended ROI (eROI), while still maintaining the same resolution and optical setup. Referring to Figure 4, the image displays a schematic representation of the alternative ways of achieving a given α . Leveraging the symmetry of the device with respect to the wind tunnel exit section, the same descent angle can be achieved by rotating the tilttable platform either in clockwise or counter-clockwise directions along the X axis, given the positioning of the centre of rotation (CoR) in the middle of rotor 1 and rotor 3. However, the optical setup does not have the same symmetrical properties as the FOV includes only half of rotor 1 and the entirety of rotor 3. In other words, rotors 1 and 2 become the front rotors for $\alpha < -90^\circ$, while, on the contrary, rotors 3 and 4 become the front rotors for $\alpha > -90^\circ$. Thus, by capturing the same descent angle both in the positive and negative

directions, the two FOVs can be stitched to obtain the mentioned eROI, which extends to $2.6R \times 5.8R$ in the streamwise and crosswise directions, respectively.

COMPUTATIONAL METHODS

The experiments were compared with computations using the unsteady free-wake panel method (UPM), which is often used for multicopter computations (Refs. 20–22). UPM was developed at DLR and allows for the simulation of flows with an arbitrary body shape (Refs. 23, 24). Rigid body motion was employed and described in UPM via a series of stacked frames of reference. The inflow was applied by adding an additional frame to the rotor head rotation frames, translating the group of rotors downward through the air at the required angle.

The code solves for potential flow with a viscous correction applied to improve the estimation of the torque values included in the post-processing (Ref. 25). This changes the reported torque values but not the wake shape. The viscous correction employs a stripwise analysis of the rotor airfoil to compute 2D sections. The boundary layer method of Eppler (Ref. 26) is used with the e^n transition model of Arnal, with $n = 9$. In UPM, lifting bodies are explicitly described as a set of airfoil contours spaced along the length of a rotor blade or wing. In this case, there are 15 spanwise panels and 95 chordwise panels. The blade root connects to a displacing body at the rotor center. In this study, the panel generation code PANGEN (a UPM module) served as a tool to prepare a computational model of a meshed blade with finite thickness and a sharp trailing edge. Sources and sinks of unknown strengths are distributed on the blade surface to enforce tangential flow at the control points. A doublet distribution along the chord simulates the lift distribution. The weighting function of the doublet strength is prescribed and depends on the airfoil thickness. The flow field is assumed to be subsonic and free of shocks, and the compressibility effects are treated approximately using the Gothert rule (Ref. 23).

The calculations used an iterative scheme to ensure pressure equality at the trailing edge and satisfy the Kutta condition. This scheme involved adding a short, zero-thickness panel to the airfoil's trailing edge along its bisector. The orientation of the Kutta panel determines the direction in which a wake element is emitted when released from the trailing edge of the blade.

After each computational step, a new free wake strip is created and added to the previous wake. At the end of five free wake strips, the wake is defined using vortex particles, which perform better than lattice wakes when circulated through the rotor. The singularity strengths evaluated for the Kutta panel determine the circulation of the vortices generated at each calculation step. The wake induction effect is included to satisfy the flow tangency condition for the next computation step. After each computation step, all the vortex particles are convected downstream.

DATA REDUCTION

A preliminary experimental campaign has been conducted, focusing on the acquisition of the aerodynamic data for each of the tested configurations. Care needs to be taken with the non-dimensionalisation of the data, since the experiments were performed with a constant rotation rate of the rotor, rather than trimming to constant thrust. The data has been non-dimensionalised with the induced velocity in hover, $V_{hov.}$, computed from the measured hover thrust for every configuration, as in (1):

$$V_{hov.} = \sqrt{T_{hov.}/2\rho A} \quad (1)$$

where A is the rotor disk area, $T_{hov.}$ the measured thrust in hover and ρ the measured air density. Horizontal and vertical velocity are computed using the following velocity decomposition, (2):

$$V_d = U_\infty \sin(\alpha) \quad V_h = U_\infty \cos(\alpha) \quad (2)$$

where U_∞ is the wind tunnel speed and α is the descent angle. The wind tunnel speed will also be referred to as translational velocity, as it represents the velocity of the device in a Lagrangian reference frame, where the device moves through space. However, a Eulerian reference frame will be considered for the remainder of the document, with the device stationary in space and the incoming dynamic pressure generated by the wind tunnel, as in the experimental setup.

Following the work of Ref. 10, the characteristic curves will be presented in terms of the induced velocity by the rotor, V_i , and in terms of the overall velocity through the rotor, which is $V_d + V_i$. Both will be non-dimensionalised with the induced velocity through the rotor in hover condition, $V_{hov.}$, which has already been introduced in eq.(1) as a scaling factor. These two quantities, V_i and $V_d + V_i$ can be respectively thought of as the rotor's induced power, P_i , and total power detracted of the profile power, $P - P_0$, *i.e.* the required power penalty stemming from profile drag. The relations only hold when the momentum theory is applicable. However, even though momentum theory is not applicable for the VRS regime, as explained in the introduction of this document, the following equations, (3), can still be used to derive the mentioned characteristic curves in VRS, where momentum theory is invalid.

$$\frac{V_i}{V_{hov.}} = \frac{P_i}{T_{hov.} V_{hov.}} \quad \frac{V_d + V_i}{V_{hov.}} = \frac{P - P_0}{P_{hov.}} \quad (3)$$

Where P , $P_{hov.}$ and $T_{hov.}$ are measured, with $P_{hov.}$ being the total power measured in hover, $V_{hov.}$ is the scaling velocity and P_0 is estimated. With equations (3), one can derive the characteristic curves for a given configuration based on performance measurements only, with additional modelling only required to give an estimate of the profile power. For the scope of this work, the approach proposed by Ref. 10 was followed: profile power has been assumed to be independent from V_d and V_h .

RESULTS AND DISCUSSION

Loads

A preliminary thrust measurement campaign has been conducted using the strain gauge balances integrated into the motor unit to explore different combinations of descent angles and incoming flow velocities. Additionally, the motor power was measured and related to the mechanical power by a dynamometer calibration of the motor and control system. Loads and power data have been acquired at 30 kHz. A spectral analysis of the thrust signals has been carried out for different flight conditions to deduce a cut-off frequency to low-pass filter the acquired data and derive a characteristic frequency of the phenomenon. The images shown in Figure 5 display the power spectral density (PSD) of the thrust signal plotted against frequency for the isolated rotor configuration at a descent angle of $\alpha = 70^\circ$. In addition, every diagram features a subplot positioned in the top-right corner that shows an enlargement of the same power spectral density distribution for the frequency range of $f \in [0; 5]$ Hz. Three flight conditions are comparatively assessed: hover, VRS and turbulent wake state (TWS).

Starting with the hover condition, presented in Figure 5a, the PSD distribution shows isolated power peaks for values corresponding to integer multiples of the 1/rev frequency in the 80 to 640 Hz range. Such behaviour is expected due to the rigid mounting of the rotor blades and hub to the support structure. Very similar results are reported in Ref. 9.

