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GYROSCOPIC EFFECTS ON WINDMILL BLADES

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ABSTRACT

The stresses due to the gyroscopic effect produced by the orientation motion along the wind direction of a horizontal-axis turbine are evaluated and analyzed. Particularly, the effects on the material and on the blade structure of a wind machine are taken into consideration. Applying the theorem of angular momentum it has been possible, through a numerical application running on a computer, to evaluate the applied moment and to analyze the state of stress versus the variation of some characteristic parameters of the system, like: tip speed ratio, available power, number of blades, angular speed of the rotor, quickness and orientation amplitude of the turbine with respect to the wind direction, etc. The cyclic variation of these stresses, with a frequency equal to the angular speed of the rotor, is studied and shown through diagrams worked out by computer. In conclusion it is shown as this variable load is a critical stress for the fatigue strength of the blade material, and may represent a vibration source for the whole windmill structure.

1 - INTRODUCTION

A gyroscope is an arbitrary rigid body free to rotate about a fixed point. The motion may be conceived as the result of three mutually orthogonal rotations: precession, nodding motion and spin. A more or less sudden variation of this vector makes, by the theorem of angular momentum, a moment equal to the first derivative of angular momentum with respect to time plus the vector product between the rotations resultant and the angular momentum.

In the same way, the windmill rotor is subjected to two rotational motion (figure 1): the first w about the rotor axis, and the second Ω about a vertical axis (exerted by a tail vane or by a servo-motor device, in order to obtain a correct orientation with the wind direction). Thus, the resultant of these motions is represented by a vector applied to the point O and free to rotate on the vertical plane containing the two rotation axes. For this reason, a horizontal-axis wind machine may be considered a gyroscope in every respect and subjected to the loads that come out of such behaviour.

Therefore, a variation of the wind direction, for example owing to a sudden gust, produces a rotation around the orientation axis and a subsequent change of the resultant of the two vectors w and Ω. This produces a stress which is extended to the whole blade, also with high loads applied to the material.

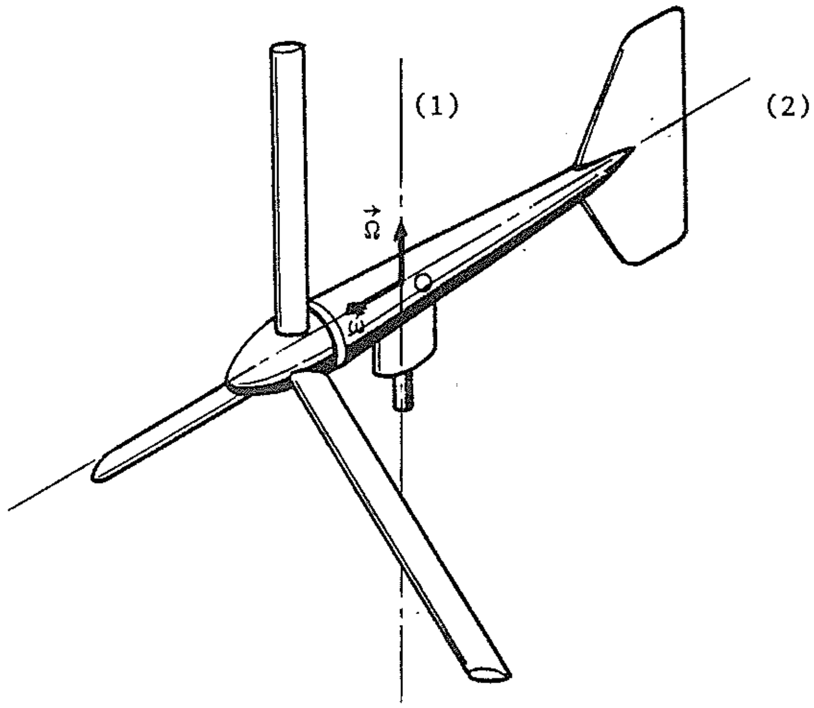


Fig.1 - Orientation axis (1) and rotor axis (2) of a wind machine.

