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ESTIMATION OF HELICOPTER PERFORMANCE BY AN EXTENDED ENERGY METHOD IMPROVED BY FLIGHT TESTS
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#### Abstract

This paper presents a useful method for obtaining helicopter performance data. Only minimal flight test data is required and the method does not require excessive computer time. The estimation of helicopter performance by the energy method yields good results for medium forward speeds. The energymethod has been extended to also include hovering, low speed, and high speed flight. It was found that only a small number of flight test data points are needed to obtain the required correction factors. These factors cover effects which are not considered in the simple downwash model, take into account ground effect influences, and correct for power losses caused by compressibility effects.

Results computed using the expanded method were compared with flight test data for five different helicopters. Calculated results agreed closely with experimental results when flight test data of sufficient accuracy was used.


1. List of Symbols

| A | area of rotor disk | $\mathrm{v}_{\mathbf{i}}$ | induced velocity |
| :---: | :---: | :---: | :---: |
| ${ }^{\text {c }}$ pc | compressibility power coefficient | $W^{1}$ | aircraft weight |
| ${ }^{\text {c }}$ T | thrust coefficient | $\dot{\text { w }}$ | acceleration along the z-axis |
| D | drag | Z | distance between tip-path plane |
| K | correction factor |  | and the ground |
| M | Mach number | $\alpha$ | angle of attack |
| m | mass of the helicopter | $\gamma$ | flight path angle |
| P | power | $\delta v_{i}$ | reduction of induced velocity |
| r | rotor blade radial position. |  | due to ground effect |
| R | radius of the rotor | $\theta_{T}$ | inclination of thrust-vector |
| T | thrust |  | from the vertical |
| $\dot{\mathrm{u}}$ | acceleration along the x -axis | $\lambda$ | inflow ratio. |
| V | airspeed | $\mu$ | tip: speed ratio |
| $V^{\prime}$ | resulted velocity in the tip-path | $\rho$ | air density |
|  | plane | $\sigma$ | rotor solidity |
|  |  | $\Omega$ | rotation frequency of the rotor |

## Subscripts

| c | compressibility | OGE | out of ground effect |
| :--- | :--- | :--- | :--- |
| cr | critical | p | profile |
| GE | ground effect | par | parasite |
| i | induced | req | required |
| IGE | in ground effect | TP | tip-path plane |

## 2. Introduction

Performance calculations of helicopters using the blade element theory yield satisfactory results. However, the computing time needed is often excessive, especially if compressibility and blade stall as well as unsteady effects and regions of reverse flow are considered. Performance determination by flight testing also yields satisfactory results but generally requires an extensive flight test.program. The energy method also yields useful performance data, generally with low computing effort, but only for medium flight speeds.

The energy method yields the power required for forward flight as the sum of individual power terms. Induced drag results as the rotor blades produce thrust. As airspeed increases the power required to overcome the induced drag, $P_{i}$, is decreased. Since there is increased air flow, the rotor needs to impart less velocity to each mass of air, and the energy required is reduced. The power required to overcome rotor blade profile drag, $P_{p}$, increases slightly with airspeed. In the lower speed region it can be assumed to be constant. The power required to overcome parasitic drag (fuselage, landing gear etc.), $\mathrm{P}_{\text {par }}$, increases as the cube of the forward velocity and exceeds considerably other power terms in the higher speed regions.

Many other effects can be accounted for, but $P_{i}, P_{p}$, and $P_{p a r}$ are the most significant. These terms are determined separately and then added together to give a useful approximation for the power required for the forward flight of a helicopter. It is found that the calculated value underestimates the actual power required. There are many reasons. Simplified models are used in determining the three power terms. There are transmission losses and some power is used by the tail rotor. Additional power is required during accelerated flight. By the addition of a miscellaneous power term equal to approximately ten percent of the total power, quite useful results are obtained in the medium speed range.

The three power terms are shown in fig. 1 for the Bell UH-1D. Also shown is their sum including the ten percent miscellaneous power term. It can be seen that the validity of the energy method decreases in the lower and upper speed regions.

