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## CURRENT STATE OF THE ART REGARDING HELICOPTER VIBRATIONS REDUCTION AND AEROELASTIC STABILITY AUGMENTATION

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## 1. INTRODUCTION

Vibrations reduction is playing an increasingly significant role as a performance improvement factor in the design of new helicopters. Vibrations affect crew performance, airframe and avionics reliability as well as maintenance costs.

There are many active and passive ways of dealing with vibrations.

Namely, they can be reduced with passive devices such as rotor and airframe absorbers or suspension systems which are being developed by most manufacturers but these, unfortunately, impose significant weight penalties amounting to 1 to 2% of the helicopter all-up weight.

Active devices such as Blade Pitch Higher Harmonic Control and Active Control of Structural Response or Force Transfer were consequently studied and demonstrated significant vibrations reduction. These devices will save weight and improve future helicopters reliability.

The second permanent challenge the engineer has to face is how to increase the aeromechanical stability of articulated, bearingless or hingeless rotors. When soft-in-plane, this type of rotor must usually be equipped with a lead-lag damper to prevent air or ground resonance instability. A large number of factors such as all-up weight, g level, main rotor rotating frequency, ground configuration etc. are playing a role in helicopter operation and dynamics engineers cannot avoid frequency coalescence between rotor and airframe in every configuration.

Damping becomes therefore necessary and can be increased with active control of aeromechanical stability. This control may then avoid the need for lead-lag dampers and contribute to a simpler and cheaper rotor design.

This paper discusses the dynamic design technique as applied by ECF.

## 2. GENERAL PRESENTATION

In forward flight, aerodynamic loads are applied to the blades and their dynamic responses to these loads generate alternate forces at the rotor hub. The fuselage dynamic response to these alternate forces then generate rotor hub displacements and the

blade dynamic response to these displacements modify the aerodynamic loads (See figure 1).

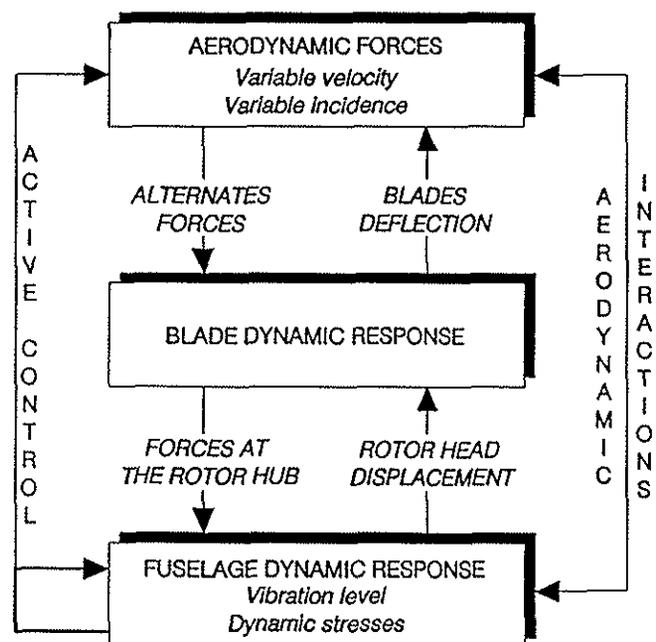


Figure 1 - Helicopter dynamic response

## 3. DYNAMIC FORCES MEASUREMENT AT THE ROTOR HUB

Evaluation of the rotor hub dynamic forces (and moments) is one of the most significant aspects as far as dynamic tuning is concerned in the development of any helicopter. ECF applies three evaluation methods :

### 3.1 Loads transmitted to fuselage interfaces

Fixed system measurements proceed with strain gauges located in the struts and in the gearbox-to-transmission deck linkage.

The rotor hub dynamic forces and moments are then determined by subtracting blades dynamic response (due to the rotor hub movement) and MGB dynamic response.

### 3.2. Rotor mast bending

Measuring in two mast sections helps determine bending moments and in-plane forces at the rotor head (by subtracting blades dynamic response due to the rotor hub movement). These measurements are very accurate for conventional rotor masts i.e. metal masts but unsuitable for the new generation masts i.e. short masts made of composite materials with highly non linear influence matrix.

#### 3.2.1. Modal Identification with strain gauges on rotor blades

ECF uses this method to determine rotor hub dynamic forces and moments by identifying the contributions of each blade mode.

This method is based on local blade moment measurements where each modal contribution is determined through its local bending moment (See figure 2). The rotor hub dynamic loads can then be computed by adding their modal values (See figure 3) and by subtracting blades dynamic response due to the rotor hub movement.