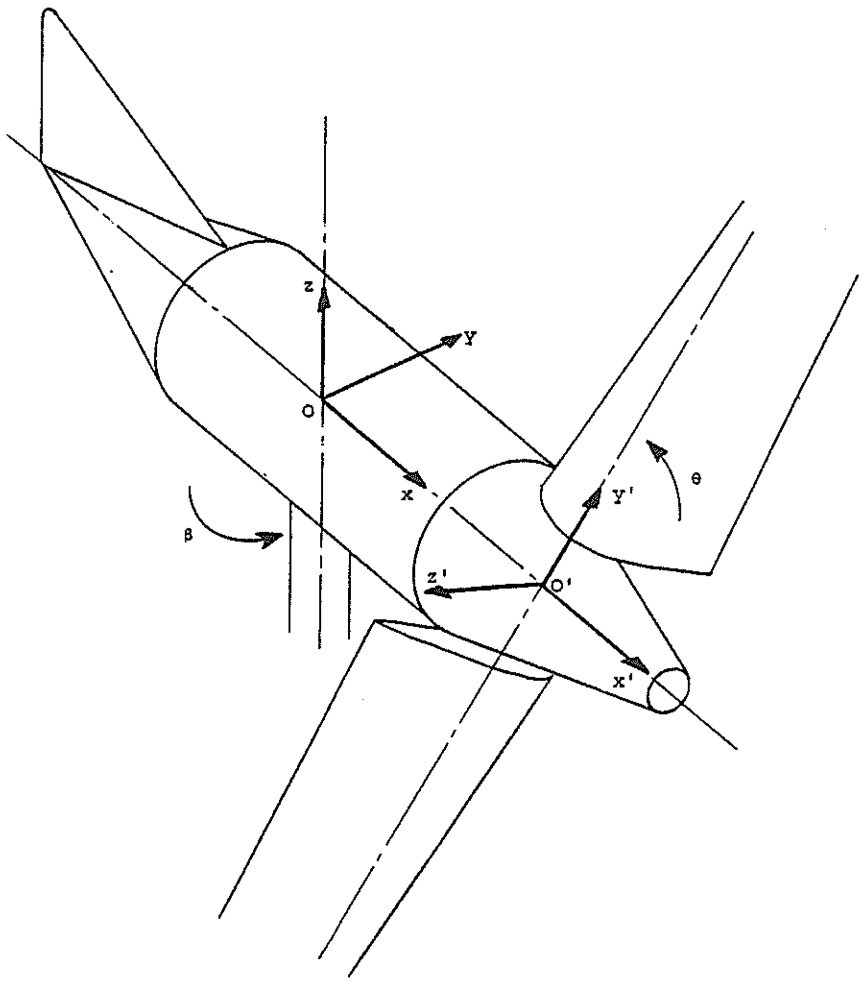


Fig.2 - Coordinate systems used for the determination of the moment due to the gyroscopic effect.

For this reason it is here suggested a method to evaluate, with sufficient approximation, the stresses due to gyroscopic effect, analyzing their variation as a function of the characteristic parameters of the turbine and of the external environment.

2-STRESSES EVALUATION

The proposed computing procedure needs two coordinate systems: the former jointed to the windmill frame turns around a vertical axis according the wind direction, the latter jointed to the rotor turns around a horizontal axis. These systems are shown in figure 2: the first one, named Oxyz, is oriented so that the axis x is placed on the rotor shaft and the axis z coincides with the orientation axis. On the other hand, the second coordinate system, represented by the trihedron O'x'y'z', is oriented so that the axes x' and y' are respectively placed on the rotor shaft and on the axis of the analyzed blade.

Let Θ be the rotation angle of the blade and ß the orientation angle of the whole turbine, the respective angular speeds are given by the following relations:

$$\vec{\omega} = \frac{d\theta}{dt}\,\vec{i}' \qquad \vec{\Omega} = \frac{d\beta}{dt}\,\vec{k}$$

If w and Ω are both different from zero, owing to the gyroscopic effect, a moment M is applied to the point O' and its expression is given by the application of the angular momentum theorem:

$$\vec{M} = \frac{d}{dt}\left[\,\vec{T}\cdot(\vec{\omega}+\vec{\Omega})\,\right] + (\vec{\omega}+\vec{\Omega}) \wedge \left[\,\vec{T}\cdot(\vec{\omega}+\vec{\Omega})\,\right] \tag{1}$$

where: T is the matrix of inertia of the blade with respect to the coordinate system O'x'y'z'; w+Ω is the global angular speed of the blade conceived as the result of the turbine rotation and orientation motion; and T·(w+Ω) is the angular momentum of the blade with respect to the system O'x'y'z'.

The solution of the expression (1) is achieved through the development of the above mentioned factors.

The matrix of inertia of a system is usually given by the expression:

$$\vec{T} = \begin{vmatrix} I_{x'} & I_{x'y'} & I_{x'z'} \\ I_{y'x'} & I_{y'} & I_{y'z'} \\ I_{z'x'} & I_{z'y'} & I_{z'} \end{vmatrix}$$

where: $I_{x'}$ is the moment of inertia of a single blade with respect to the axis x'; $I_{x'y'}$ is the product of inertia of a blade with respect to the axis x' and y', etc.

