A FRAMEWORK FOR THE OPTIMISATION OF A BERP-LIKE BLADE

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Abstract

This work presents a framework for the optimisation of certain aspects of a BERP-like rotor blade in forward flight while constraining hover performance. The proposed method employs a high-fidelity efficient CFD technique that uses the Harmonic Balance method in conjunction with artificial neural networks (ANNs) as metamodels, and genetic algorithms (GAs) for optimisation. The approach has been previously demonstrated for the optimisation of linear twist of rotors in hover (steady case) and the optimisation of rotor sections in forward flight (unsteady case), transonic aerofoils, wing and rotor tip planforms. In this paper, a parameterisation technique was defined for the BERP-like rotor tip and three of the parameters were optimised in forward flight. A specific objective function was created using the initial CFD data and the metamodel was used for evaluating the objective function during the optimisation about the shape of the final lifting surface and its performance. The results were checked using different optimisation methods and were not sensitive to the employed techniques with substantial overlap between the outputs of the selected methods. The main CPU cost was associated with the population of the CFD database necessary for the metamodel using a full factorial method and even that was reduced with the use of the Harmonic Balance method.

1 INTRODUCTION

The BERP tip was designed for high speed forward flight without compromising hover [11]. The problems associated with this flight regime, is that the effects of compressibility such as transonic flow and shockwaves become significant especially on the advancing blade. Typically, thin aerofoils are used but these tend to stall more easily at the high angles of attack which occur on the retreating side. The first step in the design of the BERP was the aerofoil selection. The aerofoils were selected such that thinner sections could be used to enable higher forward flight speeds. Camber was introduced to improve the stall capability of the blade on the retreating side. However, with camber, come increased pitching moments that were alleviated by using an aerofoil that counters the pitching moment inboards i.e. one with reflex. This resulting blade behaves well in terms of control requirements and twist loads [18].

The planform was then optimised to reduce high Mach number effects by first sweeping the tip of the blade back. This moved the aerodynamic centre of the swept part backwards causing control problems in the pitch axis. To counteract this, the swept part was translated forward which introduced a notch on the leading edge of the blade. The notch corners were smoothed to avoid flow separation. A "delta" tip was also incorporated so that a stable vortex formed at higher angles of attack on the retreating side to delay stall [3].

Some of the beneficial characteristics of the blade are that the blade stall occurs first inboards of the notch and does not spread outwards. This is because at high angles of attack, such as at the retreating side, the vortex formed travels around the leading edge and the flow over the swept part remains attached. The BERP blade shows similar performance to a standard rotor blade at low speed flight, but superior performance in forward flight due to the absence of drag rise and flow separation [3]. In hover, the FM was improved due to the minimisation of blade area and overall, there were no penalties in hover performance. At high speeds, vibration was also reduced as well as control loads for manoeuvres [3].

In terms of optimisation methods, a non-gradient method was selected since these methods are not trapped in local optima and the design space is expected to be highly uneven with lots of local optima. Genetic Algorithms (GAs) is a popular method that fits in this category. It simulate the evolutionary process involved in natural selection i.e. that an initial population exists from which newer generations are created, with each new generation increasing in 'fitness' by the processes of crossover and mutation within the individuals.

The problem with such a method is the computational cost in determining new points. Therefore these methods tend to be used for steady state or smaller design optimisations. Examples can be found in the following references [1, 19, 24, 26]. In some cases, a combination of gradient and non-gradient methods are used to reduce the computational cost but have a global method. For example, Poloni et al. [16] used a hybrid GA and gradient-based method along with an ANN to optimise the keel of a yacht for high lift and low drag, parameterised by Bézier curves. Dulikravich et al. [7] also used a hybrid method that combined a number of optimisation techniques including Genetic Algorithms, Simulated Annealing methods, gradient methods and the Nelder-Mead simplex method (NM) to create a more robust optimiser. The method switches between these techniques where they work best; for example the gradient method is employed when the variance from the GA's output is small and the NM method is used when the random behaviour of the GA is prevalent.

Régnier et al. [17] state that there are three classifications of non-gradient based optimisation viz.,

- Apriori - Decide and Search: the decision maker decides what is required and the solution is found e.g. the use of a weighting system. Only one Pareto solution is found. - Progressive and Sequential - Decide and Search: Optimisation and decision-maker are intertwined. The preferences are sequentially updated. Multiple runs are required to obtain Pareto solution.

- Aposteriori - Search and Decide: A single optimisation run provides a set of solutions that the decider can choose from e.g. Genetic Algorithms (EAs).