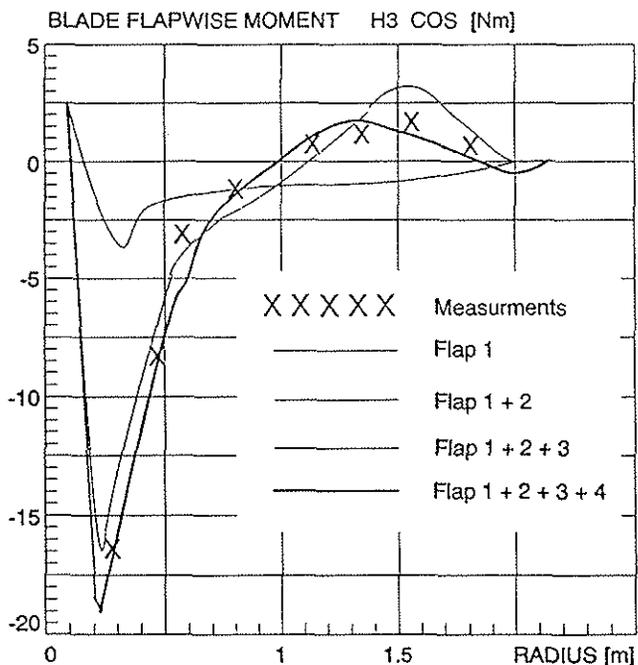
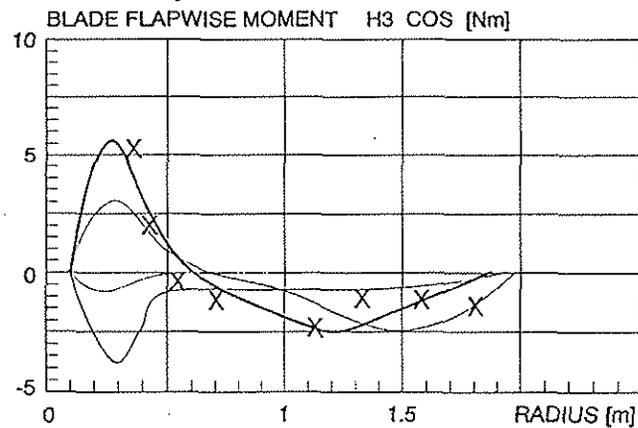


Figure 2 - Modal identification : 3/rev flapping moment radial distribution

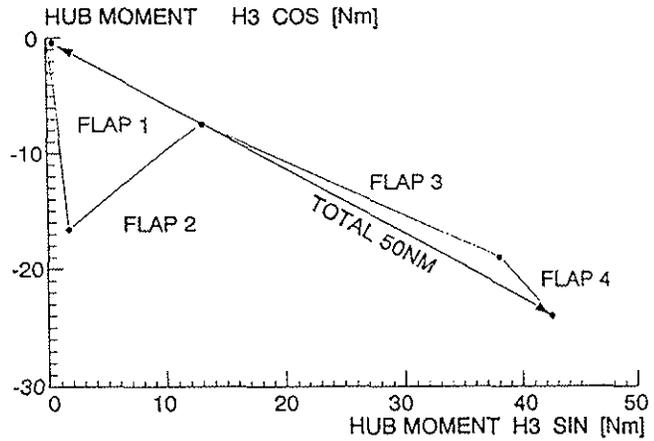


Figure 3 - 3/rev inplane moment - modal contributions

This method allows for a better understanding of the blade dynamic response to minimize rotor hub excitation. The last two methods gave satisfactory correlated results in the 349GV Gazelle research programme (See Ref. 5). Following this validation programme, the modal identification method is used for all subsequent measurements.

## 4. AERODYNAMIC AND DYNAMIC PARAMETERS

The aerodynamic parameters are mainly selected to improve helicopter performance in hover or forward flight, not for their vibration reduction capabilities. The main parameters are:

- Induced velocities
- Planform shape : rectangular or tapered
- Tip shape : swept, anhedral
- Twist

These aerodynamic parameters influence the blades dynamic properties through:

- Natural frequencies
- Generalized masses
- Modal shapes
- Modal damping

Obviously, it is difficult to understand the interferences between these two aspects (performance and vibrations) in an industrial environment. We will give an overview of the influence of each of these parameters.

### 4.1. Induced velocities

Induced velocities due to the fuselage or to the blades vortex interactions are a significant parameter. Fuselage optimization to reduce aerodynamic drag leads to design compact rotor heads. It was demonstrated during the experimental DTPX380 programme that fuselage induced velocities play a fundamental role in the rotor head aerodynamic excitation (See figures 4 and 5).

Figure 5 compares the dynamic moments calculated with and without fuselage induced velocities to those moments measured in flight.

The aerodynamic perturbations generated by the airframe increase the 4 per rev hub moment. The introduction of these perturbations in the calculations improves correlation with test results.

Another significant interaction is that generated by each blade trailing vortex. Studying this interaction with numerical calculations increases CPU costs but is proving necessary for a proper understanding of dynamic loads.

Figure 6 shows how important those calculations are for the 3-blade rotor of the 349GV Gazelle helicopter.

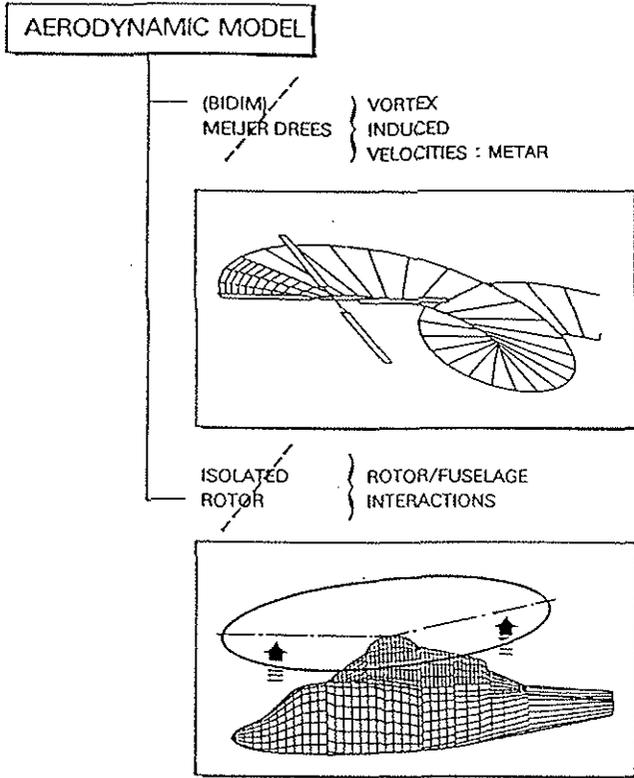


Figure 4 - Induced velocities

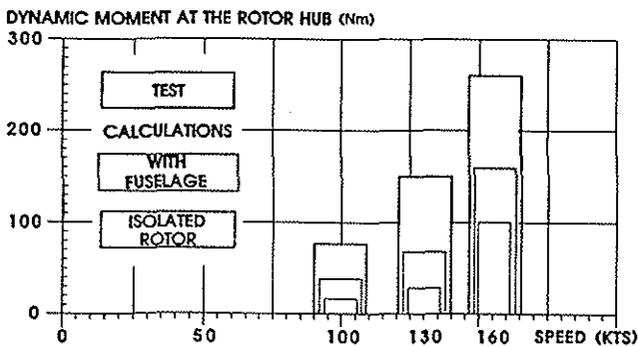


Figure 5 - Fuselage Induced velocities.