Due to the matrix of inertia symmetry, it results:

$$I_{x'y'} = I_{y'x'} \quad I_{x'z'} = I_{z'x'} \quad I_{y'z'} = I_{z'y'}$$

Moreover, the particular position of the system O'x'y'z' involves the coincidence between the axis y' and the principal axis of inertia of the blade, making possible to write:

$$I_{x'y'} = I_{x'z'} = I_{y'z'} = 0$$

As consequence, the matrix of inertia may be expressed by:

$$\vec{T} = \begin{vmatrix} I_{x'} & 0 & 0 \\ 0 & I_{y'} & 0 \\ 0 & 0 & I_{z'} \end{vmatrix}$$

The components of the vector w+Ω must be calculated with respect to the coordinate system placed on the blade. As the figure 3 is drawing, it results:

$$\vec{\omega} + \vec{\Omega} = \begin{vmatrix} \omega \\ \Omega \sin\theta \\ \Omega \cos\theta \end{vmatrix} \qquad (2)$$

Therefore, the angular momentum of the blade with respect to O'x'y'z' is given by:

$$\vec{T}\cdot(\vec{\omega} + \vec{\Omega}) = \begin{vmatrix} I_{x'}\omega \\ I_{y'}\Omega \sin\theta \\ I_{z'}\Omega \cos\theta \end{vmatrix} \qquad (3)$$

Replacing the expressions (2) and (3) in the equation (1) we obtain, following mathematical passages, the three components of the moment M, evaluated according to the trihedron stationary with the blade:

$$\vec{M} = \begin{vmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{vmatrix} = \begin{vmatrix} I_{x'}\frac{d\omega}{dt} + (I_{z'} - I_{y'})\Omega^2 \sin\theta\cos\theta \\ I_{y'}\frac{d\Omega}{dt} \sin\theta + (I_{x'} + I_{y'} - I_{z'})\omega\Omega\cos\theta \\ I_{z'}\frac{d\Omega}{dt} \cos\theta + (I_{y'} - I_{x'} - I_{z'})\omega\Omega\sin\theta \end{vmatrix} \qquad (4)$$

This expression is valid for every blade section at a distance r from the rotor shaft. Nevertheless, it is obvious that, varying this parameter, the quantities $I_{x'}$, $I_{y'}$ and $I_{z'}$ consequently change, and they have to be evaluated taking into account only the mass of the blade portion included between the examined section and the rotor tip (figure 4).

In the same figure 4, we show the three components of the moment applied to the section crossing the point O'.

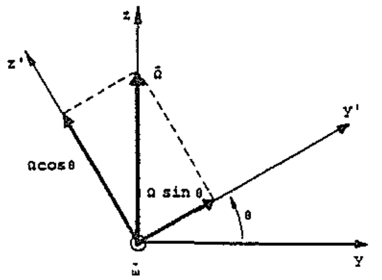


Fig.3 - Decomposition of the vector $\vec{\omega} + \vec{\Omega}$

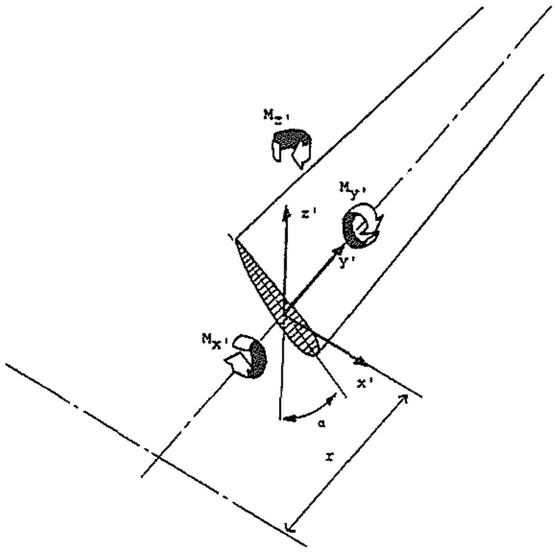


Fig.4 - Moments applied to a blade section having a distance r from the rotor axis.

As consequence of the particular tapering shape of the windmill blade, it is accettable to suppose that the moment of inertia around the longitudinal axis is negligible with respect to the others, which can be considered very similar to each other. I.e., it is possible to write:

$$I_{x'} = I_{z'} = I \qquad I_{y'} = 0$$

Following this simplification, the expression (4) becomes:

$$\vec{M} = \left| \begin{array}{c} I \dfrac{d\omega}{dt} + I\,\Omega^2 \sin\theta\cos\theta \\ 0 \\ I \dfrac{d\Omega}{dt} \cos\theta - 2I\omega\Omega\sin\theta \end{array} \right|$$

which shows the twisting component of the moment M is practically negligible in comparison with the others.

Due to the blade twist, the bending moments $M_{x'}$ and $M_{z'}$ do not act around the two principal axes of inertia of a generic section (figure 5). In fact, x' and y' have with the axis ξ and η an inclination equal to the blade setting angle α, which is function of the distance between the section and the rotor shaft.

