STOCHASTIC AEROELASTIC ANALYSIS OF COMPOSITE HELICOPTER ROTOR WITH MATERIAL UNCERTAINTY

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Abstract: This study investigates the effect of randomness in composite material properties on the aeroelastic analysis predictions. The impact of material uncertainty on the cross-sectional stiffness, natural frequencies and aeroelastic response predictions of a composite helicopter rotor blade are studied. The elastic modulii and Poisson's ratio of the composite plies are considered as random variables with a coefficient of variation of 5 percent. An analytical model is used for evaluating blade cross-sectional stiffness. Aeroelastic analysis based on finite elements in space and time is used to evaluate the helicopter rotor blade response in hover. The stochastic cross-sectional and aeroelastic analyses are carried out with Monte Carlo simulations. The blade cross-sectional stiffness matrix elements show a coefficient of variation of about 9 percent. The impact of material uncertainty on rotating natural frequencies varies with the lag, flap and torsional motions because of centrifugal stiffening. The blade tip response in hover show a considerable scattering from the baseline value. The numerical results clearly show the need to consider randomness of composite material properties in the helicopter aeroelastic analysis.

1. INTRODUCTION

In recent years, uncertainty quantification is considered as a key issue in aeroelasticity [1]. The highly multidisciplinary and complex nature of rotorcraft aeroservoelasticity has led the researchers to focus on improving the fidelity of analytical modeling, solution methods, and validation of the analysis results with experimental data [2, 3]. However, no study has focused on the uncertainty of input parameters used in the rotorcraft aeroelastic analysis and impact of these uncertainties on blade response, vibratory loads and stability predictions. In the rotorcraft aeroelastic analysis, the uncertainties can be associated with structural, aerodynamic or control parameters.

The helicopter rotor blade which plays a dominant role in the overall vehicle performance is typically made of composites. The material properties of composites used in the rotor blade design and analysis are unreliable because of the manufacturing process and lack of knowledge of precise experimental data [4-6]. Therefore, the effect of uncertainty associated with the composite material properties on rotorcraft aeroelasticity has to be evaluated. Such uncertainties can effect the interpretation of results where aeroelastic analysis results are compared to the experimental or flight test data. To the best of the author's knowledge, no research has addressed this issue.

For stochastic analysis, several methods are available [7]. Monte Carlo Simulations (MCS) are the most popular stochastic analysis technique and can be used without any modification in the existing analysis programs [5]. The rotorcraft aeroelastic analysis programs are complex in nature and need domain experts for any modification inside the program [8]. Therefore, MCS can be considered as a better choice to study the uncertainty impact on the rotorcraft aeroelastic analysis without any modification inside the program.

This study focuses on the effect of uncertainties associated with the modulus properties and Poisson's ratio of composite plies on the aeroelastic response using MCS. The uncertainty impact is studied at three stages: 1) The composite rotor blade cross sectional stiffness, 2) Free vibration characteristics of the rotor blade, and 3) The blade aeroelastic response in hover.

2. ROTOR BLADE CROSS SECTIONAL ANALYSIS

A critical aspect of helicopter rotor dynamic analysis is the calculation of equivalent 1-D beam properties for the 3-D rotor blade. For that, the cross-sectional analysis of composite rotor blade is carried out with analytical models or detailed finite element methods [9, 10]. A composite box beam is generally considered as a good representation of the helicopter blade for preliminary design studies [11]. As an initial effort to quantify the material uncertainty, the helicopter rotor blade is modeled as a thin-walled composite box beam in this study.

A direct analytical formulation presented by the Smith and Chopra [11] is used for predicting the effective elastic stiffness of the composite box beam. The analytical formulation has been used for aeroelastic analysis [12] and optimization [13] studies and is computationally efficient. The geometry and coordinates of the composite box beam are shown in Fig. 1. The deformation of the box beam is described by three displacements *u, v* and *w*, and one torsional displacement φ. The cross-sectional stiffness matrix of a composite box beam with balanced laminate as four walls can be given by the relation

$$\begin{Bmatrix} Q_x \\ M_x \\ -M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} EA & 0 & 0 & 0 \\ 0 & GJ & 0 & 0 \\ 0 & 0 & EI_y & 0 \\ 0 & 0 & 0 & EI_z \end{bmatrix} \begin{Bmatrix} u' \\ \phi' \\ w'' \\ v'' \end{Bmatrix} \tag{1}$$

Here, EA, GJ, EI_y and EI_z correspond to axial, torsional, flap (out-of-plane) and lag (in-plane) bending stiffness of the rotor blade. A balanced laminate is considered since composite rotor blades are conservatively designed to have no couplings.