Moving on to Figure 5b, the effect of the VRS regime is introduced. As previously introduced, to accentuate the effect of the VRS, the displayed PSD distribution refers to the most severe thrust reduction measured for $\alpha = 70^\circ$, which equals to $T/T_{hov.} \approx 0.8$. The PSD distribution shows the same power peaks corresponding to the integer multiples of the 1/rev frequency, as shown in 5a. However, two additional broader-banded contributions to the power spectrum are visible in the 150-300 Hz and 0-5 Hz ranges. Finally, Figure 5c shows the PSD distribution for the TWS regime. Similarly to the VRS condition, the displayed measurement point has been selected as the one that maximises the thrust variation in TWS regime, which equals to $T/T_{hov.} \approx 1.3$. Comparing Figures 5b and 5c, the PSD distributions show similar behaviours in the 150 to 300 Hz range. However, by comparing the enlargements in the top right corner, the PSD in the TWS regime lacks the power contributions in the 0 to 5 Hz range. The comparative assessment of the PSD distributions across different regimes proved that the frequency range characteristic of the VRS regime is located between 0 and 30 Hz, and, for this reason, thrust time series have been low-pass filtered at 30 Hz via a sliding average filter. In addition, the spectral analysis revealed a clear frequency associated with an increased power content in the VRS regime at $\omega \approx 1.6$ Hz, which is the highest power peak visible in Figure 5b, enlargement in the top right corner. Following the work of Ref. 10, the frequency scaling in VRS is defined as follows: $\omega_{VRS} = V_{hov.}/R$. In the current work $\omega_{VRS} \approx 35$ Hz, which leads to the definition of the following non-dimensional frequency: $\omega/\omega_{VRS} \approx 0.05$,

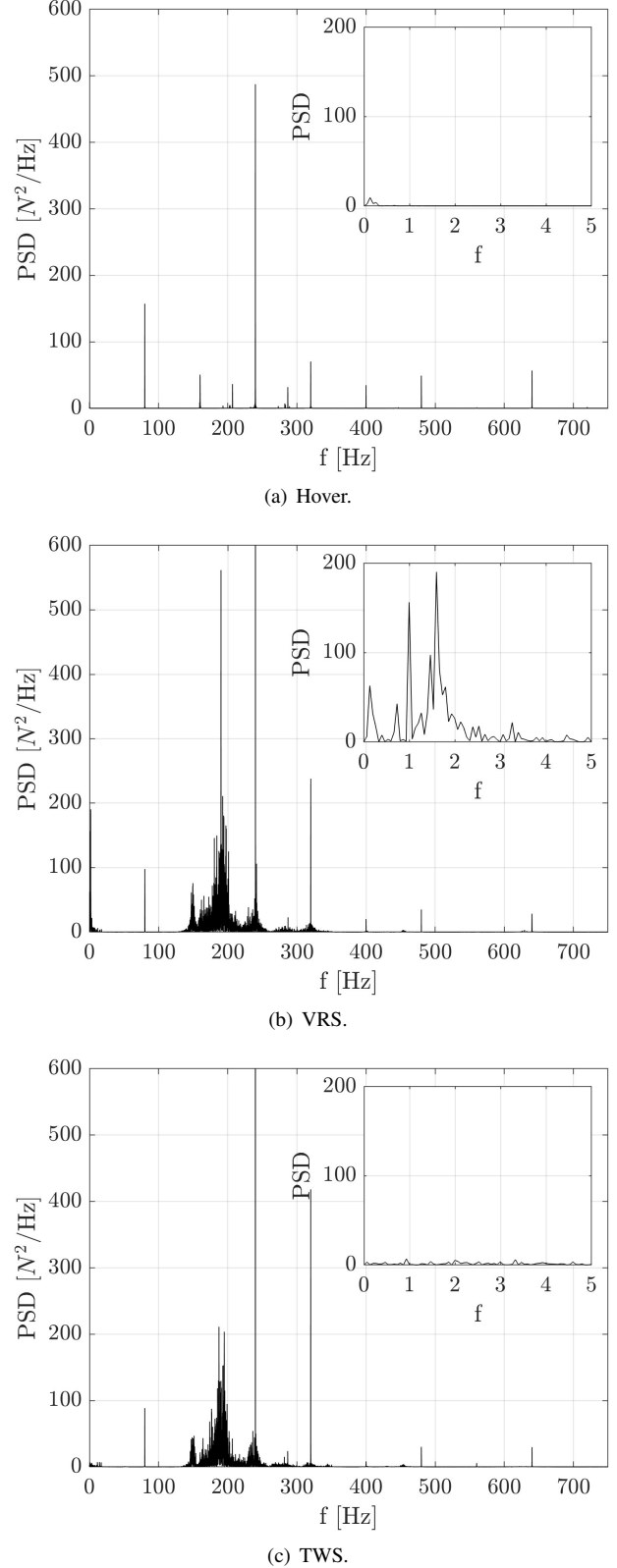


Figure 5. PSD of the thrust signal for different flight conditions: hover (a), VRS (b) and TWS (c), for the isolated rotor configuration at a descent angle of $\alpha = 70^\circ$.

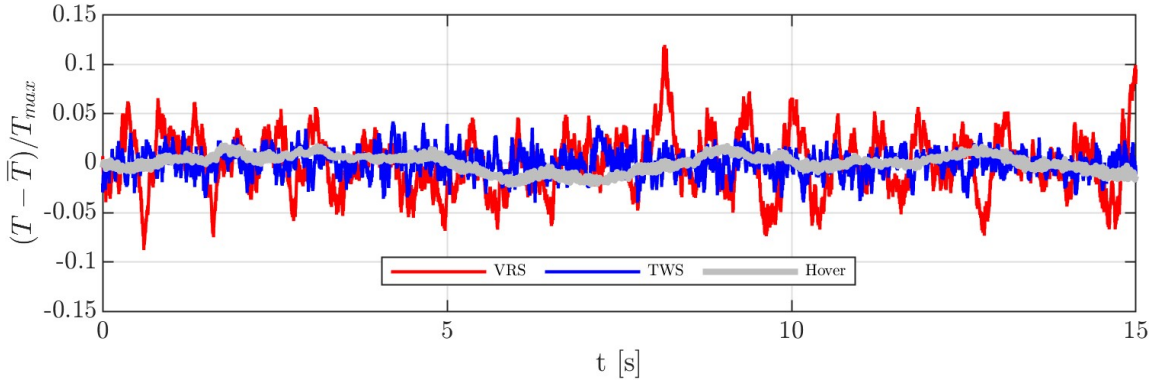


Figure 6. Low-pass filtered normalised thrust time histories for the SR configuration at $\alpha = 70^\circ$ in different flight conditions, VRS (—), hover (—), and TWS (—).

Table 1. Overview of the RMS of the low-pass filtered normalised thrust signals for different flight regimes and descent angles.