The last one is of particular interest to this project since there is no target performance required. Rather, the best design within the constraints on performance is required. Both the weighted sum and the Pareto method were used. The advantage of using a weighted sum is that it selects a small region in the database where the optimum can be found. The design variables do not change much in this area. However, this reduces the flexibility of finding another such region unless the weights are changed. Therefore the Pareto method is used to find the best compromise of the performance parameters and typically, the results of the weighted sum method should lie somewhere on the Pareto front [4]. The advantages of using a Pareto method is that it allows the designer to be able to see the best compromise before making the decision. However, this can still be a difficult task if the number of design variables is high and if the properties of the Pareto subset are not properly analysed. More information about this can be found in the paper by Daskilewicz and German [6]. Much research has been carried out in developing and comparing various evolutionary optimisation techniques and can be found in the following literature [10, 20, 23].

One way of overcoming the computational load, is to use metamodels such as Neural Networks and kriging techniques. For example, in Zhao [27], the optimisation of turbines is performed using a neural network trained model and a GA. The original data was obtained using a NS solver. Bézier curves were used to define the geometry and their control points were used as design parameters. In Imiela's work [12], he used a genetic algorithm to optimise the rotor and the MATLAB DACE kriging toolbox as a metamodel. Hover and forward flight were both considered whilst constraining structural integrity within boundaries. A number of optimisation algorithms were tested - CONGRA, a conjugate-gradient based method (works by using gradients as well as finding new search directions based on former iterations), SUBPLEX, a non-gradient based method and EGO, which uses a metamodel in a GA. The conclusions were that CONGRA does not reach the optimum fast enough, and was dependent on the step size. SUBPLEX struggled with the tip optimisation, almost producing a rectangular blade which may be due to issues with accuracy. EGO was the most efficient and the least-error prone.

Kriging and ANNs are popular methods used due to their versatility and accuracy. Glaz et al. [8,9] are in favour of such methods but suggested that since no single metamodel is generically the best, it may be useful to use a combination of metamodels instead. Celi [5] also mentioned that the "connection between predictions and accuracy of the optimisation may be more subtle than appears at first glance." Also in the work done by Liu et al. [13], it was shown that for more complicated cases, kriging out-performs other methods. Marcelet and Peter [15] compared four different metamodels for their

performance. It was found that kriging and ANN were two of the most accurate methods.

The aim of the current work is to quantify the improvements that a BERP-like tip can have on a typical high speed forward flying rotor. Such rotors tend to have swept tips and thin sections outboards on the rotor. The base rotor selected here is made of two sections, the HH-02 and the NACA 64A-006 at the tip and has a sweep of 20 degrees initiated at r/R =0.92. It has an AR of approximately 13.7 and linear twist of -9 degrees. The optimisation was carried out primarily for forward flight, although hover conditions were also analysed and constrained. The optimisation method used is based on a Genetic Algorithm (GA) coupled with a Neural Network metamodel (ANN). This evolutionary type optimisation technique ensures a higher probability of obtaining the global optimum compared to its gradient-based counterpart [14]. The efficiency lost in using this method is regained by use of the metamodel. The Neural Network method was selected due to its accuracy, robustness and efficiency. More details of the ANN used and the effect of the number of layers, neurons and outputs can be found in references [?,?,21]. Kriging was also coded and used to compare and produced similar results. All the codes used, were built in house and more details about them can be found in reference [?, ?] that cover rotor aerofoil optimisation and the UH60-A blade optimisation.

2 NUMERICAL METHODS

The time-marching method, even when parallel computing is used, can take days of clock time for a rotor to be fully analysed. Another technique that can be used to obtain the performance of the rotors to the same accuracy (provided a sufficient number of modes is used), is the Harmonic Balance Method (HB) [25]. With HB, the time taken to perform the same calculation can be about an order of magnitude less than the time taken using time-marching. This greatly improves the efficiency of the optimisation process, making it a more usable technique in the rotor design. The method is demonstrated in this paper, and the results obtained use four modes resulting in flow snapshots at every 10 degrees of azimuth for a 4-bladed rotor. The method has been shown to give results of similar accuracy to time marching methods in Woodgate and Barakos [25]. A brief summary of the method is given in this paragraph.

HB represents the governing equations in the frequency domain. Therefore, if Eqn 1 represents the governing equations where Q(t) is the solution and R(t) is the residual; these are assumed to be periodic.

$$F(t) = \frac{dQ(t)}{dt} + R(t) = 0 \tag{1}$$

Then, expressing the solution as a Fourier series with a fixed number of modes, N_H :

$$Q(t) = \hat{Q}_o + \sum_{n=1}^{N_H} \left(\hat{Q}_c \cos\left(\omega nt\right) + \hat{Q}_s \sin\left(\omega nt\right) \right), (2)$$

$$R(t) = \hat{R}_o + \sum_{n=1}^{N_H} \left(\hat{R}_c \cos\left(\omega nt\right) + \hat{R}_s \sin\left(\omega nt\right) \right), (3)$$

$$F(t) = \hat{F}_o + \sum_{n=1}^{N_H} \left(\hat{F}_c \cos\left(\omega nt\right) + \hat{F}_s \sin\left(\omega nt\right) \right)$$
(4)