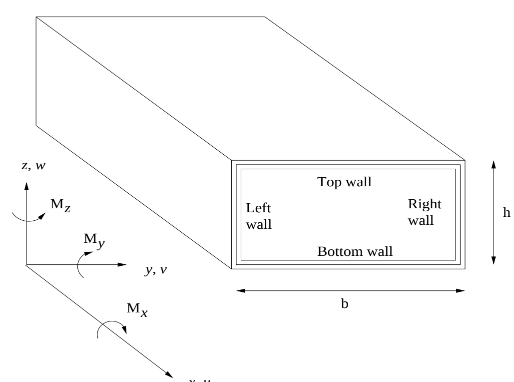

Figure 1. Composite box beam

3. NONLINEAR AEROELASTIC MODEL

A comprehensive aeroelastic analysis code based on the finite element method is used to evaluate the helicopter blade response. The rotorcraft structure is modeled as a nonlinear representation of composite elastic rotor blades coupled to a rigid fuselage. The rotating elastic rotor blade is modeled as a slender elastic beam undergoing flap bending w, lag bending v, elastic twist φ, and axial deflection u with a rotational speed of Ω. The effect of moderate deflections is included by retaining second order non-linear terms. Governing equations are derived using a generalized Hamilton's principle applicable to non-conservative systems.

$$\int_{\psi_1}^{\psi_2} (\delta U - \delta T - \delta W) d\psi = 0 \tag{2}$$

Here, δU is the virtual strain energy and δT is the kinetic energy contributions from the elastic blade. Also, δW is the virtual work variational from the external aerodynamic forces acting on the blade and $\psi = \Omega t$ is the azimuth angle around the rotor disk. The unsteady aerodynamics and free wake models are used to calculate the aerodynamic forces [14]. The blade is discretized into beam finite elements each with fifteen degrees of freedom. These degrees of freedom correspond to cubic variations in axial elastic and (flap and lag) bending deflections, and quadratic variation in elastic torsion. The finite element equations are reduced in size by using normal mode transformation. This results in the non-linear ordinary differential equation with periodic coefficients as given below.

$$M\ddot{p}(\psi) + C\dot{p}(\psi) + Kp(\psi) = F(p, \dot{p}, \psi) \tag{3}$$

Here M, C, K, F and p represents the finite element mass matrix, damping matrix, structural stiffness matrix, finite element force vector and modal displacement vector, respectively. Nonlinearities in the model occur due to Coriolis terms and moderate deflection assumptions in the strain-displacement relations. These equations are then solved using finite element in time in combination with the Newton-Raphson method. The above equations govern the dynamics of the rotor blade. The solutions to the equations are then used to calculate rotor blade loads using the force summation method, where aerodynamic forces are added to the inertial forces. The blade loads are integrated over the blade length and transformed to the fixed frame to get hub loads. The steady hub loads are used to obtain the forces acting on the rotor and combined with fuselage and tail rotor forces to obtain the helicopter rotor trim equations:

$$F(\Theta) = 0 \tag{4}$$

These nonlinear trim equations are also solved using the Newton-Raphson method. The helicopter rotor trim equations and the blade response equations in (3) and (4) are solved simultaneously to obtain the blade steady response and hub loads. This coupled trim procedure is important for capturing the aeroelastic interaction between the aerodynamic forces and the blade deformations. Further details of the analysis are available in [14].