RMS	VRS	TWS	Hover
$\alpha = 90^\circ$	0.0707	0.0298	0.0259
$\alpha = 70^\circ$	0.0873	0.0310	0.0136
$\alpha = 40^\circ$	0.0515	0.0225	0.0152

which, again, is consistent with the data obtained by Ref. 9. Figure 6 reports on the normalised thrust time series signals for three different flight conditions, VRS, hover, and TWS at $\alpha = 70^\circ$. The plotted normalised thrust is defined as follows: $T - \bar{T} / T_{max}$, where $\bar{T}$ is the thrust signal averaged over the entire sample and T_{max} is the maximum thrust measured in the sample. Figure 6 is presented to highlight the oscillatory nature of the thrust signal in different flight conditions. Thrust reference values are reported in the form of an annotation in the top left corner of each subplot, with the VRS and TWS conditions presented as variation over the hover case. Each curve displays the low-pass filtered signal at 30 Hz. Comparing the thrust signal measured in VRS with the thrust signal in hover and TWS conditions, a significant reduction in fluctuations can be observed. The severity of such fluctuations has been quantified using the root mean square and reported in table 1, where the RMS of the normalised low-pass filtered thrust signal is presented for $\alpha = [40^\circ, 70^\circ, 90^\circ]$, proving that the trends showed for $\alpha = 70^\circ$ in Figure 6, extend to the entire span of tested descent angles. The RMS in VRS is 2.4 to 2.8 times larger than in TWS and 2.7 to 6.4 times larger than in hover. Moreover, for the thrust signal measured in VRS, fluctuations show a low-frequency periodicity, which is lacking in hover and the TWS regime. Such low-frequency periodicity is associated with the accumulation and shedding of vorticity in the recirculating wake of the rotor in VRS. Even though the flow in TWS still manifests relevant unsteady behaviour, the rotor wake does not experience the mentioned periodical build-up and shedding of vorticity underneath the rotor, which causes the low-frequency content in the thrust

signal in VRS. Data acquired during the aerodynamic load campaign has been condensed into a single scatter plot which combines information about the flight condition, namely V_d , V_h and α , and the relative value of thrust reduction/increase over the hover case. Such a graph will be referred to as a flight path plane (FPP) since it describes the mean thrust variation for different combinations of horizontal and descent velocities. The adoption of this kind of graph is customary (Refs. 7, 9–11, 27–30) for the analysis of the VRS regime since it provides an overview of the most critical flight paths in descending flight. Figure 7 shows the FPP scatter plots for two different configurations: SR, subplot (a), and QR, subplot (b). Every circle marker identifies a single data entry in the test matrix of the aerodynamic load campaign, and it is associated with the mean thrust value computed over a 30 s acquisition window. For the QR configuration, Figure 7b, thrust values are also averaged across the four rotors. The mentioned circle markers are also colour-coded, with red, white and blue indicating reduction, no variation or increase in thrust over the hover case, respectively. The solid and dashed black lines connecting the dots are obtained via Delaunay triangulations and respectively represent a mean thrust reduction threshold of 5% and 15%. The mentioned Delaunay triangulations provide a quantification of the VRS boundaries: the innermost region, identified with the solid black line, defines the so-called *deep* VRS region where thrust is severely reduced and control authority over the air vehicle is compromised. Indeed, the most crucial boundary is the outermost, identified with the dashed line and termed *shallow* VRS. In this region, only a modest reduction in thrust is observed, while attitude control can already be impaired, to a degree which mainly depends on the vehicle's architecture. Furthermore, the VRS boundary computed using the model developed in Ref. 10, is displayed with square gray markers. Starting with Figure 7a, the *deep* VRS region is confined roughly to $V_d/V_{hov.} \in [-1.5; -1.1]$ and to $V_h/V_{hov.} \in [0; 0.5]$, whereas the *shallow* VRS region extends further, reaching vertical velocities as low as $V_d/V_{hov.} = -0.8$, while extending to significantly higher horizontal velocities of up to $V_h/V_{hov.} = 0.8$. A progressive entrance and a rather sudden exit from the VRS regime can be observed for the SR configuration: starting from the hover condition and

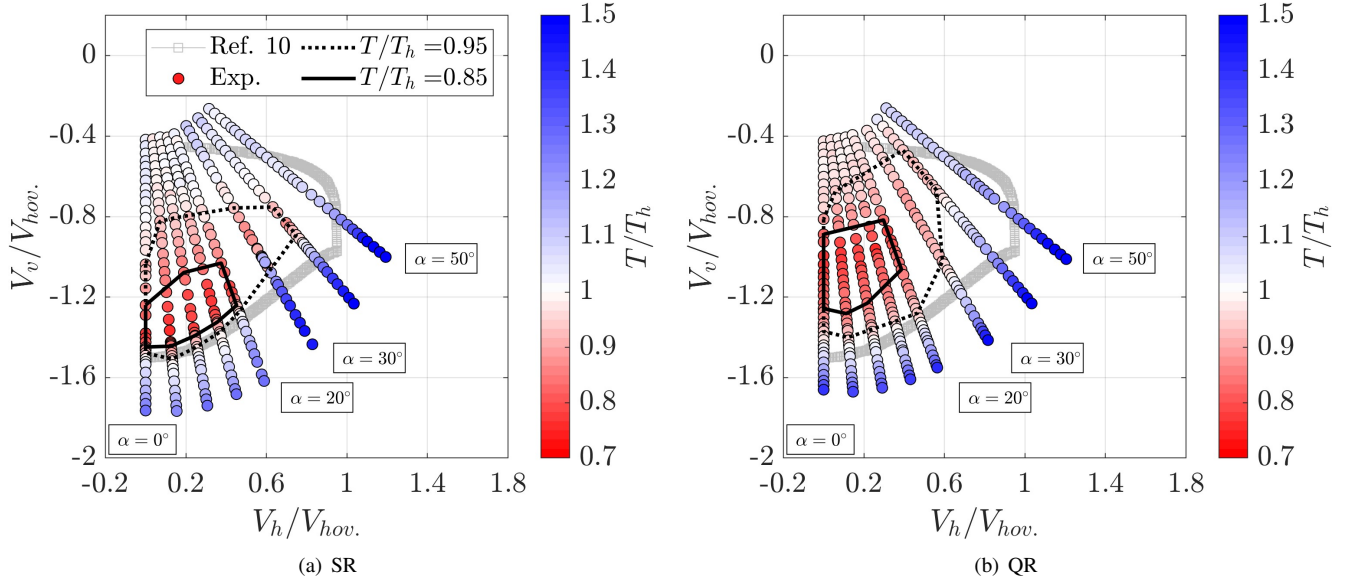
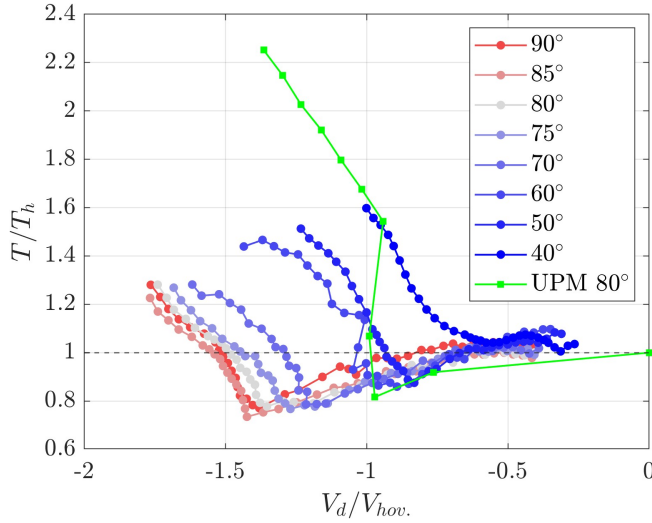