A Fourier transform of Eqn.4 gives

$$\hat{F}_o = \frac{\omega}{2\pi} \int_o^{2\pi/\omega} F(t)dt = \hat{R}_o$$
⁽⁵⁾

$$\hat{F}_c = \frac{\omega}{\pi} \int_o^{2\pi/\omega} F(t) \cos(wnt) dt = \omega n \hat{Q}_s \hat{R}_c \quad (6)$$

$$\hat{F}_s = \frac{\omega}{\pi} \int_o^{2\pi/\omega} F(t) \sin(wnt) dt = -\omega n \hat{Q}_c \hat{R}_s \quad (7)$$

This gives a system of equation N_T equations, where $N_T = 2N_H + 1$, for the Fourier series coefficients:

$$\hat{R}_o = 0 \tag{8}$$

$$\omega n \hat{Q}_s \hat{R}_c = 0 \tag{9}$$

$$-\omega n \hat{Q}_c \hat{R}_s = 0 \tag{10}$$

This is expressed in matrix form as:

$$\omega M \hat{Q} + \hat{R} = 0 \tag{11}$$

where M is an $N_T \times N_T$ matrix.

Eqn 11 can then be solved by applying pseudo time stepping with implicit and explicit integration and the appropriate turbulence models etc. [25]. The time marching method of HMB can be found in references Steijl et al. [2,22].

3 PARAMETERISATION TECHNIQUE

This parameterisation technique allows for the following design features to vary: (i) the sweep angle, (ii) the gradient of the BERP notch, (iii) the spanwise position of the notch. To do this, both the leading and trailing edges of the BERP tip are modified. Referring to Figure 2, the leading edge is defined by three equations and the trailing edge by two.

For the leading edge, the first part is defined by a sigmoid curve that represents the notch region. The sigmoid equation is given as:

$$y = \frac{\Delta y}{1 + e^{-g(x - x_o + \Delta x/2)}} \tag{12}$$

where Δy is the notch height i.e. the notch length in the chord-wise direction, Δx is the total width of the notch i.e. the notch length in the span-wise direction, g is the gradient of the notch and x_o is where the notch starts from. The x coordinate of the notch maximum is defined by the user and is kept constant except when the notch position needs to be varied. The g value is varied to change the gradient.

The second part is used to define the sweep. It represents the part of the leading edge after the notch as a parabola:

$$y = -a(x - x_1)^2 + \Delta y + y_{add}$$
 (13)

where a is the gradient of the parabola used to alter the sweep, x_1 corresponds to the notch end and the beginning of sweep, Δy is the notch height and y_{add} is an additional y offset value to ensure that the y ordinate of the parabola starts at the same position as the notch height. The value of y_{add} is computed automatically, once x_1 and Δy are known.

The third part describes the delta tip which joins to the trailing edge. It is represented as a polynomial of order 2.5:

$$y = -b(x+c)^{2.5} - \Delta y'$$
(14)

where b is the gradient of the delta tip, c is the centre where the gradient of the curve becomes 0 (used to match the gradient of the curve to the previous parabola) and $\Delta y'$ is the additional y displacement required to match the curve to the previous parabolic curve. See Figure 2.

The two parameters, g and a can be changed independently and the rest of the parameters appearing in the equations are automatically adjusted so that the curves match at the point and the gradient level. These are the values of Δx , $\Delta y'$, c. The initial x co-ordinate, x_o is modified with the gradient of the parabola so that the tip point occurs at the same place for a required sweep. This is why for different notch positions, different sweep parameters are used to obtain the same sweep distribution. The gradient b is dependent on the trailing edge curve as well. Therefore the trailing edge must be determined first. The gradient of the trailing edge curve can also be modified independently of the sweep gradient of the leading edge. This allows the tip point of the blade to move in the y-direction which inherently modifies the chord distribution as well.

The trailing edge is defined first by a linear curve that has the same gradient as the leading edge sweep parabola or a scaled value of it, if required, and then by a polynomial of order 3.5 that is matched to the point and gradient of the sweep curve that comes before. The trailing edge curve must be specified before hand, as the tip point is required to find the gradient of the delta polynomial so that the leading and trailing edge curves meet at a single point. So the first curve for the trailing edge is given as

$$y = -a(x - x_1) + \Delta y + y_{add_{TE}} \tag{15}$$

And the latter part of the trailing edge is given by

$$y = -b_{TE}(x + c_{TE})^{3.5} - \Delta y'_{TE}$$
(16)

where all the values and constants correspond to the trailing edge parameters except for the sweep parameter, a which is exactly the same as the leading edge sweep. The gradient of the trailing edge can be increased or decreased relative to the leading edge sweep gradient by scaling it with a factor. Figure 3 show the examples used to build the design space for the optimisation.