In order to lead this situation to the case of straight bending, it is necessary to consider the components of the bending moment with respect to the principal axes of inertia:

$$M_\eta = M_x \cdot \sin\alpha + M_z \cdot \cos\alpha$$

$$M_\xi = M_x \cdot \cos\alpha - M_z \cdot \sin\alpha$$

Let I_ξ and I_η be respectively the geometrical moments of inertia of the blade section referred to the longitudinal and transversal axis, u and v the maximum distances between a point belonging to the section and the barycenter measured along the axis ξ and η (figure 6). The limit values of the applied stresses are:

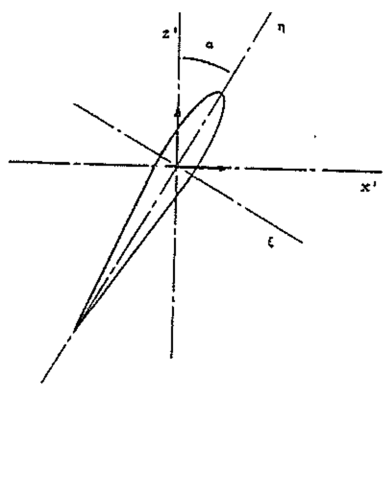
$$\sigma_\eta = \frac{M_\xi}{I_\xi} v \qquad \sigma_\xi = \frac{M_\eta}{I_\eta} u$$

Introducing the expressions previously evaluated, it results:

$$\sigma_\eta = \frac{Iv}{I_\xi}\left[\left(\frac{d\omega}{dt} + \Omega^2\sin\theta\cos\theta\right)\sin\alpha + \left(\frac{d\Omega}{dt}\cos\theta - 2\omega\Omega\sin\theta\right)\cos\alpha\right]$$

$$\sigma_\xi = \frac{Iu}{I_\eta}\left[\left(\frac{d\omega}{dt} + \Omega^2\sin\theta\cos\theta\right)\cos\alpha - \left(\frac{d\Omega}{dt}\cos\theta - 2\omega\Omega\sin\theta\right)\sin\alpha\right]$$

From the above equations we obtain that the stresses σ_ξ and σ_η are also dependent on the functions $\Theta=\Theta(t)$ and $\beta=\beta(t)$.



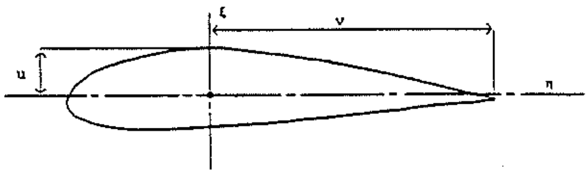


Fig.6

Fig.5 - Position of the principal axes of inertia of a blade section with respect to the coordinate system O'x'y'z'.

In order to simplify the next computing steps, although without losing generality in the calculation procedure, we assume the orientation of the rotor axis is oscillating according to the following angular variation:

$$\beta = \beta_0 \sin\omega_1 t$$

(i.e., considering only the first term of Fourier series decomposition of a generic oscillation motion to obtain the alignment with the wind direction). Moreover, let the rotational speed of the rotor w be constant.

These hypotheses permit to write:

$$\theta = \omega t \qquad \frac{d\omega}{dt} = 0 \qquad \Omega = \frac{d\beta}{dt} = \beta_0\omega_1\cos\omega_1 t \qquad \frac{d\Omega}{dt} = -\beta_0\omega_1^2\sin\omega_1 t$$

Replacing in the previous expression, we obtain:

$$\sigma_\eta = \frac{Iv\beta_0\omega_1}{I_\xi}[\beta_0\omega_1\cos^2\omega_1 t\sin\omega t\cos\omega t\sin\alpha - (\omega_1\sin\omega_1 t\cos\omega t + 2\omega\cos\omega_1 t\sin\omega t)\cos\alpha]$$
$$\sigma_\xi = \frac{Iu\beta_0\omega_1}{I_\eta}[\beta_0\omega_1\cos^2\omega_1 t\sin\omega t\cos\omega t\cos\alpha + (\omega_1\sin\omega_1 t\cos\omega t + 2\omega\cos\omega_1 t\sin\omega t)\sin\alpha] \qquad (5)$$

A precautionary analysis of the stresses applied to every blade section due to the gyroscopic effect is represented by the solution and checking of the following expression:

$$\sigma = |\sigma_\xi| + |\sigma_\eta|$$

which results function of the following parameters:

1) half-amplitude oscillation β_0 of the orientation motion around a vertical axis;

2) angular speed w_1 of the orientation motion around a vertical axis;

3) blade angular position $\Theta = wt$ assumed during the rotation around the shaft;

4) blade angular speed w around the rotor shaft;

5) distance r between the examined section and the rotor shaft (in fact, the values of the moments of inertia I, I_ξ and I_η, the entities u and v, and the setting angle α, depend on this parameter).

3 - NUMERICAL APPLICATIONS

The expressions carried out in the previous paragraph allow a quite easy evaluation of the stresses due to the gyroscopic effect applied to a generic blade section. Nevertheless, it is possible to do that only if we exactly know the geometry, the structure and the material of each blade: of course if we know the employed airfoil and the variation of the blade chord and setting angle versus the distance from the rotor axis.

Both the chord and the setting angle are function of several parameters, like: the available windmill power, the average wind speed on the installation area, the number of blades, the employed airfoil and, at last but not least, the "tip speed ratio" u_o/V, being u_o the circumferential speed at the blade tip and V the upstream wind velocity at five diameters in front of the turbine.