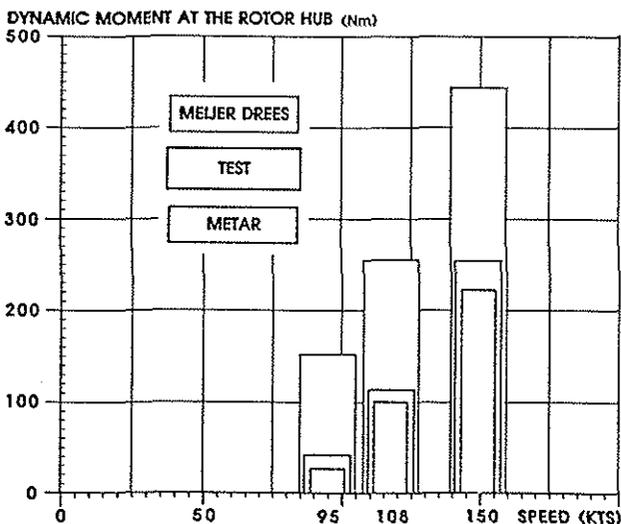


Figure 6 - Blades vortex induced velocities.

## 4.2. Blade characteristics

### 4.2.1 Number of blades

Alternate loads in the rotating system are transmitted to the fixed system. For isotropic rotors, only the  $b^{th}$  harmonics components are transmitted. The other harmonics cancel out. The number of blades is thus a highly significant factor as far as vibrations are concerned. General conclusions can then be drawn: the higher the number of blades, the lower the dynamic loads at the rotor head. The X380 research programme illustrates this. Figure 7 shows the in-plane moment excitations of the 4-blade Dauphin and 5-blade X380 helicopters. X380 excitations are reduced by a factor 2.

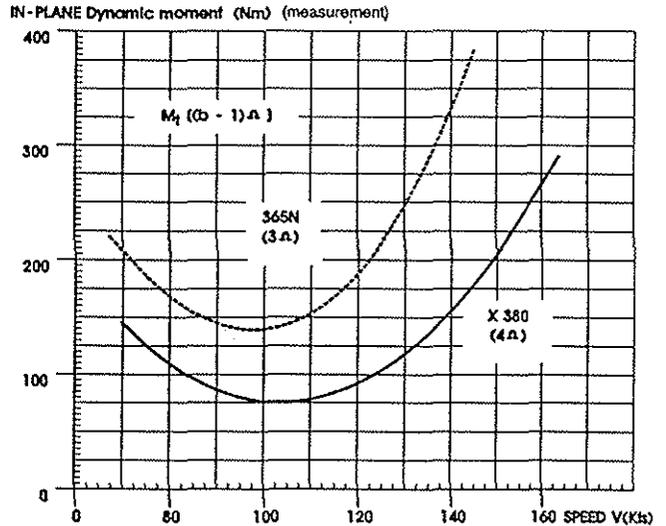


Figure 7 - In-plane moments as a function of speed for 4 blades and 5 blades Dauphin.

These loads are transmitted to the airframe via the suspension device. The fuselage response also depends on the fuselage transmissibility which varies with the excitation frequency.

The fuselage transfer is more than three times lower at 30 than at 24 Hz (See figure 8) and produces a satisfactory vibration level (0.15g approx. at 160kt) without any suspension system.

$Y_F$  (Vertical acceleration for 1000 N.m hub pitch excitation)

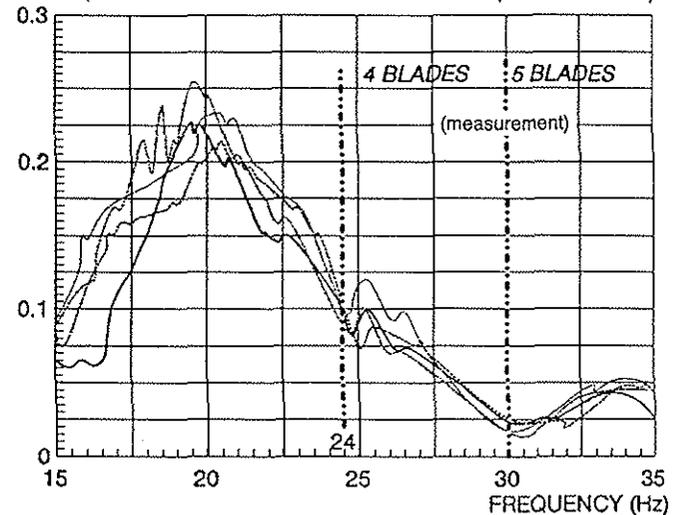


Figure 8 - DTPX380 fuselage response

### 4.2.2 Planform, tip shape and twist

The latest aerodynamic studies are producing new blades that

are no longer rectangular but tapered with evolving tips. Their twist can be modified and an anhedral added to improve their performances in hover or at high speed. The platform influences the spanwise distribution of the aerodynamic loads as well as the dynamic properties of the blades. Tapering leads, for example, to low generalized masses for those modal shapes where dynamic response and vibration level are increased. The different results available in the literature confirm that high twist is favourable for hover and low speed performance. The linear aerodynamic theory shows that higher harmonics blade flatwise loads are proportional to twist.