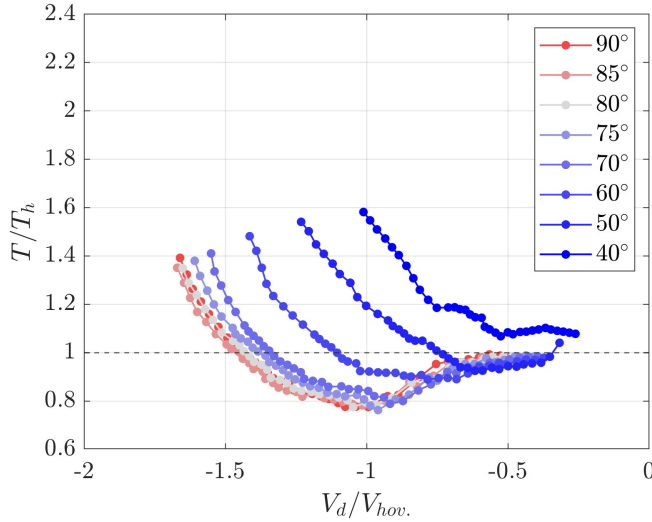
Figure 7. Flight path plane scatter plots coloured with mean thrust value modifications over the hover case for the SR (a) and QR (b) configurations. Red ($\bullet$), white ($\circ$) and blue ($\bullet$) indicate a reduction, no variation and increase in thrust over the hover case, respectively. Black lines indicate Delaunay triangulation for a thrust reduction threshold of 0.95 and 0.85, indicated by a solid (—) and dashed line (---), respectively. Grey markers ($\blacksquare$) indicate the VRS boundary computed by Ref. 10.

increasing the descent velocity, the *shallow* and *deep* VRS boundaries are significantly further apart than when exiting VRS, with the two boundaries almost overlapping at $V_d/V_{hov} = -1.5$. By comparing Figure 7a and 7b, a configurational effect can be deduced: moving from the SR to the QR configuration, both deep and shallow VRS regions extend over a larger area and happen at significantly smaller descent velocities. The deep VRS region for the QR configuration extends across the following velocity ranges: $V_d/V_{hov} \in [-0.8; -1.3]$ and $V_h/V_{hov} \in [0; 0.4]$. The *shallow* VRS region is larger as well and, most importantly, is shifted towards reduced descent velocities. Despite the different extent of the VRS regions, the maximum thrust reduction is similar for both configurations. In addition, both configurations show $\alpha = 50^\circ$ as the last descent angle, which is susceptible to VRS. Figure 8 displays non-dimensional thrust values for the SR and QR configurations measured at different descent velocities. Figure 8a also presents load data obtained via UPM simulations at $\alpha = 80^\circ$ for the SR configuration. In Figure 8, as V_d/V_{hov} is reduced, for a given α , thrust exhibits a non-monotonic behaviour. Comparing Figures 8a and 8b, thrust evolves differently for the SR and QR: the former shows an almost linear thrust reduction with α , while the latter presents a non-linear behaviour. For most descent angles, $V_d/V_{hov} \approx -1.4$ marks the critical descent rate at which minimum thrust is generated, $T/T_h \approx 0.75$, and the trend is inverted. As the descent rate is further increased, thrust starts to be restored. After this limit, thrust quickly grows, showing, again, an almost linear behaviour. The same cannot be said for QR, as a clear turnover point where thrust starts growing again cannot be identified. Finally, the thrust distribution

computed via UPM, for the SR configuration, shows that only a qualitative comparison is possible, as thrust is overestimated and the turnover point is not correctly predicted. Moving on to Figure 9, the experimental data measured during the aerodynamic load campaign are presented in terms of the non-dimensional induced velocity and non-dimensional required power for the SR and QR configurations. Experimental data, plotted with circular, square and triangular markers, are presented alongside the analytical expressions obtained via momentum theory, shown with solid black lines in the range: $V_d/V_{hov} \in [-5; -2] \cup [0; 4]$. Non-dimensional induced velocity and required power distributions computed via momentum theory are also shown in the range $V_d/V_{hov} \in [-2; 0]$, with dashed black lines, where momentum theory is invalid. In addition, an existing experimental correlation is displayed with a thick solid black line, (Ref. 31). Although the underlying hypotheses are not verified, momentum theory still offers reasonable results for low descent velocities in the range $V_d/V_{hov} \in [-1; 0]$. As shown in subplots 9a and 9c, the extension of momentum theory, the data from 31, and the current experimental data all show good agreement. Moving on to subplots 9b and 9d, data for the QR configuration is presented. Deviations from the literature can be noted for both the induced velocity and required power distributions; the markers appear shifted toward the right, and, consequently, peak induced velocity is reached earlier and with steeper gradients in terms of vertical velocity, although the maximum measured induced velocity is approximately the same as in the SR configuration, $V_i/V_{hov} \approx 2$. As thoroughly discussed by Ref. 10, the non-dimensional power distribution is a measure of the required power to generate a set level of thrust.



(a) SR



(b) QR

Figure 8. Non-dimensional thrust plotted against descent velocities for different descent angles, for the SR (a) and QR (b) configurations.

In this sense, the negative correlation between vertical and induced velocity reveals the unstable nature of the VRS region: as an air vehicle happens to fly within the unstable portion of the V_i/V_d plane, an increase in descent rate causes an increase in induced velocity, i.e. an increase in power required to generate the same thrust level. If not counteracted, the described condition can induce a further increase in descent rate, which, in turn, further increases the required power for a given thrust level. In this sense, comparing Figures 9a and 9b, one can deduce that moving from the SR to QR, the configurational effect is essentially detrimental since the slope of the unstable region is steeper, thus amplifying the unstable positive feedback between descent rates and induced velocity. One could argue that, given the steeper slope, the unstable region is smaller for the QR than the SR configuration. At the same time, however, one would still need to cross the unstable region to get to the extended stable region in the V_i/V_d plane,

which could represent a potentially hazardous manoeuvre.

Flow fields

As previously discussed in the introduction to this paper, the flow field around a rotor in VRS undergoes a major topology modification compared to the hover condition. This section aims to quantitatively measure the aforementioned flow field modifications by comparatively analysing the SR and QR configurations, operating at varying velocities and descent angles. Firstly, the reader is referred to Figure 10, where time average flow fields around the SR configuration at a descent angle of $\alpha = 90^\circ$ are presented through non-dimensionalised velocity magnitude contours with streamlines superimposed. Velocity magnitude is computed using all three velocity components measured through S-PIV and non-dimensionalised via the tip speed, $V_{tip} = \omega R$. Figure 10 shows six different subplots displaying flow fields for increasing descent velocities. Each subplot is accompanied by an annotation on the right, indicating the non-dimensional descent velocity, which utilises the previously introduced V_{hov} as the velocity scale and is computed for each different configuration based on the generated thrust. In addition, each subplot also features an annotation on the top which indicates the mean thrust variation over the hover case.

Starting with the subplot in the top left corner of Figure 10, it shows the flow field generated by a rotor in hover: two distinct jets generated by the rotor blades can be identified, although these quickly coalesce into a single structure as early as $X/R = 0.5$.

Increasing the descent velocity from $U_\infty/V_{hov} = 0$ to $U_\infty/V_{hov} = -0.57$, leads to the subplot in the top right corner: only subtle variations with the hover case can be identified, as predicted by momentum theory, which still holds for small descent velocities. Similarly, a negligible variation in thrust is observed, with a 3% increase over the hover case. The flow field is still dominated by the jet-like structure issued from the rotor; however, a slight reduction in the velocity generated in the wake can be seen, alongside a premature divergence of the wake. Big recirculation regions start to appear on both sides of the primary rotor wake. A further increase in the descent velocity, from $U_\infty/V_{hov} = -0.57$ to $U_\infty/V_{hov} = -0.86$, intensifies the trend. The flow field shown in the subplot in the middle row on the left essentially shows the same flow topology, although the mentioned modifications are more pronounced. The velocity in the wake is more significantly reduced, the wake diverges more, and the recirculation regions have moved upwards towards the rotor plane.