Changing conveniently these parameters, it is possible to analyze several blade shapes and the respective variation of the stresses due to gyroscopic effect.

For this purpose, a software package has been carried out and set up by the author. Operating on a personal computer, it allows to calculate and video-record, by means of the expressions (5), the variation of the stresses versus the previous listed parameters.

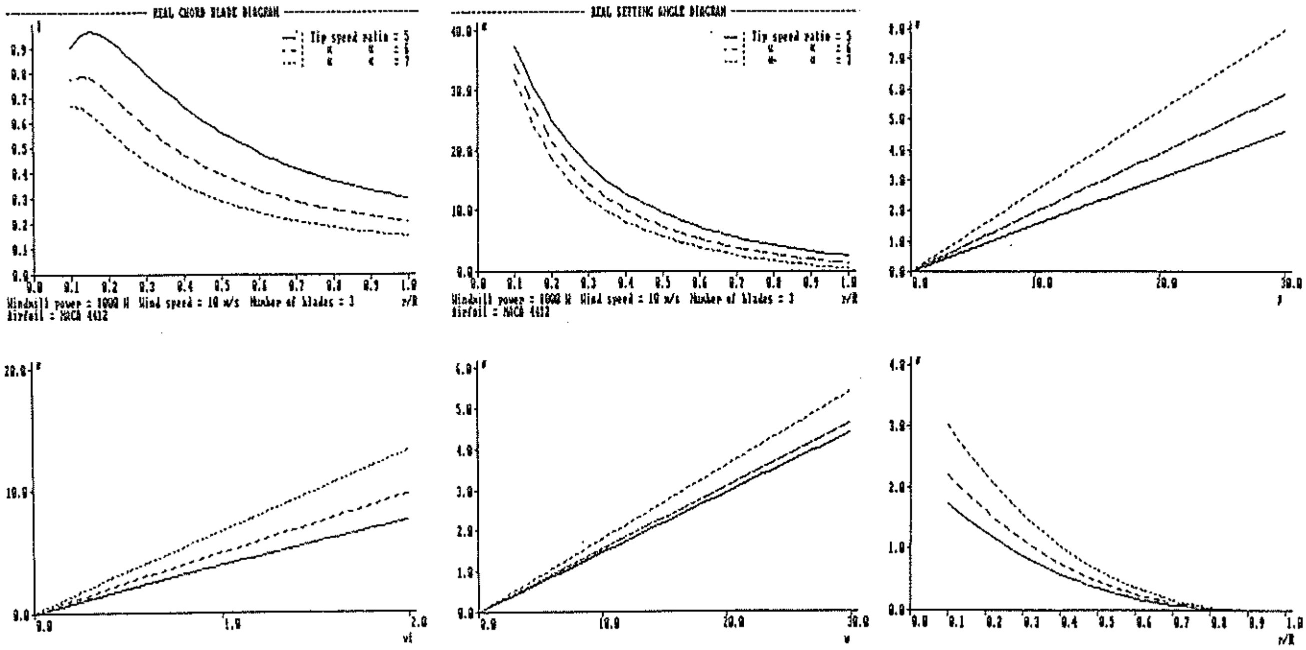
The airfoil chord and the setting angle values have been established through the vortex theory of Glauert. But, this method leads, very often, to higher values of chord and setting angle in the neighborhood of the rotor hub; moreover the blade shapes are twisted. These considerations constrain, mainly for machining reasons, to straighten the diagrams shown in the figures 7, 8, 9 and 10, taking, near the blade tip for the setting angle and the chord, values very close to the optimal ones because of the importance of the swept area per length unit of blade (in practice, for usual airfoils like Gottingen 623, NACA 4412 and NACA 23012, the maximum setting angle will be chosen between 10° and 15° at a distance 0.2R from the rotor shaft). So we obtain the following relations:

$$\alpha(r) = \alpha_1 r + \alpha_0 \qquad \ell(r) = \ell_1 r + \ell_0$$

which may be replaced in the expressions (5) respectively to compute the entities $\sin\alpha$, $\cos\alpha$, I , I_ξ , I_η , u and v.

Each of the previous figures, in addition to the variation of the blade chord (expressed in meters) and the setting angle (degree), displays the variation of the stress σ (Kg/mm^2) versus the parameters: half-amplitude oscillation β_o (degree); angular speed w_1 of the orientation motion (rad/s); distance r/R between the section and the rotor shaft; and blade angular position Θ as assumed during the rotation around the shaft (degree).