#### 4.2.3 Structural properties of the blades

The current blade design methodology is an optimization of aerodynamic performances as well as a change in internal structure to improve dynamic behaviour. The simplest methodology involves retaining a margin between blade modal frequencies and hub excitation frequencies ( $n \cdot$  main rotor frequency). It is possible to increase the generalized mass or shift the modal frequency of the modes the most critical for vibrations with tuning masses. Optimization techniques involve local stiffness and mass adjustments to reduce globally aerodynamic excitations and blade response to get low N per rev hub loads (moment, vertical and lateral shears). This type of optimization sponsored by DRET and STPA is undertaken by ECF in cooperation with ONERA. A preliminary study has shown that the dynamic moment can be reduced by more than 50% at the flight optimization point (See figure 9).

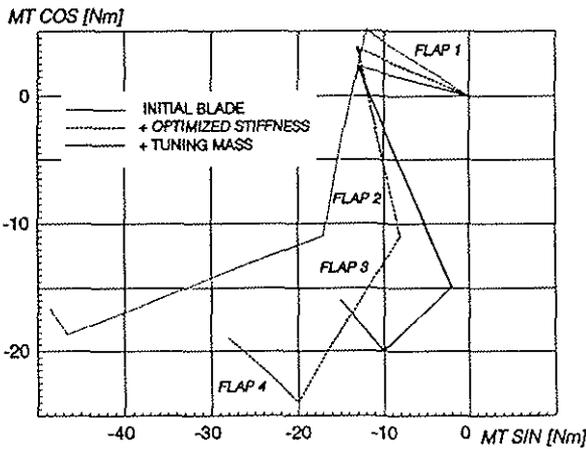
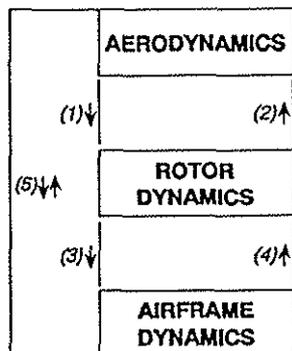


Figure 9 - Dynamic optimization, 3/rev rotor hub in-plane moment

## 5. HELICOPTER VIBRATORY RESPONSE

The dynamics of the whole helicopter is described in Fig. 10 below.



where :

- (1) are the aerodynamic loads applied to the rotor blades
- (2) is the influence of rotor dynamics on aerodynamics
- (3) are the forces applied by the rotor to the airframe
- (4) are the displacements applied by the airframe to the rotor
- (5) is the coupling between airframe dynamics and aerodynamics

Figure 10 - Helicopter dynamics

Predicting the helicopter vibratory response then imposes taking together into account :

- aerodynamics
- rotor dynamics
- airframe dynamics

It is assumed in this chapter that :

- there is no interaction between aerodynamics and airframe dynamics (coupling (5) is not taken into account)
- aerodynamics is not modified by the rotor hub dynamics (aerodynamics is taken into consideration with the isolated rotor). Nevertheless, this effect can be taken into account with numerical R85 code (See Ref. 2) but it requires high computing CPU time.

The models routinely used by ECF for helicopter vibratory response prediction are presented below.

### 5.1 Airframe dynamics

The helicopter airframe dynamic behaviour is represented in free-free configuration by its modal features which are:

- Natural frequency  $\omega_{Sj}$
- Generalized mass  $m_{Sj}$
- Modal damping ratio  $\xi_{Sj}$
- Modal shape  $X_{Sj}$

It can be pointed out here that only the first airframe modes are of interest because the helicopter vibratory excitations are of the low frequency type.

These modal features can be identified during shake tests in the laboratory or computed from a finite element model of the airframe (See figure 11).

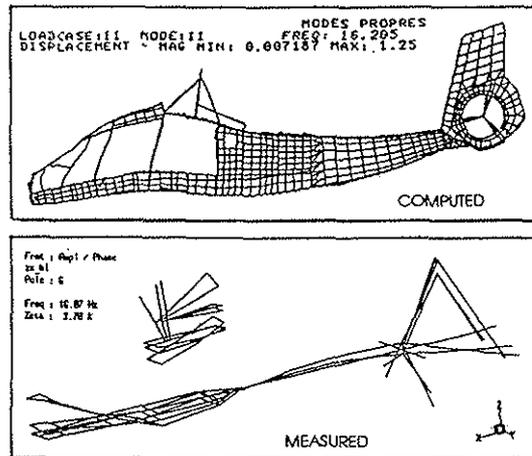


Figure 11: DGV airframe mode

### 5.2 Rotor dynamics

The rotor dynamic behaviour is modeled in a fixed coordinate system by its impedance at the rotor hub. This impedance, a  $6 \times 6$  complex matrix, is determined with the following equation:

$$Z_R(\omega) = \frac{\partial f}{\partial u_T} \Big|_{u_T = 0}(\omega)$$

where  $f$  is the vector of the dynamic rotor hub excitations in the fixed coordinate system (6 complex components) and  $u_T$  is the vector of the rotor hub dynamic displacements (6 complex components).

The  $Z_R(\omega)_{ij}$  term represents the influence of a unit variation of the

$j^{\text{th}}$  component of the rotor hub displacement (translation or rotation) on the  $j^{\text{th}}$  component of the rotor hub excitation (force or moment).