4 GRID AND GEOMETRY GENERATION

Figure 3 shows how a swept rotor tip can be transformed into a BERP-like tip and the effect of varying the parameters. The base rotor is made up of two sections - the HH-02 inboard (up to r/R = 0.92) and the NACA 64A-006 at the tip (r/R = 1). The aerofoil is linearly blended towards this latter section. This rotor has a rectangular tip swept back by 20 degrees, shown in Figure 4. When the BERP planform is applied to this tip, the non-linear variation of the chord means that the thickness changes non-linearly as well. To maintain the thickness so that it decreases linearly, from 9.6% (thickness of the HH-02) to 6% (thickness of the NACA 64A-006), sections are cut from the base rotor such that when they are scaled to the chord length required, they have the thickness value that satisfies the linear variation. The problem with this method is that the point of maximum thickness for each of these sections varies non-linearly and therefore the blade surface appears to be bumpy. To overcome this, the notch section is blended from the HH-02 to a NACA-64A section of the appropriate thickness and then the rest of the tip is built with NACA-64A sections of the required thickness so that at the tip, the thickness is 6%.

Another issue that arises is that the HH-02 aerofoil has a tab whilst the NACA 64A-006 does not. So a tab is introduced for the NACA64A and is kept constant till the tip, where the tip is rounded off. The tab is introduced by cutting the aerofoil curves at about 20% from the trailing edge and then rotating the latter part of the curves in the longitudinal axis of the blade so that it adds the required thickness for the tab. The curve is then blended by point, tangent and radius to the main part of the curve to remove any kinks.

Also, the twist is removed from where the BERP tip begins, to avoid having a dihedral trailing edge as shown in Figure 5. The trailing edge point is kept at a constant z-value and each section is twisted so that its chord line (not necessarily its quarter chord point) intersects (or its extension intersects) the reference pitch axis. This prevents dihedral from occurring.

A number of ICEM replay files are used to automate these steps, but some manual intervention is required in the creation of these grids.

4.1 Tip Anhedral

The tip anhedral is implemented as follows. Let's assume that 10 degrees of anhedral is to be implemented starting from the station r/R = 0.918 to the tip. Then, for each station in between these two stations (r/R = 0.918 and r/R = 1.0), there will be a Δz distance by which that section should be translated downwards (Figure 6) which is found as:

$$\Delta z = \Delta x tan\theta \tag{17}$$

Once each station has been translated, then the stations are joined by curves and surfaces are created.

5 FLIGHT CONDITIONS

The hover flight conditions for the blade were based on a rotor tip Mach number of 0.65 which was assumed from a tip speed of approximately 220m/s at ISA sea level conditions. The chord of the blade was 1.75 ft, therefore the Reynolds number was approximately 8 million. The weight of the air-craft was approximated by assuming a pressure altitude of 0 ft, a free-air temperature of 20° C, a wheel-height of 80 ft to ensure out-of-grounds operations and a torque factor of 1 with 100% rpm. The weight capability at these conditions is approximately 20,000 lb. This results in a C_T of 0.018 based on a rotor radius of 24 ft.

$$C_T = \frac{T}{1/2\rho A(\Omega R^2)} = \frac{9000 \times 9.81}{0.5 \times 1.225 \times \pi (7.3^2) \times 220^2} = 0.018$$
(18)

For forward flight, the conditions are for high speed at reasonable thrust level. Therefore, the advance ratio, $\mu = 0.34$, C_T = 0.0122. This was obtained from typical maximum speed, minimum thrust for that speed based on empty weight + 20% or about 6.2 tonnes.

6 HOVER RESULTS

The objective for this case, was to obtain a high FM over a wide thrust setting. Therefore the FM over a range of thrusts was obtained for the original rotor and a BERP variant. Figure 7 shows the comparison of these blades for increasing C_T/σ . The BERP rotor with the same twist and anhedral has a better FM at low thrust, but at higher thrust values, its performance drops to below that of the original rotor. With a higher twist, some of the performance of the original rotor is redeemed and with an anhedral of 20 degrees implemented, an improved rotor in hover is obtained.

The reason for this performance trend is that for the BERP rotor, the loading of the blade increases steeply where the BERP section starts as can be seen in Figure 8. Also the loading inboards is lower than the original rotor. With increasing twist, the inboard loading is increased which improves the FM. The anhedral, reduces the outboard loading and thus the performance of the BERP blade matches the original rotor blade.

7 FORWARD FLIGHT RESULTS

The original BERP rotor used aerofoils specifically designed for its overall performance [3]. However, the optimisation here only changes the planform while using generic aerofoil sections. To compare the effect of the aerofoil on the planform, steady flow 2D results were obtained at a number of sections and azimuths and compared to the flow of the section on the blade at the same conditions. This was done by obtaining the section at the rotor and running as a steady 2D aerofoil to obtain the same lift coefficient at the same Mach and Reynolds conditions experienced on the rotor. In this way the downwash angle can be estimated as well even with some caution since 3D effects are not included.