Further increase in descent velocity to $U_\infty/V_{hov} = 1.14$, leads to the subplot in the middle row, on the right. Two flow structures now coexist in the investigated region of interest: the centre of the recirculation regions has come into view, showing the ring vortex almost in its entirety, while the jet issuing from the rotor is even more spread out, reaching a lower velocity. The described flow modification leads to a net 6% thrust reduction. The last row of subplots shows the flow fields achieved for the highest descent velocities, $U_\infty/V_{hov} = -1.29$,

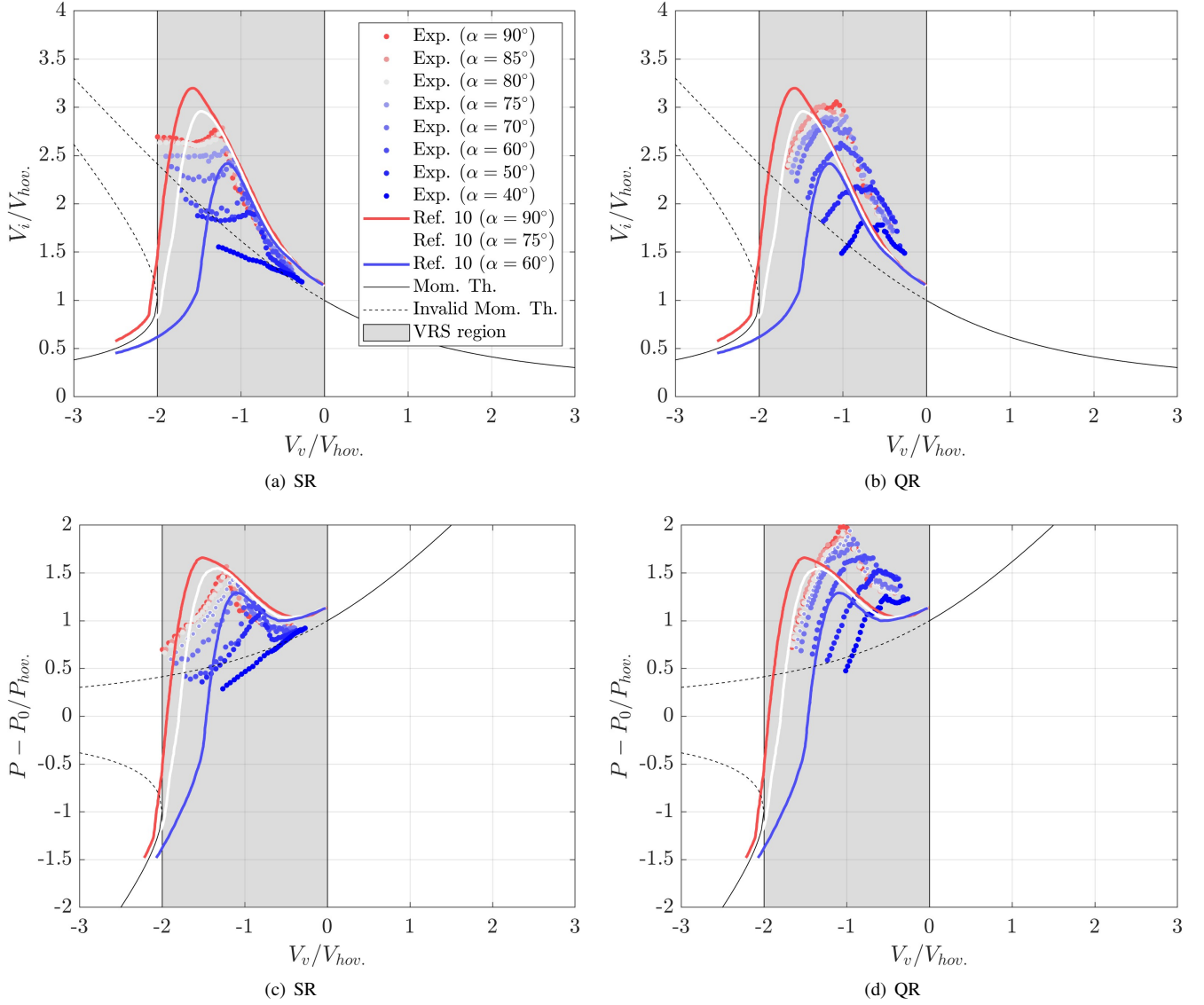


Figure 9. Non-dimensional induced velocity and non-dimensional required power for the SR and QR configurations, from current experimental data ($\bullet, \bullet$), momentum theory (—), invalid momentum theory (---), VRS model from (Ref. 10) (—, —).