The different terms of this Impedance can be obtained in different ways :

- Analytical computing based on a knowledge of the modal features of a cantilevered rotor blade (aerodynamic damping is taken into account)
- Numerical computing with an isolated rotor dynamic behaviour code (code R85, see Ref. 2)
- Rotor rig testing in the laboratory with rotor hub displacements excitations and rotor hub reactions measurements

It can be noted here that the diagonal terms of the rotor hub Impedance matrix represent the rotor dynamic mass multiplied by  $\omega^2$ . Figures 12 and 13 give an example of the DGV rotor in-plane and out-of-plane dynamic masses as a function of the reduced frequency. It can be pointed out that the rotor dynamic mass can be very different from its mass and this shows how important it is to take the rotor dynamics into account in the helicopter vibratory response prediction.

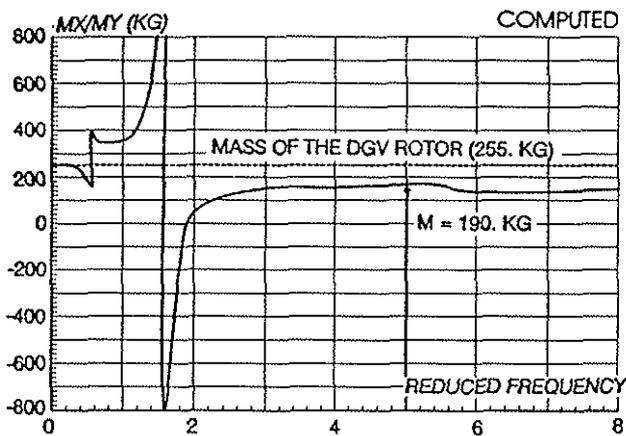


Figure 12: DGV rotor in-plane dynamic mass

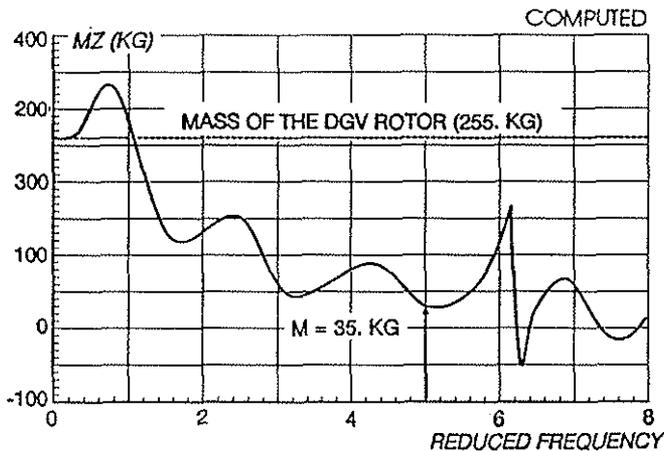


Figure 13: DGV rotor out-of-plane dynamic mass

### 5.3 Aerodynamics

Aerodynamics is represented by the rotor hub excitation vector (forces and moments) in a fixed coordinate system and with isolated rotor condition ( $u_T = 0$ ).

This vector, which is dependent on both aerodynamics and rotor dynamics, can be derived from :

- Numerical computing with an isolated rotor dynamic

behaviour code (code R85, see Ref. 2)

- Flight test measurements with strain gauges on rotor blades or shaft which need to be corrected by the airframe dynamics ( $u_T \neq 0$  in flight)

### 5.4 Rotor-airframe coupling and vibratory response

The method used here is essentially based on a linearization of the rotor hub dynamic loads (See Ref. 3) and the following assumption is considered :

$$f(u_T, \omega) \approx f_0(\omega) + Z_R(\omega) u_T$$

Both the eigen values of the rotor-airframe system (stability analysis) and the forced response of the rotor-airframe system (flight vibratory response) can then be computed (See figure 14) from the data given in the sections above.

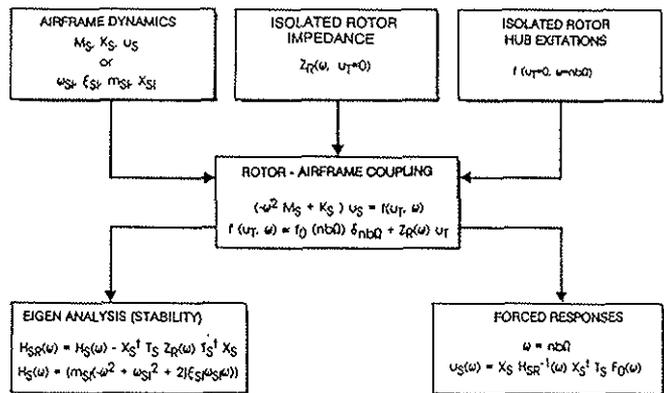
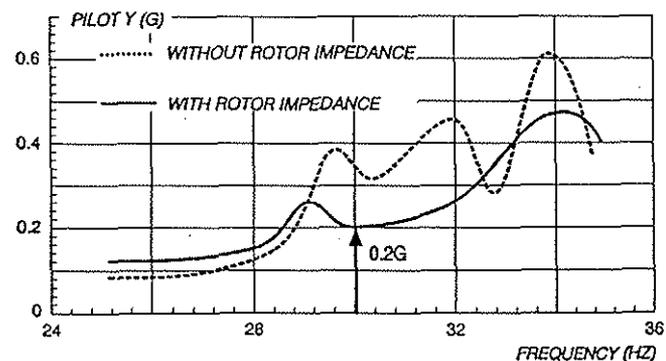
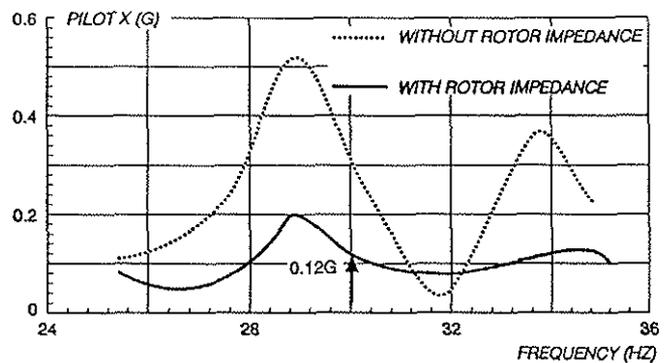


Figure 14: Rotor-airframe coupling method in a fixed coordinate system

Figure 15 below shows the DGV vibratory response computed, with or without rotor-airframe coupling, in the pilot seat at 190 kt. It can be noted here how important it is to take rotor-airframe coupling into account to predict the helicopter vibratory response.