Figure 9 shows the comparison of the aerofoil with the rotor section at r/R = 0.75 at 4 azimuth angles. It can be seen that the 3-dimensional effects on the rotor section play an important role in reducing the geometric angle via the downwash increment. The aerofoils also have large pitching moments especially more outboards on the retreating side as shown in

Figure 10 and 11. This shows the importance of selecting good aerofoils for a BERP-like rotor and hence why much effort was put in to selecting the RAE aerofoils for the actual BERP rotor. A dM/dt optimisation similar to the one discussed in [?] prior to the planform optimisation would have likely produced a better rotor. However, regardless of the rotor sections, these aerofoils have been used in rotorcraft and this case only serves to highlight the planform optimisation. The high suction peaks of Figures 10 and 11 at 190 and 280 degrees of azimuth show the limitation of the employed aerofoils since they are pushed to high lift and high suction peaks.

7.1 Tip Sweep Effects

Table 2 shows the effect of sweep on the performance parameters of the BERP rotor. There is considerable loss in thrust with increased sweep. Therefore, for all cases shown, the rotor was trimmed to give the same thrust of approximately $C_T/\sigma = 0.09$. These results comparing the effect of sweep are shown in Figures 12 - 14 for the results with the highest notch gradient and the most inboard and outboard notch positions. With more sweep, it can be seen that the distribution of the lifting load is reduced at the back and outboards on the advancing side, and is increased at the front of the disk and more inboards on the advancing side. The load is also distributed more inboards in the spanwise direction at the notch for the blade with a more inboard notch.

The pitching moment is mostly negative on the advancing side and mostly positive on the retreating side. With increased sweep, the magnitude of the moment increases (Figure 15) since the moment is calculated about the pitch axis. Also as with M^2C_n , a more inboard notch spreads the moment peaks out.

The torque distribution shows a drop in C_Q where the notch of blade is. C_Q is highest at the back of the disk and lowest at the tip in the advancing side. These extremes increase in magnitude with more sweep. Overall, on the advancing side, the torque reduces with increased sweep and on the retreating side reaches maximum value. Figure 15 shows a 3D view of the load distributions comparing low and highly swept blade tips for the most inboard and outboard notch BERP-like tips. The torque distribution shows that the effect of sweep is increased when the notch is more outboard. The advantage of high sweep on the advancing side and low sweep on the retreating side is also shown. The moment distribution has similarities to the torque distribution and its magnitude is much higher at the front and back for the more swept tips. The lifting load distributions differ much less than the other performance parameters.

Figure 16 compares the blade loads at 4 azimuth positions and shows the effect of sweep at each location. As can be seen, high sweep offloads the tip at the back of the disk and increases it at the front. On the advancing side, lift is maintained till just after the notch, where the highly swept blade loses lift quickly. The torque is low for higher sweep for most of the blade. At the tip, after the notch region, however, it increases rapidly. The pitching moment follows a similar pattern although the sweep does not have much of an effect inboards.

7.2 Effect of Notch Offset

Figure 12 - 14 also compare the loads when the BERP area of the blade is increased i.e. when the notch is more inboards. Again, these are trimmed results although not much variation with notch position occurs in thrust with or without trimming as shown in Table 3. The effect of this parameter is to amplify the effect of the sweep parameter. For example, the redistribution of lift so that it is reduced at the back and increased at the front caused by sweep is larger in magnitude when the notch starts more inboard. The same can be seen for moment in Figure 13 where the region on the edge of the disk where moment is higher is thinner for the more outboard notch. For torque, the general trend is an increase with radial position. Where the notch occurs there is a drop in torque and then a continued increase followed by another drop where the anhedral occurs. With a more outboard notch, the torque continues to rise prior to reaching the notch for longer, therefore the latter part of the curve is higher. This can be seen in Figure 14 where the value of the reduced region at the notch is not as low when the notch is more outboard. Also, the torque further out from the notch is higher for the rotor with the more outboard notch. More inboards, on the advancing side, a decrease in torque is observed over a larger region and this brings the total value of the torque down as shown in Table 3. Figure 17 shows that the total torque on the blade is most affected by notch position on the retreating side.

The table 3 also shows that the peak-to-peak moment decreases (also seen in Figure 16) but the absolute average moment over a full revolution increases, the further outboard the notch is.

7.3 Effect of Notch Gradient

Figure 18 compares the performance of different notch gradients. The notch gradient has a smaller influence on the design than the other parameters tested. The difference in the lift and moment distribution do not change much. With a higher notch gradient, there is a slight increase in the pitching moment, evident from Table 4 where the average moment is slightly higher and the peak-to-peak value is lower, suggesting that a higher notch gradient provides better performance. The torque is not affected much at low sweep, but at higher sweep, notch gradient has slightly more influence on the torque as shown in Table 4 and Figure 18.