Particularly: figure 7 shows three curves evaluated by changing the tip speed ratio u_o/V; figure 8 displays three curves relative to different available power values (expressed in watts); figure 9 shows three curves



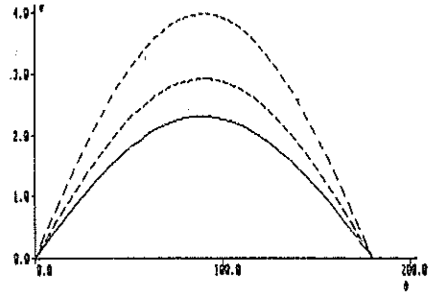
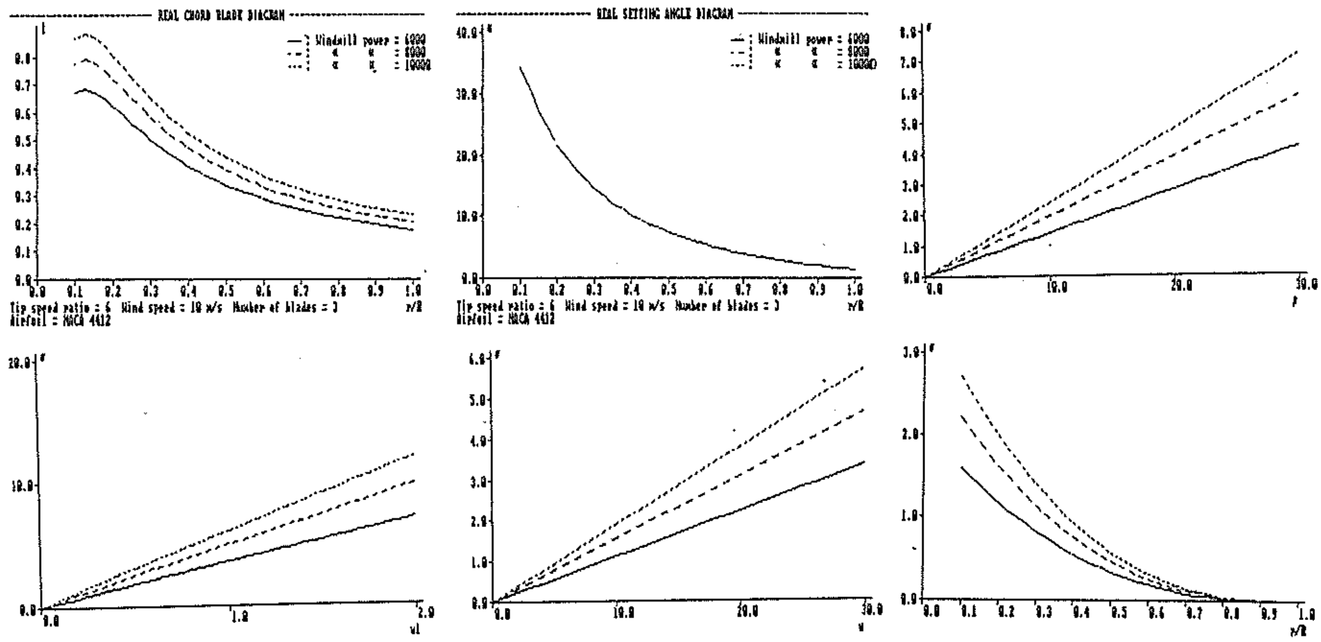


Fig.7 - Variation of the stress due to gyroscopic effect as function of the tip speed ratio. Available power: 8000 W; project wind speed: 10 m/s; number of blades: 3; airfoil: NACA 4412.



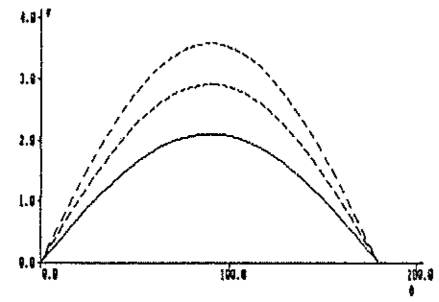


Fig.8 - Variation of the stress due to gyroscopic effect as function of the available power. Tip speed ratio: 6; project wind speed: 10 m/s; number of blades: 3; airfoil NACA 4412.

corresponding to a given value of the project wind speed (expressed in m/s); finally figure 10 represents three curves relative to two, three and four blades windmill, respectively.

An analysis of the obtained results leads to the following considerations:

a) The state of stress σ is linearly growing with the increasing half-amplitude oscillation β_o, the oscillation angular velocity w_1 and the blade angular velocity w. Particularly, we may see stress values which reach 10÷15 Kg/mm^2 when w_1 assumes very high values, also for the little rotors considered in this paper. These stress values are markedly greater than those ones reached in the other diagrams. It follows that an unexpected variation of the wind direction, for instance in consequence of a sudden gust, causes a critical situation for the blade integrity.