4. NUMERICAL RESULTS

The effect of material uncertainties on the cross-sectional stiffness, natural frequencies of the rotating blade, and aeroelastic response of the composite rotor blade are studied. A baseline analysis is carried out initially with the mean values of material properties and the results from non-deterministic analysis are compared with these baseline results. The rotor blade considered in this study is a uniform blade equivalent of the BO-105 rotor blade [12]. The BO-105 rotor blade properties are given in Table 1. The baseline box beam has a breadth of 0.144 m, height of 0.081 m and ply orientation of $[\ 0_3\ /\ (15/\text{-}15)\ _3\ /\ (45/\text{-}45)\ _2\ ]\ _s$. Each wall of the box beam is therefore made of balanced symmetric laminate with 26 plies and each ply is 0.127 mm thick. The graphite/epoxy material properties are given in Table 2.

Table 1. Baseline hingeless rotor properties

Number of blades	4
Radius, R (m)	4.94
Hover tip speed, ΩR (m/s)	198.12
Mass per unit length, m_0 (kg/m)	6.46
Lock number, γ	6.34
Solidity, σ	0.1
C_T/σ	0.07
$EI_y/m_0\Omega^2R^4$	0.00834
$EI_z/m_0\Omega^2R^4$	0.02317
$GJ/m_0\Omega^2R^4$	0.00382
m/m_0	1.0

Table 2. Material properties of graphite/epoxy

Material properties	Mean
E_1 (MPa)	141.96e3
E_2 (MPa)	9.79e3
G_{12}(MPa)	6.00e3
ν_{12}	0.42

4.1 Stochastic Cross Sectional Properties

The impact of material uncertainty on the rotor blade cross sectional properties is evaluated initially. Most of the studies on uncertainty analysis of composite structures generally consider E_1, E_2, G_{12} and ν_{12} as statistically independent random variables and assume a coefficient of variation (c.o.v.) of 5 to 10% for each of these properties [15, 16]. For this study, the composite material properties E_1, E_2, G_{12} and ν_{12} are considered as independent random variables with a normal distribution and a c.o.v. of 5 percent is assumed for each. The 5000 MCS of material properties are shown in Fig. 2

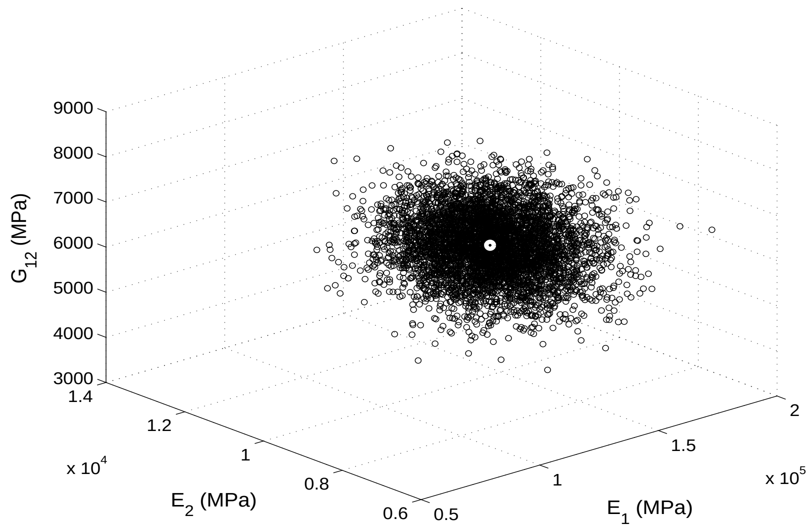

Figure 2. MCS of material properties

The composite box beam stiffness is evaluated with the non-deterministic values of the material properties using the 5000 MCS [11]. The MCS of flap, lag and torsional cross sectional stiffness are shown in Fig. 3. In contrast to the scatter in the material properties (Fig. 2), the stochastic flap, lag and torsional stiffness are almost clustered around a single line in the stiffness design space (Fig. 3). However, the impact of material uncertainty on the cross sectional properties depend on the type of laminates and plies angles. The histograms of the cross-sectional stiffness are shown in Fig. 4. The flap, lag and torsional stiffness show a c.o.v. of 9.33, 9.33, and 8.39% respectively as given in Table 3a. The stiffness values are scattered around $\pm$20 % percentage around the baseline values with respect to the material uncertainties.