Figures 19 and 20 show the integrated loads of pitching moment and torque over the blade during one revolution for varying design parameters. The peak-to-peak moment value reduces with further outboard notch positions and the average moment tends to be more centred around zero when sweep is higher. The torque seems to be mostly affected by sweep and on the advancing and retreating side. The torque is reduced more on the advancing side than the increase on the retreating side. This is because it alleviates the compressibility effects. On the retreating side, the differences are more subtle. This data suggests that a highly swept blade would be optimal. Having a higher notch gradient would also improve the moments and having a notch more inboards would amplify the effects of the sweep. The quantities of the design parameters that make up the optimum design are obtained in the next section.

8 PLANFORM OPTIMISATION

The original population obtained from the CFD results contained 27 points. The design parameters selected were the average pitching moment (C_{mavg}), peak-to-peak pitching moment (ΔC_m) and the torque coefficient (C_q) . The objective function weights were determined such that on average, each of these parameters had the same influence. This was determined using the data from the original population which was obtained using the high-fidelity CFD solver. First each design was scaled with the baseline design case. The baseline case chosen was the BERP most similar to the typical fast flying rotor as shown in Figure 21. The parameters for it are NE = 12, NG = 25, SW = 0.25. The average ratio of $C_{m_{avg}}$ to ΔC_m was found to be 2.7893:1 and the average ratio of ΔC_m to C_q was found to be 0.9548:1. Therefore the ratio of C_{mavg} to ΔC_m to C_q is obtained as: 2.6634 : 0.9548 : 1.0000. The weight for C_q was then calculated as:

$$w_{C_q} = \frac{2.6634}{2.6634 + 0.9548 + 1} = 0.5767 \tag{19}$$

Hence the weight of C_{mavg} and ΔC_m are given as:

$$w_{C_{maya}} = 0.5767/2.6634 = 0.2165 \tag{20}$$

$$w_{\Delta C_m} = 0.5767/0.9548 = 0.6040 \tag{21}$$

 C_q was also used as a constraint. Since the rotors were trimmed to a $C_T/\sigma = 0.09$, C_T/σ did not need to be constrained.

From this data, it was determined that the most influential design parameter was the sweep, followed by the notch position and then the gradient. ANNs were trained for each of the performance parameters as shown in Figure 22. These parameters were used to find the optimum blade using a GA which was compare with the Pareto front shown in Figures 23 to 25.

The comparison of the optimum with the original baseline blade is shown in Table 5 for the trimmed rotors. A much better avg M²C_m was obtained and also a slightly better $\Delta M^2 C_m.$ The performance of the resulting optimum relative to the baseline design is shown in Figures 26, 27 and 28. The black line indicates the contour line of the baseline design and the red is the optimum blade design. On the moment plot, it can be seen that the optimised blade has larger areas of lower moment especially on the retreating side but also on the outboard region of the advancing side. A similar trend can be seen for the torque plot. Figure 28 shows the difference in the objective function components between the optimum and the baseline design. For the regions of higher OFV, the areas enclosed by red are larger showing that the optimised blade increases the area where performance is good and vice versa for areas of poorer performance.

The hover performance of the optimum blade was measured and compared to the baseline in Table 5. The results were obtained at a collective of 13 degrees. The C_T/σ is slightly less than the baseline design mostly due to the added solidity, but the FM obtained was higher. Figure 29 compares the C_P distribution for the BERP reference and optimised blade. It can be seen that the optimised one spreads the loading at the tip over more of the span. Therefore, overall, the optimised blade has better performance than the baseline blade especially in terms of moment where the average pitching moment was reduced to approximately a fifth of the baseline designs.

9 SUMMARY AND CONCLUSIONS

This paper describes the optimisation of a BERP-like rotor planform. The optimisation technique is based on a genetic algorithm that uses predictions from an Artificial Neural Network that has been trained using high-fidelity CFD data from the HMB solver. The optimisation was carried out for forward flight while constraining hover performance by modifying twist and anhedral in hover. A parameterisation method was defined to modify the sweep, the notch gradient and the notch position of the BERP-like tip. A full factorial method was used to obtain the sample space, which contained 27 design points. The objective was to improve the compressibility effects on the advancing side of the rotor and the stall performance in the retreating side. This objective was captured using the average pitching moment of the peak-to-peak pitching moment of the rotor over a full revolution. In addition torque was optimised for and constrained. All the results were trimmed to the same thrust. The outcome was a substantially improved pitching moment performance with slightly reduced torque for the same thrust as the baseline design. This was obtained using high sweep and notch gradients with the notch position at approximately r/R = 0.86. Hover performance was not compromised.