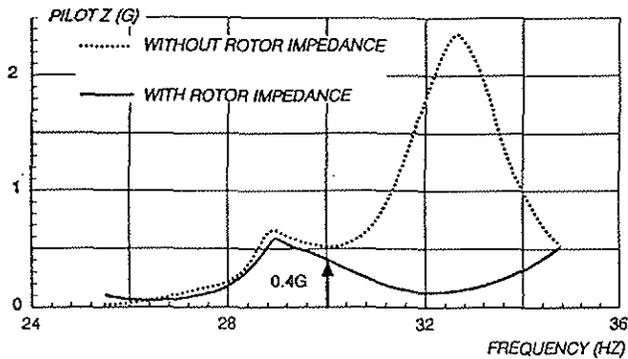


Figure 15: Example of DGV vibratory response

and flapping masses vibrations to generate inertia forces counteracting the main rotor excitations.

### Optimization and Performances Predictions

The performances of passive vibration control systems are optimized in four steps:

- 1) Simplified modelization of helicopter dynamics. This usually is an analytical modelization with a low number of degrees of freedom which helps both to understand the physical principles of the system as well as to roughly estimate the optimum masses and/or stiffnesses.
- 2) The system, including masses and stiffnesses previously determined, is included in the full helicopter modelization (see Chapter 5 above) to obtain the helicopter modal features, and the vibration level can then be computed with the following equations :

$$u_S(\omega) = X_S H_S(\omega)^{-1} X_S^T T_S F_O(\omega)$$

$$H_S(\omega) = (m_{Sj}(-\omega^2 + \omega_{Sj}^2 + 2j\xi_{Sj}\omega_{Sj}\omega))$$

- 3) Performances are optimized with parametric studies applied to the system masses (vector  $\Delta m_C$ ) and/or stiffnesses (vector  $\Delta k_C$ ) by computing the modified vibration level with the following equations :

$$u_S(\omega, \Delta m_C, \Delta k_C) = X_S H_S(\omega, \Delta m_C, \Delta k_C)^{-1} X_S^T T_S F_O(\omega)$$

$$H_S(\omega, \Delta m_C, \Delta k_C) = (m_{Sj}(-\omega^2 + \omega_{Sj}^2 + 2j\xi_{Sj}\omega_{Sj}\omega) - X_{kC}^T \Delta k_C X_{kC} + \omega^2 X_{mC}^T \Delta m_C X_{mC})$$

- 4) The optimum masses and stiffnesses determined above are included in the full helicopter modelization, returning to the second step backwards whenever a more accurate optimization is required, which is then used to predict the system performances.

### Performances

The passive vibration control systems can help to obtain a satisfactory helicopter vibration level but their main drawbacks are :

- The weight penalties they impose
- The fact that they cannot adapt themselves to a change in structural configuration (fuel weight, stubwing loads, crew and passenger weight etc), flight conditions (speed, curve etc), or mission (transport, firing etc), so that the helicopter vibratory level may deteriorate and prove unsatisfactory in some flight and structure configurations.

### 6.2 Active vibration control

The active vibration control systems developed by ECF are:

- The Blade Pitch Higher Harmonic Control, (See Ref. 1)
- The Active Control of Structural Response (ACSR) or Force Transfer (ACFT), (See Ref. 4)

### Operating principles

These different systems control some actuators to minimize the response of some sensors. These systems are mainly composed of three elements :

- 1) The sensors that are either accelerometers in the cabin and/or a firing sight or gun or also gauges on the MGB - fuselage links
- 2) The computer receiving the sensors response and calculating

## 6. PASSIVE AND ACTIVE VIBRATIONS CONTROL

Although airframe dynamics, rotor dynamics and aerodynamics were optimized, the helicopter overall vibration level can prove unsatisfactory and needs to be controlled and improved with an external device. These systems are of two types, passive and active, and the following chapters are a description of the main vibration control systems used or developed by ECF.

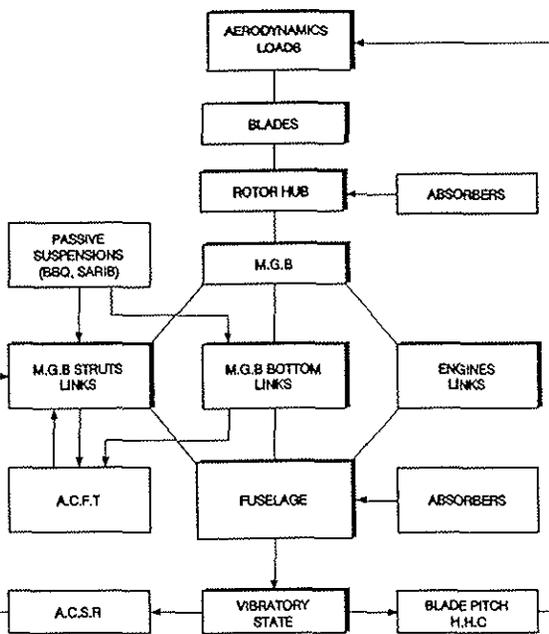


Figure 16: Helicopter passive and active vibration control

### 6.1 Passive vibration control

The two main passive vibration control systems used by ECF are dynamic absorbers and MGB suspension systems.