The sensitivity of blade stiffness to each of the composite material properties is also studied. The MCS is carried out with five different cases. For the case I, randomness is considered in all the four material properties and for the remaining four cases, each one of the material properties is considered as the random variable and remaining material properties are assumed with its mean value. The c.o.v of the blade stiffnesses for five cases are given in Table 3a. The randomness in the longitudinal Young's modulus E_1 show the highest impact on the cross sectional stiffness. However, the randomness of the other material properties (E_2, G_{12} and ν_{12}) have very less impact on the cross sectional stiffness.

Table 3a. Stochastic cross sectional stiffness

Random variables	C.O.V of stiffness (%)			
	EA	GJ	EI_y	EI_z
$E_1, E_2, G_{12}, \nu_{12}$	9.33	8.39	9.33	9.33
E_1	9.28	8.16	9.28	9.27
E_2	0.08	0.18	0.08	0.08
G_{12}	0.87	1.85	0.87	0.87
ν_{12}	0.17	0.26	0.17	0.17

Now, to study the influence of the ply angle sequence in material uncertainty propagation, the baseline box beam with a different ply angle sequence is considered. For example, uncertainty impact on the box beam stiffness with a ply sequence of $[0 / (45/-45)_6]_s$ is studied. This ply sequence has more number of 45 degree plies than the baseline ply sequence. The c.o.v of the stiffness are shown in Table 3b. For this ply angle sequence, the randomness in shear modulus G_{12} show a higher impact on the stiffness than the Young's modulus E_1 of the baseline ply angle sequence.

Table 3b. Stochastic cross sectional stiffness

Random variables	C.O.V of stiffness (%)			
	EA	*GJ*	EI_y	EI_z
$E_1, E_2, G_{12}, \nu_{12}$	7.18	9.87	7.183	7.18
E_1	4.47	9.86	4.48	4.48
E_2	0.13	0.22	0.13	0.13
G_{12}	5.55	0.13	5.54	5.54
ν_{12}	0.007	0.32	0.03	0.006