The range where satisfactory results are obtained is restricted in the lower speed region by limitations on the momentum theory. The momentum theory is used to estimate the induced velocity, $v_{i}$, which is used to determine $P_{i}$. To use the momentum theory, it is necessary that the stream-tube area remain finite both ahead of and behind the disk. For the hovering helicopter, the stream-tube area is infinite above the disk because the velocity is zero. This represents one limit for the momentum theory. For the vertically descending helicopter, another limit occurs when the induced velocity is equal to the descent rate.

The range where satisfactory results are obtained using the energymethod is also restricted in the higher speed ranges. Compressibility effects occur at the tip of the advancing blade.

In the United States, correction factors for the induced velocity and the ground effect ( $K_{i}$ and $K_{G E}$ ) have been applied to produce an improved performance model. This improved model was used with very satisfactory success during height-velocity and takeoff maneuver investigations (ref. 1 and 2). A constant power term containing both the profile drag and miscellaneous powers was assumed. Good agreement between predicted and flight test results was achieved.

Beginning with this improved performance model, modifications and extensions were made to allow its application over the entire helicopter speed range.

## 3. Performance Equations

The performance of a helicopter can be described by five algebraic equations.

### 3.1. Total Power Required Equation

The total power required, $P_{\text {req }}$, can be written as the sum of the power required to overcome induced drag, parasitic drag, and profile drag, the power required to compensate for compressibility effects, $P_{C}$, and a rest term, $\mathrm{P}_{\text {rest }}$, which includes miscellaneous power losses.

$$
P_{\text {req }}=P_{i}+P_{p a r}+P_{p}+P_{c}+P_{\text {rest }} \text {. }
$$

### 3.2. Force Balance Equations

The force equations describe the trajectory of the helicopter as it accelerates parallel or perpendicular to the flight path. Thrust and inclination of the thrust vector are determined by these equations. Several simplifying assumptions are made to facilitate this study.
1.) It has been demonstrated by experiment that the resultant rotor vector is generally inclined slightly aft of the tip-path plane. As this inclination usually does not exceed one degree, it is assumed that the thrust vector is always perpendicular to the plane of the tips.
2.) Forces resulting from the horizontal tail were neglected because they are small in comparison with fuselage forces.
3.) The fuselage drag acts at the center of gravity of the helicopter, parallel to the free stream velocity.
With these assumptions the force equations in an earth-fixed coordinate system are (fig. 2):

$$
\begin{aligned}
T \cdot \sin \theta_{T} & \stackrel{\mp}{=} D \cdot \cos \gamma-m \cdot \dot{u}-0 \\
-T \cdot \cos \theta_{T} & +D \cdot \sin \gamma \stackrel{m}{=} \cdot \dot{w}+W=0
\end{aligned}
$$

### 3.3. Momentum Equation

The simple momentum theory assumes the following:
1.) The rotor has an infinite number of blades and can be considered as an actuator disk with a uniform flow through the disk.
2.) The induced velocity is perpendicular to the plane of the tips.
3.) The induced velocity at the airscrew disk is one-half of the total increase in velocity imparted to the air column.
The momentum theory gives the relation between the resulting thrust vector $T$, the induced velocity, the angle of attack of the tip-path plane $\alpha_{T P}$, and the airspeed $V$. The resulting airspeed in the tip-path plane can be taken from figure 3.

$$
V^{\prime}=\sqrt{\left(v_{i}-V \cdot \sin \alpha_{T P}\right)^{2}+\left(V \cdot \cos \alpha_{T P}\right)^{2}}
$$

The momentum equation can be written as

$$
T=\dot{m} \cdot \Delta V=\left(\rho \cdot A \cdot V^{\prime}\right) \cdot\left(2 \cdot v_{i}\right)
$$

Expressing induced velocity as a function of the tip speed ratio $\mu$ and the inflow ratio $\lambda$ and rearranging gives an equation of fourth order in $\lambda$.

$$
\lambda_{T P}^{4}-2 \cdot \frac{W}{\Omega R} \cdot \lambda_{T P}^{3}+\left(\frac{V}{\Omega R}\right)^{2} \cdot \lambda_{T P}^{2}-2 \cdot \frac{W}{\Omega R} \cdot \mu_{T P}^{2} \cdot \lambda_{T P}+\left[\left(\frac{W}{\Omega R}\right)^{2} \cdot \mu_{T P}^{2}-\left(\frac{C_{T}}{2}\right)^{2}\right]=0
$$

The solution of this equation yields the inflow ratio $\lambda$, from which the induced velocity can be estimated.