The time required to obtain all the data was reduced by using the Harmonic Balance method in parallel. With the Harmonic Balance method, the clock time for obtaining the CFD data is reduced by an order of magnitude. Using the HB, each calculation took approximately 3/4 of a day when started from a previous solution, whereas for the same case with TM, for two revolutions so that the data is periodic, would take approximately a week of clock time. The azimuth resolution for the HB was every 10 degrees. This makes the method more practical to rotor optimisation on forward flight.

The ANN predictions were quite accurate. The error convergence was set to 0.01 and the maximum error in the overall combined objective function between the CFD data and the ANN predictions was 2.7% which was for a design that had a 16% change in design parameters. This shows that the ANN was a reliable metamodel. The overall optimisation was limited to the aerodynamic performance of various planform designs and did not include rotor section optimisation.

Future work on this method includes optimising the rotor sections as well as the planform, possibly including the fuselage in the optimisation to reduce the effective downwash on the fuselage aerodynamics.

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r/R = 0.750			
Azimuth (deg)	rotor geometric angle	2D incidence for	"downwash" (deg), A - B
	(deg), A	same C _n (deg), B	
10	13.8571	5.40	8.46
100	15.6783	3.07	15.61
190	12.1430	7.00	5.14
280	17.0265	8.80	8.23
r/R = 0.862			
Azimuth (deg)	rotor (deg)	aerofoil (deg)	downwash (deg)
10	22.6805	4.80	17.88
100	14.6000	1.03	13.57
190	10.3370	4.30	6.04
280	15.8830	6.80	9.08
r/R = 0.918			
Azimuth (deg)	rotor (deg)	aerofoil (deg)	downwash (deg)
10	10.108	2.30	7.81
100	9.1738	0.55	8.62
190	8.3903	3.80	4.59
280	10.916	6.15	4.77

Table 1: Table showing the effective downwash angle at 3 stations on the BERP-like blade; before the notch, in the middle of the notch and after the notch. Figures 9 to 11 correspond to this data. Rotor stands for the rotor section pitch angle (A), aerofoil for the equivalent C_n aerofoil angle (B) and "downwash" is the difference between A and B.

NE	NG	SWEEP	C_T/σ	C _Q	avg M ² C _M	$\Delta M^2 C_M$
After ti	After trimming					
11.75	28	0.09	0.0905	0.000192	-0.00253	0.01030
11.75	28	0.13	0.0900	0.000191	-0.00164	0.01029
11.75	28	0.21	0.0899	0.000186	-0.00020	0.01115

Table 2: Sweep effects on performance comparison. NE is the notch position parameter and NG is the notch gradient parameter. avg $M^2 C_M$ is over one revolution and $\Delta M^2 C_M$ is the peak-to-peak amplitude over one revolution.

NE	NG	SWEEP	C _Q	avg M ² C _M	$\Delta M^2 C_M$	
After Trimming						
11.50	28	0.185	0.000186	0.00030	0.01156	
11.75	28	0.21	0.000186	-0.00020	0.01115	
12.00	28	0.25	0.000184	-0.00047	0.01062	

Table 3: Example of BERP spanwise notch position performance comparison. NE is the notch position parameter and NG is the notch gradient parameter. avg M^2C_M is over one revolution and ΔM^2C_M is the peak-to-peak amplitude over one revolution. The sweep values differ because the gradient of the parabola differs when the position of notch changes, but in actual fact, the sweep is the maximum sweep on all three rotors and it is the same amount of sweep.

NE	NG	SWEEP	C_T/σ	C _Q	avg M ² C _M	$\Delta M^2 C_M$
12.00	25	0.10	0.09088	0.000189	-0.00256	0.00961
12.00	28	0.10	0.09066	0.000189	-0.00246	0.00957
12.00	35	0.10	0.09108	0.000189	-0.00236	0.00949
11.75	25	0.21	0.08976	0.000188	-0.00030	0.01096
11.75	28	0.21	0.08981	0.000186	-0.00020	0.01115
11.75	35	0.21	0.08981	0.000187	-0.00011	0.01102

Table 4: Example of BERP spanwise notch gradient performance comparison. NE is the notch position parameter and NG is the notch gradient parameter. avg $M^2 C_M$ is over one revolution and $\Delta M^2 C_M$ is the peak-to-peak amplitude over one revolution.

NE	NG	SWEEP	C_T/σ	CQ	avg M ² C _M	$\Delta M^2 C_m$
11.75	35	0.21	0.08318	0.000171	-0.00037	0.01078
After trim		0.08981	0.000187	-0.00011	0.01102	
Baselin	ie		0.09045	0.000186	-0.00052	0.01083

Hover Performance Comparison						
NE	NG	SWEEP	C_T/σ	FM	Collective (deg)	
11.75	35	0.21	0.2896	0.6873	13	
Baseline		0.3039	0.6543	13		

Table 5: BERP and baseline case (NE = 12, NG = 25, SW = 0.25) performance comparison related to Figure 26. NE is the notch position parameter and NG is the notch gradient parameter. avg M^2C_M is over one revolution and ΔM^2C_M is the peak-to-peak amplitude over one revolution



Figure 1: The HH-02 and the NACA 64A-006 aerofoils used for the baseline blade design.