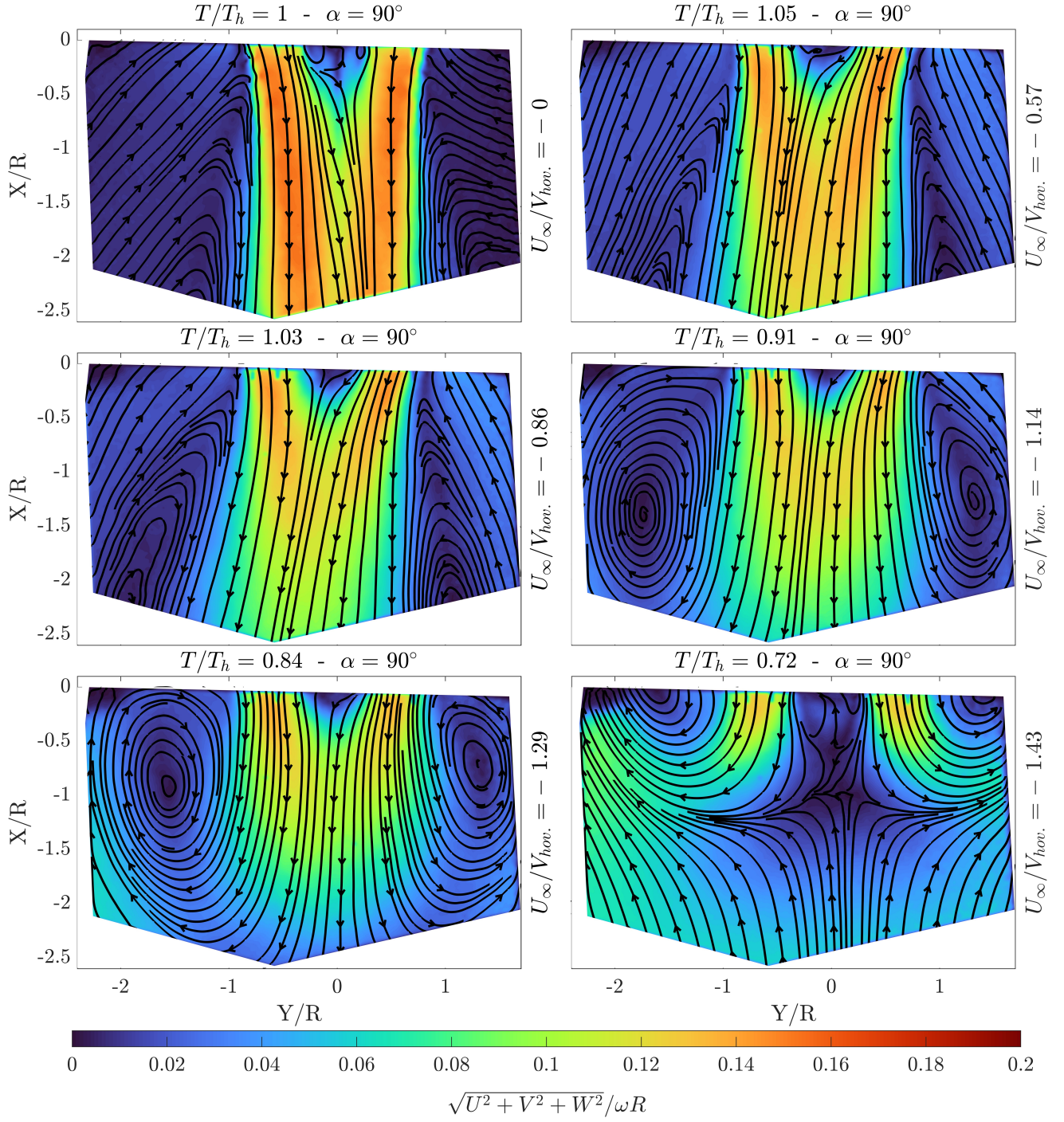


Figure 10. Time averaged S-PIV flow fields displayed as non-dimensional velocity magnitude contours with streamlines superimposed for the SR configuration, $\alpha = 90^\circ$ and multiple descent velocities.

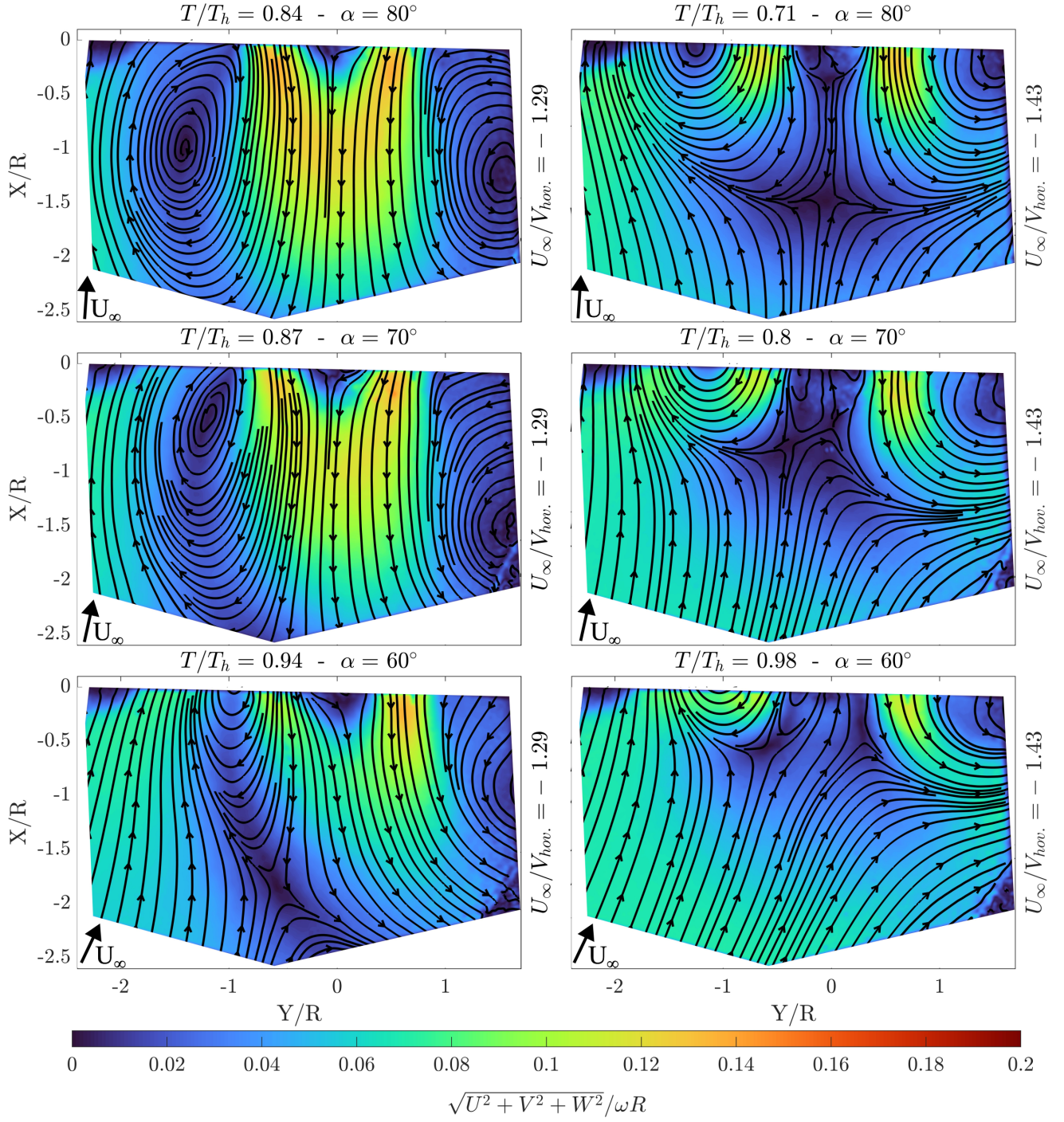
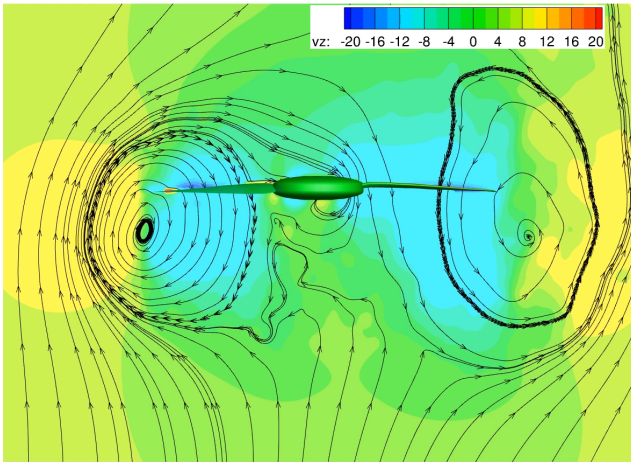
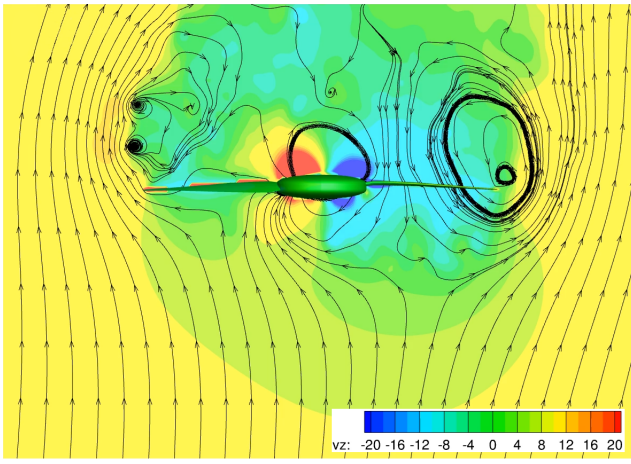


Figure 11. Time averaged S-PIV flow fields displayed as non-dimensional velocity magnitude contours with streamlines superimposed for the SR configuration, $\alpha \in [60^\circ, 70^\circ, 80^\circ]$ and multiple descent velocities.