### 3.4. Ground Effect Equation

As the helicopter approaches the ground, the induced velocity required to produce a given thrust is reduced with a resultant decrease in induced power (ref. 3). The change in induced velocity is given by the expression

$$
\delta v_{i g}=\frac{1}{16} \cdot \frac{v_{i}}{\left(\frac{Z}{R}\right)^{2} \cdot\left[\left(\frac{V}{v_{i}}\right)^{2}+1\right]}
$$

where $Z / R$ is the dimensionless ratio of tip-path plane height to blade radius. It can be seen from fig. 4 that the ground effect equation is not valid for small heights. Therefore, the computation in the theoretical model is made with a constant value for $\delta v_{i g}$ if $Z / R$ is lower than 0.5 . This assumption is not critical. $Z / R=0.5$ is about the minimum which can be reached by most helicopters.

## 4. Correlation of the Performance Model with Flight Test Data

In the lower speed region, inexact results of the induced velocity are obtained by the application of the momentum equation, which causes discrepancies between flight test and theoretical model results. To correct this, it was first attempted to multiply the induced velocity by the correction factor $K_{i}$. Because this method did not yield good results for any cases, and in consequence of literature investigations (ref. 4), a linear dependence for this factor on airspeed was also introduced. The induced power now can be written as

$$
P_{i}=T \cdot v_{i} \cdot K_{i} \cdot\left(1-\mu_{T P}\right)
$$

Figure 5 shows the power-required curve for the BELL UH-1D. The term $P_{\text {rest }}$ contains the profile-drag power and the miscellaneous power which are assumed to be constant. After the introduction of this new correction factor, good correlation was obtained between flight test results and the model for airspeeds up to $25 \mathrm{~m} / \mathrm{s}$. In the upper speed region, the assumption of a constant profile-drag power is not valid. Therefore, power losses to profile drag were computed separately.

Current helicopters have rotor blade tip speeds and forward flight speeds that can cause the rotor to encounter compressibility effects. To improve the computed results in the upper airspeed range power losses caused by compressibility effects were calculated. The correction method is based on
the estimation of the blade radius outboard of which the blade section free stream Mach number is greater than the blade section local critical Mach number. Additionally, the computation of compressibility effects is not made around the entire 360 degrees of azimuth; instead, the advancing blade is considered only in the azimuth-position $\psi=90^{\circ}$. This method yields satisfactory accuracy. The compressibility effects power coefficient $c_{p c}$ can be written as

$$
c_{p c}=\left.\sigma \cdot K_{c} \cdot\left(M-M_{c r}\right)\right|_{\substack{r=R \\ \Psi=90^{\circ}}} \cdot(1+\mu)^{2} \cdot\left(1-\frac{r_{c r}}{R}\right)^{2}
$$

where $K_{c}$ is the compressibility correction factor.
Figure shows the single power terms as a function of the airspeed, with and without compressibility correction for the SIKORSKY CH-53 D/G. It can be seen in these graphs that compressibility effects not only occur at high forward speeds, but can be present in the lower speed region too. The tip speed of the advancing blade is lower, but the critical Mach number is also lower because of the increased collective pitch of the blades at low airspeeds. The combined effect can sometimes cause increased compressibility effects. The introduction of the compressibility correction not only yields an improvement in the computed results in the upper airspeed region, but also a reduction of the rest term $\mathrm{P}_{\text {rest. }}$. Since this factor can be interpreted as giving information about the accuracy of the method, it shows that the compressibility correction is a practicable way to improve the mathematical model.