Figure 2: (a) Notch gradient, (b) sweep and (c) delta parameter equation definitions for the BERP planform.



Figure 3: Visualisation of the three parameter changes to the geometry surfaces in ICEMCFD.



Figure 5: Effect of twisting about the quarter chord point (top) or about the quarter chord line (bottom).



Figure 6: Generation of anhedral for the blade tip.



Figure 7: Figure of Merit vs. thrust coefficient of the original blade and the BERP variants with varying twist and anhedral.



Figure 8: Lift distribution along the span with varying twist at 13 degrees of collective.



Figure 9: Aerofoil comparison to section on BERP-like rotor at the same conditions before the notch, green is the 2D aerofoil and red is the rotor section, r/R = 0.75.



Figure 10: Aerofoil comparison to section on BERP-like rotor at the same conditions in the middle of the notch, green is the 2D aerofoil and red is the rotor section, r/R = 0.862.



Figure 11: Aerofoil comparison to section on BERP-like rotor at the same conditions after the notch, green is the 2D aerofoil and red is the rotor section, r/R = 0.918.



Figure 12: M^2C_n for the BERP-like rotors with fixed parameters: NE = 11.5 and NE = 12, NG = 35 and variable sweep parameters. The black line indicates the $M^2C_n = 0$ line.



Figure 13: M^2C_m for the BERP-like rotors with fixed parameters: NE = 11.5 and NE = 12, NG = 35 and variable sweep parameters. The black line indicates the $M^2C_m = 0$ line.



Figure 14: M^2C_q for the BERP-like rotors with fixed parameters: NE = 11.5 and NE = 12, NG = 35 and variable sweep parameters. The black line indicates the $M^2C_q = 0$ line and the white line indicates the approximate middle value, $M^2C_q = 0.2$.

NE = 11.5









 M^2C_m



Figure 15: M^2C_n , M^2C_m and M^2C_q for the BERP-like rotors with fixed parameters: NG = 28 and variable sweep parameters for different notch positions. Red is lowest sweep, blue is highest sweep. The arrow shows the free stream direction.







Figure 16: Comparisons at azimuth 0, 90, 180 and 270 degrees of the M^2C_n , M^2C_m and M^2C_q for BERP-like rotor with fixed parameters: NE = 11.5 and 12, NG = 28 and variable sweep parameters.



Figure 17: Comparisons of the integrated loads, M^2C_m and M^2C_q for BERP-like rotor with fixed parameters: NE = 11.5 and 12, NG = 35 and variable sweep parameters.



Figure 18: M^2C_n , M^2C_m and M^2C_q for the BERP-like rotors with fixed parameters: NE = 11.75 and SW = 0.21 with varying NG. The black line indicates a contour level = 0 line and the white line indicates the approximate middle value for $M^2C_q = 0.2$.



Figure 19: M^2C_m integrated over the full blade at each azimuth for the BERP-like rotors.







Figure 21: The baseline BERP-like rotor in comparison to a swept tip design. The parameters for this rotor are NE = 12, NG = 25, SW = 0.25.



Figure 22: ANN predictions with training data and GA selection shown for each of the performance parameters. The white dots are the GA optimal selection and the black dots are the CFD training data for the ANNs. The dashed line is where the contour level = 1 i.e. the value for the baseline design.



Figure 23: Pareto front points compared with GA selection; red is NE = 11.5, green is NE = 11.75, blue is NE = 12. The white dots are the GA optimal selection and the cyan dots are the Pareto front solutions.



Figure 24: Pareto front for the BERP-like design.



Figure 25: (a) Pareto front points compared with GA selection; red is NE = 11.5, green is NE = 11.75, blue is NE = 12, (b) OFV contour colour map in the design space. The white dots are the GA optimal selection, the cyan dots are the pareto selection and the black dots are the CFD training data for the ANNs.



Figure 26: Optimum (red contour lines) compared to reference (black contour lines) for M^2C_m and M^2C_q .



Figure 27: Optimum (red contour lines) compared to reference (black contour lines) for M^2C_m and M^2C_q .



Figure 28: OFV for the baseline and BERP-like rotor where the black contour lines represent the reference rotor with parameters: NE = 12, NG = 25, SW = 0.25, and the red lines represent the optimised rotor with parameters: NE = 11.75, NG = 35 and SW = 0.21 where $OFV = -0.2 \times M^2 C_m - 0.6 \times M^2 C_q$.



Planform comparison

Figure 29: C_p and planform distribution of the reference (blue) and optimised (red) BERP variant at high thrust (collective = 13 degrees).