#### Operating Principles

The dynamic absorbers located in the rotor hub or the cabin generate inertia forces via a flapping mass counteracting the main rotor excitations (rotor hub) or the structural response to the main rotor excitations (cabin).

The MGB suspension systems (BBQ and SARIB) modify the MGB - fuselage links (flexibility and/or flapping masses) for the MGB

the commands minimizing a function of this response (called performance criteria)

3) The actuators receiving commands from the computer. These are series mounted with the flight actuators in the Blade Pitch Higher Harmonic Control and they replace the MGB struts in the Active Control of Structural Response and Force Transfer

It can be pointed out here that the Blade Pitch Higher Harmonic Control and the Active Control of Structural Response or Force Transfer are fundamentally different in that :

- The Blade Pitch Higher Harmonic Control modifies the external source (aerodynamic loads) to minimize the performance criteria
- The Active Control of Structural Response or Force Transfer modifies the structural dynamics to minimize the performance criteria

### Optimization and Performance Prediction

#### ACSR/ACFT

The performances of the Active Control of Structural Response (ACSR) or Force Transfer (ACFT) are predicted with a global linear modelization.

The command minimizing the performance criteria PI :

$$PI = u_m^h W_u u_m$$

with the following behaviour measurement law:

$$\begin{aligned} u_m &= u_{m0} + B_m v \\ u_{m0} &= X_m H_S^{-1} X_S^t T_S f_0 \\ B_m &= X_m H_S^{-1} X_V^t \end{aligned}$$

is provided by

$$v = - (B_m^h W_u B_m)^{-1} B_m^h W_u u_{m0}$$

#### Blade Pitch HHC

The blade pitch HHC performances are predicted with a local linear modelization.

The command minimizing the performance criteria PI :

$$PI = u_m^h W_u u_m$$

with the following behaviour measurement law :

$$\begin{aligned} u_m(v_n + \Delta v) &= u_{m0}(v_n) + B_m(v_n) \Delta v \\ u_{m0}(v_n) &= X_m H_S^{-1} X_S^t T_S f_0(v_n) \\ B_m(v_n) &= X_m H_S^{-1} X_S^t T_S Y(v_n) \\ Y(v_n) &= \left. \frac{\partial f_0}{\partial v} \right|_{v_n} \end{aligned}$$

is provided by :

$$v = \lim_{n \rightarrow \infty} \frac{1}{n} (\sum_{l=1}^n \Delta v_l)$$

$$\Delta v_n = - (B_m^h(v_n) W_u B_m(v_n) + W \Delta v)^{-1} B_m^h(v_n) W_u u_m(v_n)$$

### Performances

The active vibration control systems offer higher performances than passive ones (see Figure 17 below) essentially because they are capable of adapting to changes in structural and/or flight configuration. The main drawbacks of these systems are their technological complexity and costs (actuator costs).

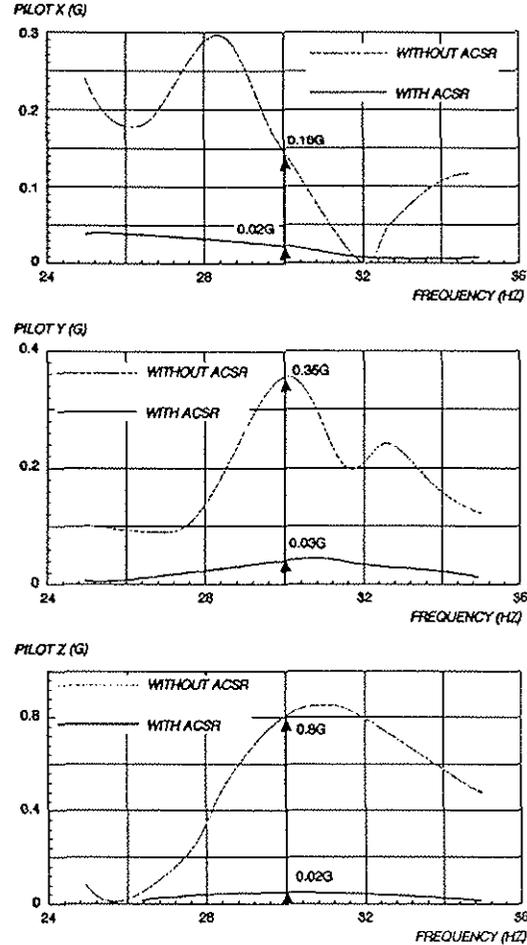


Figure 17: DGV2 passive and active (ACSR) forced response

## 7. AEROMECHANICAL STABILITY AUGMENTATION BY ACTIVE CONTROL

Rotor fuselage instabilities are a permanent engineering challenge and this chapter primarily discusses ground and air resonance, although other instabilities such as flap-lag coupling and flutter would also deserve attention.

Past efforts were mainly focussed on passive mechanical devices such as lead-lag dampers or under-carriages to avoid coalescence of fuselage body modes with rotor lead-lag modes. Optimizing involves moving the 1st regressive lead-lag frequency away from the helicopter's modal frequencies, both on the ground and in the air.

A wide variety of configurations is usually encountered with different weights, inertiae etc. for a given helicopter. The fuselage modal frequencies vary significantly and in many cases, it is not possible to obtain large gaps between the fuselage's rigid modes and regressive lead-lag modes. Sufficient damping must then be provided at both the fuselage and rotor to avoid the instability.

It is better to have, for ground resonance purposes, a high 1st

lead-lag frequency which avoids overdamping the rotor.

This high frequency is obtained in light helicopters with elastomeric dampers which do not impose heavy load, stress, weight or volume constraints. This technology offers many advantages in terms of cost and maintenance. The problem with operating rotors at a high 1st lead-lag frequency is that air resonance may occur and this problem is more acute with rigid rotors because damping is then mainly structural.