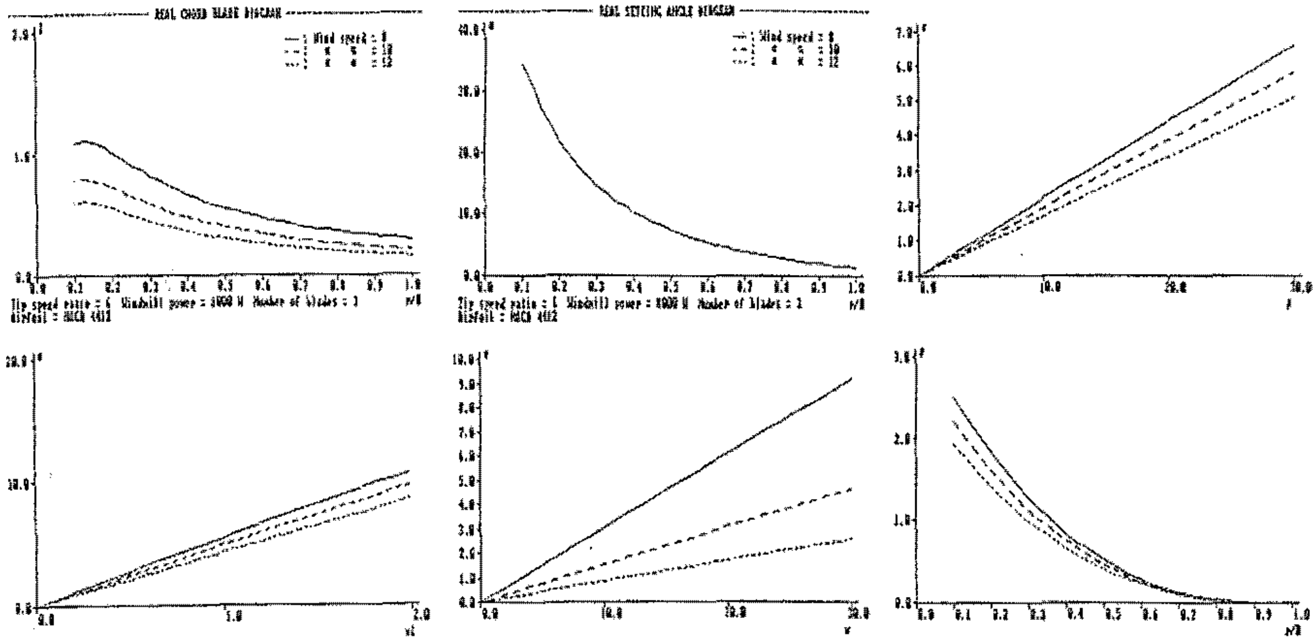
b) The diagrams, which show the values σ versus the ratio r/R, demonstrate that the sections in the neighborhood of the rotor hub are mostly stressed, while the strain owing to the gyroscopic effect is practically equal to zero near the blade tip. This contributes to increase still more the loads applied to the sections near the hub, which are already hardly stressed by aerodynamic and centrifugal effects.

c) The curves relative to blade angular position Θ show a sinusoidal variation with stress values equal to zero, corresponding to the positions 0° and 180° (horizontal position), and with maximum stress values corresponding to the positions 90° and 270° (vertical position). This cyclic variation, with stress peak over 4 Kg/mm^2, added to the strains due to the aerodynamic and centrifugal effects, produces a fatigue stress whose outcome is necessary to consider during the blade structural verification. Moreover, it can represent a dangerous source of vibratory motion such that to involve the whole windmill structure, and to diverte some of the power transmitted by the rotating shaft into building up and maintaining transverse vibration.

d) Figure 7 particularly shows how the stresses enhance when the tip speed ratio u_o/V of the wind machine grows up. In fact, a high value of this factor leads to a progressive reduction of the chord and, of course, of the resisting section area, making a stress increase.

e) A look to figure 8 emphasizes how the stresses due to the gyroscopic effect linearly grow up when the available windmill power enhances. This is explained, any other condition being equal, considering that an available power increase is obtained by growing up the rotor diameter according to the well-known Betz' formula:

$$P = 0.2\ D^2\ V^3$$



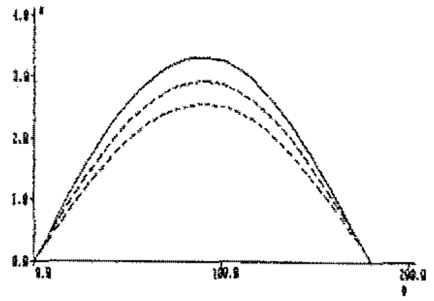
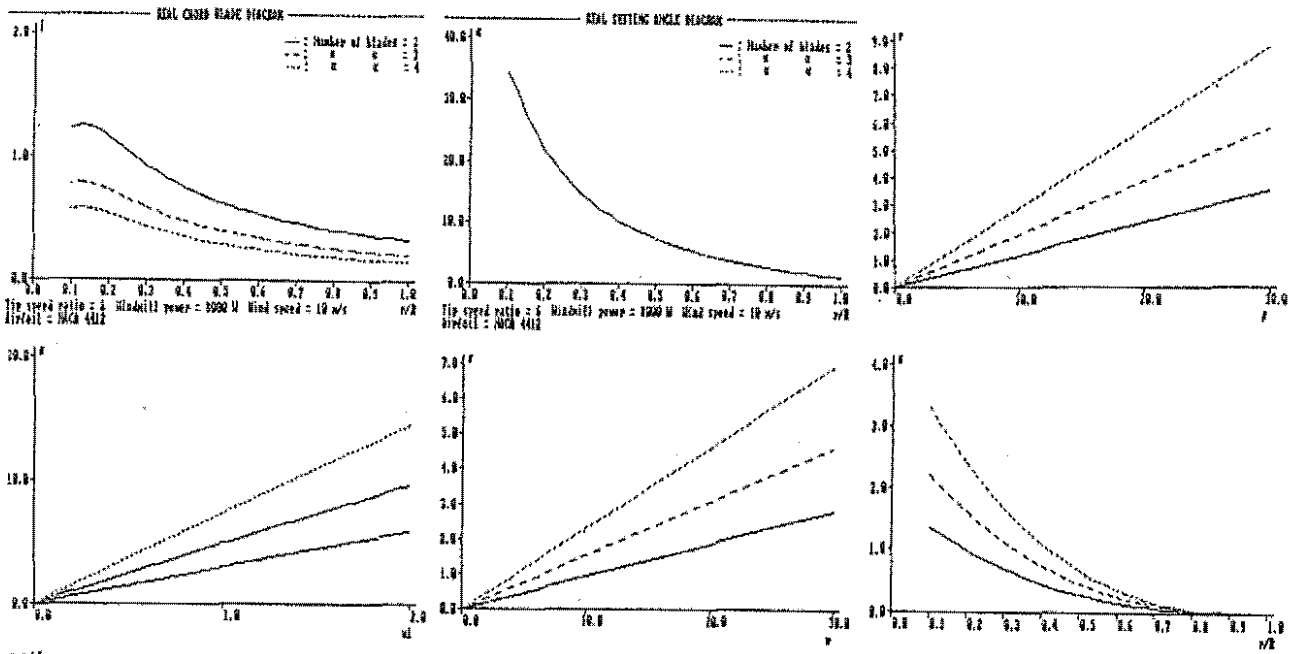


Fig.9 - Variation of the stress due to gyro scopic effect as function of the project wind speed. Tip speed ratio: 6; available power: 8000 W; number of blades: 3; airfoil NACA 4412.



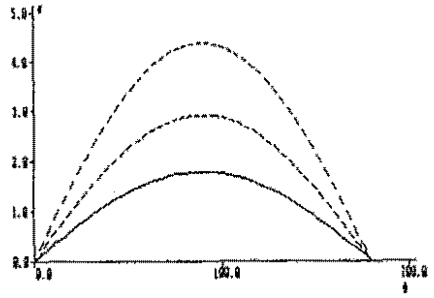


Fig.10 - Variation of the stress due to gy-roscopic effect as function of the number of blades. Tip speed ratio: 6; available power: 8000 W; project wind speed: 10 m/s; airfoil: NACA 4412.

where P is the power, D is the diameter and V is the project wind speed. Since the bending moment applied to each section is proportional to the mass of the blade portion included between the tip and the section (figure 4), we obtain that a diameter increase makes a stress growth.

f) How figure 9 shows, an increase of the project wind velocity indirectly induces a decrease of the stresses due to the gyroscopic effect. In fact the more the average wind velocity is high in a given place, the more the required rotor diameter is small, in order to obtain the same power transmitted by the rotating shaft. Therefore, also the produced stresses are smaller.

g) Conditions being equal, the stresses σ increase with the number of blades the wind machine has been built (figure 10). In fact, the first diagram shows a drastic reduction of the airfoil chord when the number of blades enhances. That leads to a reduction of the resisting section area and, consequently, to an increase of the stresses applied to the blade.

4 - CONCLUSIONS

The adopted computer process has allowed to estimate, with sufficient approximation, the stresses due to the gyroscopic effect applied to a generic blade section of a horizontal-axis wind turbine.

The analysis of the outcome has brought to the conclusion that such stresses, predominantly with bending component, may reach not negligible values also for small wind machines. Their cyclic variation, with a frequency equal to the angular speed of the rotor, makes a fatigue stress on the blade structure an it may represent a vibration source for the whole windmill.

It has been demonstrated how the more dangerous conditions occur in the neighborhood of the rotor hub, when the blade is in vertical position. Moreover the load tends to enhance with the increase of the turbine rotational speed and of the rapidity and the orientation amplitude of the rotor with respect to the wind direction. Also the geometrical and design parameters of the rotor influence such state of stress: the progressive increase of the tip speed ratio, of the rotor diameter and of the number of blades, produces a remarkable increase of the strains due to the gyroscopic effect.

5 – BIBLIOGRAPHY

1) D. LE GOURIERES – "Wind Power Plants, Theory and Design" – Pergamon Press, 1982.

2) H. GLAUERT – "Windmills and Fans, Aerodynamic Theory" – Dover Publ. Inc. N.Y., 1963.