(a) SR



(b) QR

Figure 12. Instantaneous flow fields simulated in UPM and displayed as vertical velocity with streamlines superimposed for the SR configuration and two different descent velocities, $U_\infty/V_{hov.} = -0.71$ (a) and $U_\infty/V_{hov.} = -1.14$ (b) and $\alpha = 80^\circ$.

on the left, and $U_\infty/V_{hov.} = -1.43$, on the right. A descent velocity of $U_\infty/V_{hov.} = -1.29$ causes the ring vortex to be pushed closer to the rotor plane, where its effect on the rotor performance is stronger, since it causes an 11% thrust loss over the hover case. Simultaneously, the presence of the mentioned ring vortex prevents the wake of the rotor from developing, and a major velocity reduction can be observed in the wake. Finally, moving to $U_\infty/V_{hov.} = -1.43$, the ring vortex has partially left the region of interest again, by shifting upwards. The ring vortex is now roughly positioned along the rotor plane. In this configuration, the effect of the ring vortex on the rotor performance is even more severe: a 19% reduction in thrust is observed. At this descent rate, the effect of the incoming dynamic pressure on the wake of the rotor is so strong that the downwash generated by the rotor blades barely develops and immediately diverges in the radial direction and does not merge at all. The mentioned repositioning of the ring vortex across the rotor plane reveals a stagnation point, which

Table 2. Overview of the translational velocity decomposition for different descent angles.

	$V_d/V_{hov.}$	$V_h/V_{hov.}$	$U_\infty/V_{hov.}$
$\alpha = 80^\circ$	-1.27	0.224	-1.29
$\alpha = 70^\circ$	-1.21	0.441	-1.29
$\alpha = 60^\circ$	-1.12	0.645	-1.29
$\alpha = 80^\circ$	-1.41	0.355	-1.43
$\alpha = 70^\circ$	-1.34	0.489	-1.43
$\alpha = 60^\circ$	-1.24	0.715	-1.43

marks the lower boundary of the vortex ring.

Once the behaviour of the SR configuration in vertical descent ($\alpha = 90^\circ$) has been discussed for different descent velocities, the effect of different descent angles is investigated, as in Figure 11. The flow fields generated by the SR configuration at the two highest descent velocities are compared for $\alpha \in [60^\circ, 70^\circ, 80^\circ]$. As $\alpha \neq 0$, two velocity components make up the overall translational velocity, as presented in the decomposition shown in eq. (2). The same velocity decomposition is also shown in Figure 3 but in the reference frame of the quadrotor, where the translation velocity component is shown in black and is indicated with $V_{tot.}$, whereas eq. (2) reports on the velocity decomposition in the wind tunnel reference frame. For the sake of simplicity, U_∞ has been taken as a reference value for different descent conditions, as it represents the wind tunnel operating condition. For these reasons, care needs to be taken while comparing different descent angles for equal $U_\infty/V_{hov.}$ values. For the sake of clarity, table 2 provides an overview of the V_d and V_h values obtained for different descent angles. Firstly, the reader is referred to the first row of Figure 11, where $\alpha = 80^\circ$ is considered. To correctly interpret the flow structure visible in Figure 11, it is important to recall that the relative position of the rotor plane and the cameras does not change with different α : this implies that the rotor plane is always positioned at $X/R = 0$. In other words, the S-PIV data presented is shown in the reference frame of the device, with the incoming flow from the wind tunnel changing direction accordingly to α . Starting with the first column on the left, flow fields measured at $U_\infty/V_{hov.} = -1.29$ and different descent angles are displayed. Increasing α , *i.e.* moving from top to bottom in Figure 11, causes the ring vortex to tilt in the $X/R - Y/R$ plane. Such rotation also introduces an asymmetry in the vortex ring structure: the front side of the structure gets smaller and is pushed towards the rotor plane, while the rear side is pushed away from the rotor plane and stretches further in the Y/R direction. Increasing the descent rates causes the vortex ring to be pushed against the rotor plane, which, in turn, affects the development of the rotor downwash underneath the rotor; a similar effect is caused by the approach of the ring vortex towards the rotor plane, due to an increase in α . As α increases, the downwash from the front rotor blade is suppressed, while the one generated by the rear blade is preserved. In the flow field measured at the highest descent angle, $\alpha = 60^\circ$, shown in the bottom left corner of Figure 11, the stagnation point is visible, located at roughly

$X/R \approx -2$ and $Y/R \approx -0.5$. This velocity contour proves that the stagnation point moves along the boundary of the vortex ring as the descent angle changes.

As the descent rate is increased from $U_\infty/V_{hov.} = -1.29$ to $U_\infty/V_{hov.} = -1.43$, the flow field topology changes significantly, as shown in the right column of Figure 11. Starting with the flow field in the top right corner, and comparing it with the flow field on the left, the increase in descent rate causes the stagnation point to come into view in the ROI, as it is pushed towards the rotor plane. As per the previous descent rate, the vortex ring around the rotor plane appears rotated in the $X/R - Y/R$ plane, accordingly to α . Increasing the descent angle to $\alpha = 70^\circ$, causes the stagnation point to move upwards even further. One could argue that the flow topology is progressively shifting towards the TWS regime, with the recirculating regions mostly developing above the rotor plane. Analogously, the stagnation point below the rotor plane is not as well defined as in the previous case, with the jets generated by the two rotor blades evolving independently from one another. The described evolution from the VRS to the TWS regime can be explained by the increase in the horizontal velocity component associated with the increased angle of descent. The encounter with VRS is essentially characterised by the periodic accumulation and shedding of vorticity underneath the rotor; however, provided that the horizontal velocity component is sufficiently high, such accumulation of vorticity is prevented and the flow topology migrates to the TWS regime. A further increase in descent angle, to $\alpha = 60^\circ$, leads to the flow field presented in the bottom right tile of Figure 11. The stagnation region is almost completely suppressed, and the recirculation regions are barely visible within the ROI, indicating that the evolution from the VRS to the TWS regime is definitive.

Figure 12 displays a slice through the plane of rotation of the instantaneous flow field simulated in UPM. The flow field is displayed as vertical velocity with streamlines superimposed, for the SR configuration at $\alpha = 80^\circ$ and for two different descent velocities, $U_\infty/V_{hov.} = -0.71$, Figure 12a, and $U_\infty/V_{hov.} = -1.14$, Figure 12b. The simulated flow fields are qualitatively similar to the data measured in the S-PIV campaign: a region of recirculating flow forms around the rotor and, as descent velocity is increased, the wake is shifted above the rotor.

Moving on to the QR configuration, flow fields are presented in Figure 13 using the eFOV, for $\alpha = 80^\circ$ and $\alpha = 70^\circ$ in the left and right columns, respectively. Three different descent velocities are reported for every descent angle, namely $U_\infty/V_{hov.} = [-1, -1.14, -1.29]$, as indicated in the annotations on the right. Before addressing each flow field individually, it can be noted that for the current rotor separation distance, a single vortex ring structure wraps around the device. Ref. 17 conducted a parametric study on separation distance for rotors in axial descent and found similar results, proving that for separation distances small enough, the ring vortices generated by each rotor merge, forming a large unified structure around the multirotor configuration.