For theoretical investigations at lower flight altitudes, it is necessary, especially in the lower speed region, to insert another correction factor. This ground-effect constant $K_{G E}$ takes into account the discrepancies between the theoretical and flight test values for the change in the induced velocity.

$$
K_{G E}=\left(\frac{\delta v_{i g}}{v_{i}}\right)_{\text {flight test }} /\left(\frac{\delta v_{i g}}{v_{i}}\right)_{\text {theory }}
$$

Some rearrangement yields this correction factor.

$$
K_{G E}=\frac{\frac{P_{O G E}-P_{I G E}}{P_{O G E}-\bar{P}}}{\frac{1}{16} \cdot\left(\frac{R}{Z}\right)^{2}}
$$

$\overline{\mathrm{P}}$ is the sum of the profile-drag power, compressibility effects power, and the rest term $P_{\text {rest. }} \bar{P}$ is assumed to be constant for hovering flight, both in and out of ground-effect. Practical experience has shown that the groundeffect correction factor is usually between 1.5 and 2.5. Figure shows the influence on the power-required curve of this factor.

To evaluate the coefficients $K_{i}$ and $K_{c}$, as well as the rest term $P_{r e s t}$, three flight test power data points are necessary. However, care should be taken to avoid grouping the three points too close together since measurement errors can easily invalidate the results. The ground effect correction factor is found separately by simply measuring the power required to hover in and out of ground-effect.

Using this improved performance model, forward flight performance for the BELL UH-1D, SIKORSKY CH-53, BOELKOW BO-105, ALOUETTE II and SIKORSKY S-58 was calculated, and the results were compared with available performance data. As an example, figure 8 shows the power-required curve for the BELL UH-1D. The results show good agreement for all airspeed regions. Similar results have already been presented in fig. 6. Of course, the accuracy of the theoretical model is highly dependent on the quality of the data used to get the correction factors.

## 5. Influence of Helicopter Flight Altitude and Gross Weight on the Correction Factors

The correction factors can be assumed to be constant for some cases, for instance take-off and landing. In many instances however, changes in aircraft gross weight caused by fuel consumption and air density variations resulting from changes in flight altitude must be considered in the computations. To find out the dependence of the correction factors on these variables, a parameter sensitivity investigation was performed.

Fig. 9 shows the results of this investigation. It can be seen that the correction factors for the induced power and for the compressibility effects power increase linearly with aircraft flight altitude. The rest term also is linear, but decreases with altitude.

The influence of the helicopter weight is slightly more complicated. $K_{i}$ still increases linearly with helicopter weight, but $K_{c}$ appears to vary with the square of the weight. This indicates that compressibility effects ary strongly dependent on helicopter gross weight. Also it can be seen that the rest term decreases with the square of the gross weight.

Considering these dependencies in the computation, performance determinations are now possible in cases, where gross weight and air density have no constant values.

## 6. Application of the Performance Model to Climbing Flight

By showing that these correction terms are also valid for climbing flight, the determination of climb performance was greatly simplified. Fig. 10 shows the computed engine power required as a function of airspeed for various rates of climb. From this graph, the maximum rate of climb as a function of engine power available was determined and plotted as a function of airspeed. The results show good agreement with flight test data. The assumption that the correction factors are valid for both horizontal forward flight and climbing flight seems to be justified.

## 7. Conclusions

For many theoretical investigations of helicopters it is necessary to know the rotor power required. Performance calculations using the blade element theory yield satisfactory results; however, in some cases the computing time needed is considerable. It has been shown in this paper that the application of the energy method also yields satisfactory results with small computing effort when correction factors obtained from flight test data are used.

This new extension of the energy method by flight test data can be used in the low, medium, and high airspeed regions, and is valid for climbing flight. Additionally, since gross weight and density altitude variations are also accounted for; the scope of application of the performance model has been essentially extended.
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Fig. 1 Performance estimation by the energy method


Fig. 2 Forces, acting at the helicopter


Fig. 3 . Velocities, relative to the tip path plane


Fig. 4 The influence of tip plane height on induced velocity


Fig. 5 Performance estimation by the energy method with induced velocity correction


Fig. 6 Influence of ground-effect correction on power-required curve


Fig. 7 Influence of compressibility on power-required curve


Fig. 8 Performance estimation by the energy method with induced velocity - and compressibility correction


Fig. 9 Influence of helicopter flight altitude and gross weight on the correction factors



Fig. 10 Climb performance