Stress, weight and volume constraints prohibit using elastomeric dampers in heavier helicopters and the common approach in this case is to use fully or partially hydraulic dampers providing a high damping ratio at relatively low stiffnesses. The penalties associated with this technology are high development and maintenance costs.

Optimizing at landing gear level involves selecting the landing gear geometry as well as the tyres and dampers' dynamic stiffness in accordance with other constraints e.g. taxiing and crash for proper positioning of rigid body modes relative to regressive lead-lag frequency.

Damper optimization covers chamber volume, oil volume and orifice lamination to obtain high damping ratios at the low frequencies and displacements typical of ground resonance.

Those efforts notwithstanding, ground and air resonance may still occur because of poor equipment maintenance, improper servicing or even failure for some components.

Active control systems have been and are still being investigated extensively to overcome the potential problems associated with passive systems.

Several criteria are applied to those systems for comparison purposes:

- Damping augmentation versus frequency displacement
- Rotor damping versus fuselage damping augmentation
- Individual blade control versus swashplate control
- Rotating system versus fixed system control

Several criteria will apply to the type of control that will be used:

- Cost
- Safety
- Performance
- Ruggedness
- Design qualities

The challenge is obviously to get rid of lead-lag dampers to obtain cheaper and lighter designs.

The development of active control of ground resonance is now part of Eurocopter research programme and its validity shall be demonstrated in the near future according to the principles presented in Fig. 18 below.

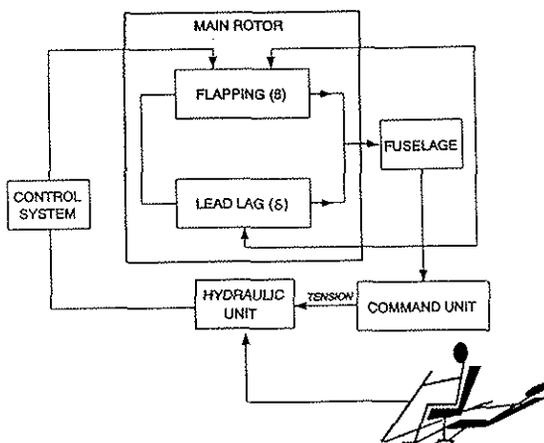


Figure 18: Active control of ground resonance (principle)

The theoretical calculations that were made show (Fig. 19) that the system is unstable without and stable with active control.

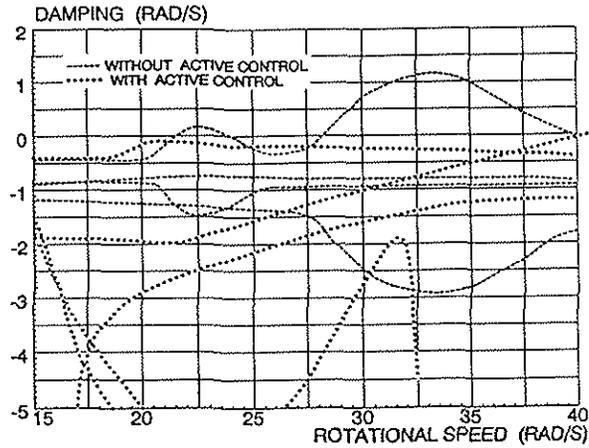


Figure 19: Active control of ground resonance (simulations)

## 8. CONCLUSION

The research work undertaken by Eurocopter France is mainly an attempt to improve the current understanding of helicopter dynamics, stability and aeroelastic response.

The main idea is that the helicopter, with inherently low vibrations and proper stability, is proving cheapest, most reliable and least maintenance intensive.

Active controls are, for example:

- Active Control of Structural Response or Force Transfer
  - Higher Harmonic Control
  - Active Control of Ground Resonance
- are currently under development and a full scale demonstration programme is now in progress.

## 9. REFERENCES

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## 10. NOTATIONS

- $(A)^t$  : transpose of the matrix A
- $(A)^h$  : transpose and conjugate of the matrix A
- $M_S$  : airframe mass matrix
- $K_S$  : airframe stiffness matrix
- $u_S$  : airframe D.O.F. vector
- $u_T$  : rotor hub D.O.F. vector
- $u_m$  : controled measured vector
- $u_{m0}$  : uncontroled measured vector
- $W_U$  : ponderations matrix on the measures
- $W_W$  : ponderations matrix on the pitch blade command variations
- $\omega_{Si}$  : airframe natural frequency ( $i^{\text{th}}$  mode)
- $\zeta_{Si}$  : airframe modal damping ratio ( $i^{\text{th}}$  mode)
- $m_{Si}$  : airframe generalized mass ( $i^{\text{th}}$  mode)
- $X_{Si}$  : airframe modal shape ( $i^{\text{th}}$  mode)
- $X_{mC}$  : airframe modal shape on the D.O.F. related to mass modifications
- $X_{kC}$  : airframe modal shape on the D.O.F. related to stiffness modifications
- $X_m$  : airframe modal shape on the measured D.O.F.
- $X_v$  : airframe modal shape on the controled D.O.F.
- $v_{mC}$  : mass modifications vector
- $v_{kC}$  : stiffness modifications vector
- $H_S$  : airframe modal transfer
- $H_{SR}$  : coupled airframe-rotor modal transfer
- $T_S$  : airframe rotor hub location matrix
- $Z_R$  : Isolated rotor Impedance
- $Y$  : sensivlity of the rotor hub excitations to the pltch blade command
- $f$  : rotor hub excitations
- $f_0$  : Isolated rotor hub excitations
- $v$  : command vector
- $b$  : number of blades