Starting with the first row of Figure 13, the effect of differ-

ent descent angles on the flow field topology is presented for $U_\infty/V_{hov.} = -1$. As described for the SR configuration, increasing the descent angle causes the vortex ring to rotate in the $X/R - Y/R$ plane. Comparing the top left and top right tiles of Figure 13, the rotation of the vortex ring structure can be appreciated by noting the change in location of the recirculation regions at the front and back of the device.

Similar considerations apply to the second row of Figure 13 which shows data for $U_\infty/V_{hov.} = -1.14$. The recirculation regions are smaller and located closer to the rotor plane.

Increasing the descent rate to $U_\infty/V_{hov.} = -1.43$ leads to the flow fields presented in the last row: the stagnation point becomes visible in the eROI, while the ring vortex spreads over a smaller area and is placed even closer to the rotor plane. The rotation caused by the descent angle is less pronounced, although still visible. Comparing the thrust reduction levels for all three configurations, it can be noted that, as the ring vortex collapses towards the rotor plane and the quadrotor starts shifting into the TWS regime, thrust increases again.

CONCLUSIONS

The VRS phenomenon for multicopters has been the subject of an experimental investigation conducted in the Rotor Test Facility Göttingen (RTG) of the German Aerospace Center (DLR), with a focus on understanding its aerodynamic characteristics through thrust and power measurements, complemented by S-PIV measurements. These measurements were conducted for multiple descent rates in the range $U_\infty/V_{hov.} \in [0; -1.7]$ and for multiple descent angles in the range $\alpha \in [0; 50^\circ]$. Two configurations were comparatively assessed: the SR is used as reference configuration while the quadrotor configuration, QR, is tested to investigate the interactional effects. In addition, numerical simulations were conducted using the in-house developed code UPM.

The aerodynamic load campaign indicated that multirotor platforms exhibit increased susceptibility to the vortex ring state phenomenon. In particular, the quadrotor configuration demonstrated a broader region of the flight envelope characterized by significant mean thrust reduction when compared to the isolated rotor configuration. Nonetheless, comparable levels of mean thrust reduction were observed across all configurations, with the maximum reduction approaching approximately 30% in the worst case. A configurational effect has also been noted in the dynamics of the onset, development and decay of the VRS regime: the QR configuration has been shown to fall in and out of the VRS regime more gradually, if compared to the SR configuration. This was deduced by comparing the shape and relative location of the *deep* and *shallow* VRS regions in the flight path planes for the SR and QR configurations, with the former showing the lower end of the deep and shallow VRS regions almost overlapping. In addition to the mean thrust reduction, a PSD analysis revealed that thrust signals measured in the VRS regime show a low-frequency periodicity as well as increased thrust fluctuations, which is consistent with the current state of the art.

The S-PIV campaign clarified the evolution of the flow field

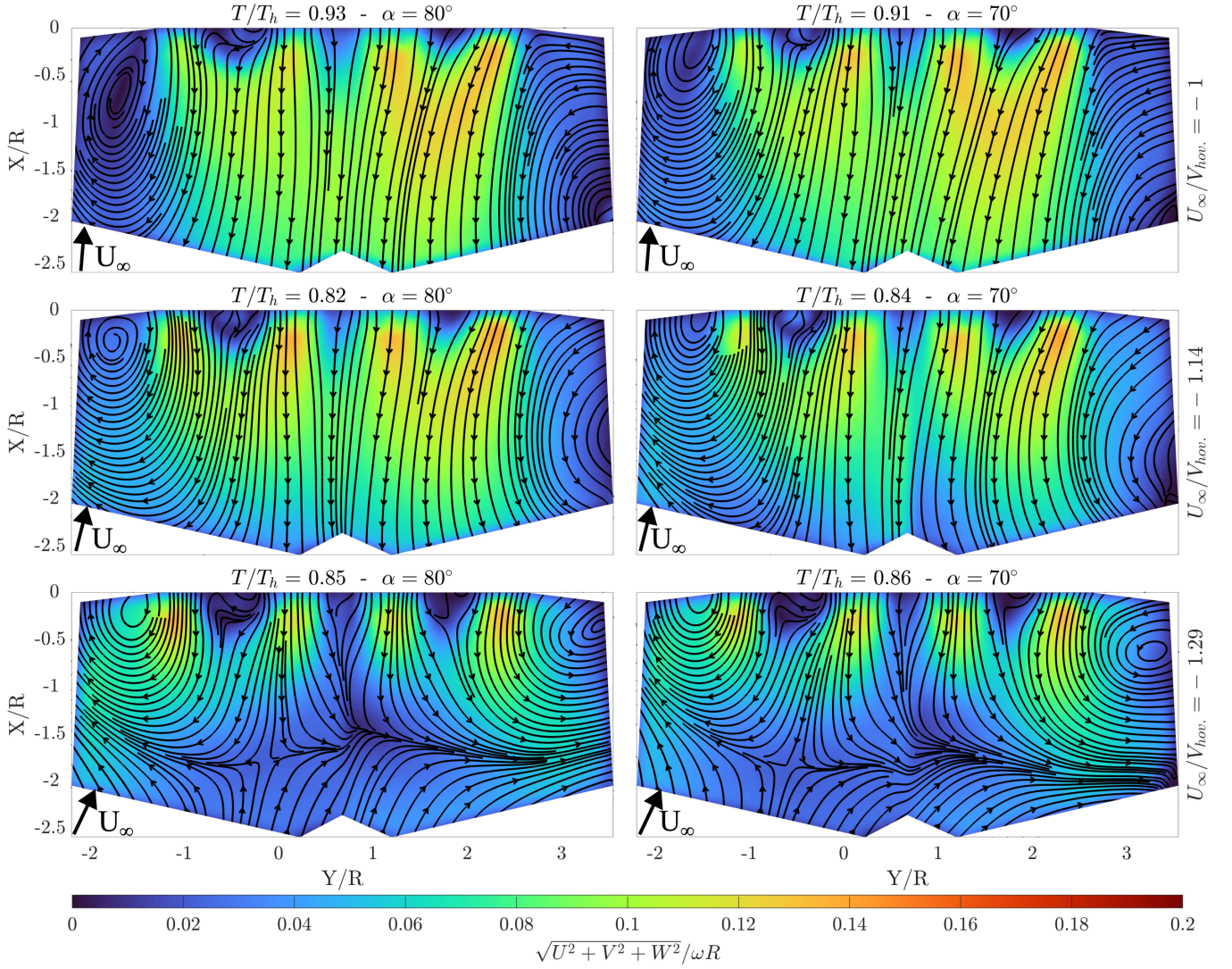


Figure 13. Time averaged S-PIV flow fields displayed in the eFOV as non-dimensional velocity magnitude contours with streamlines superimposed for the QR configuration, $\alpha \in [70^\circ, 80^\circ]$ and multiple descent velocities.

topology in the VRS and TWS regimes, depending on the descent rate and descent angles. More specifically, increasing α caused the ring vortex to rotate in the $X/R - Y/R$ plane and to deform: the ring vortex at the front of the device is reduced in size and pushed towards the rotor plane, while, on the contrary, the side of the vortex ring located at the back is enlarged and pushed away from the rotor plane.

Tandem and side-by-side 2-rotor configurations were measured as well and will be analysed in future work.

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