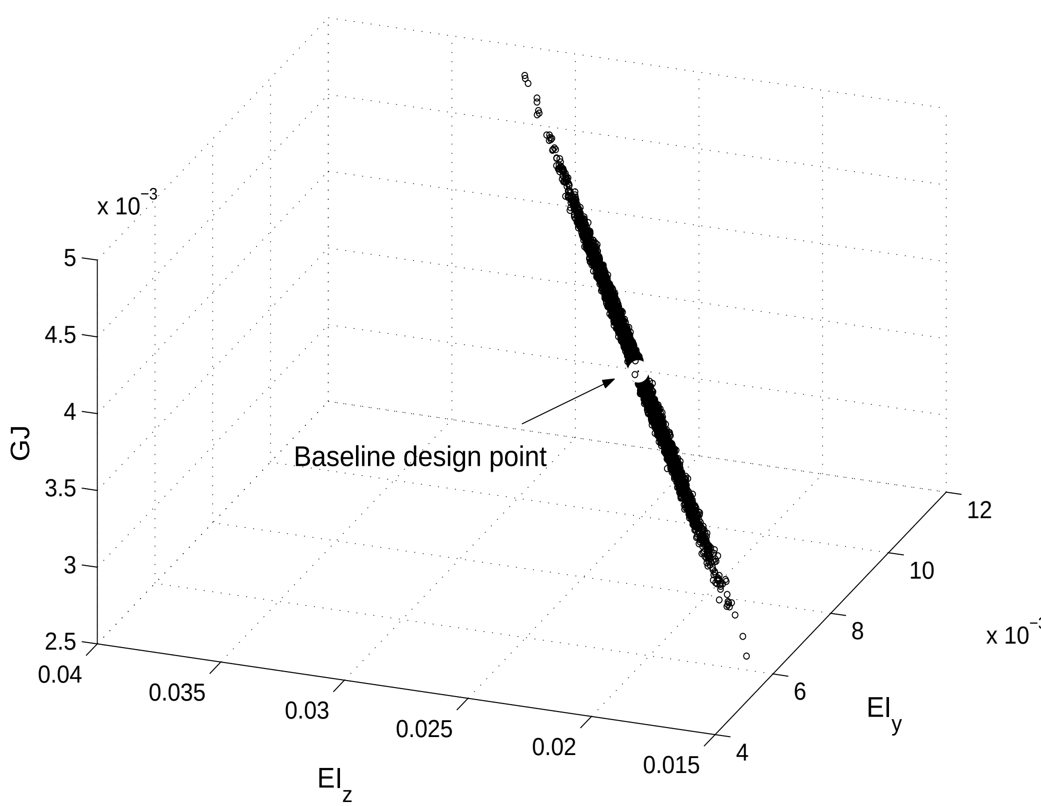

Figure 3. MCS of cross sectional stiffness (non-dimensional)

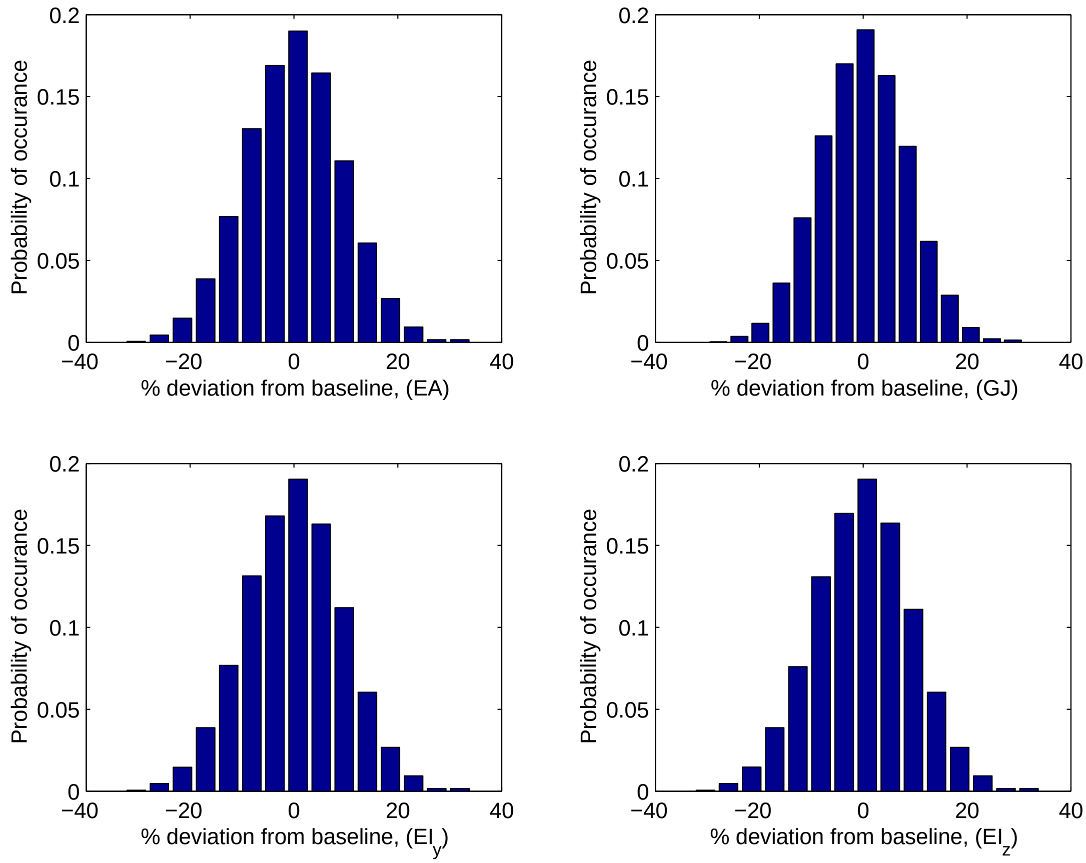

Figure 4. Histogram of cross sectional stiffness

3.2 Stochastic Natural Frequencies

The stochastic stiffness values calculated in the previous section are used to evaluate the stochastic non-rotating and rotating natural frequencies of the composite rotor blade. The natural frequencies of the rotating blades are calculated by solving the following Eigenvalue problem

$$K\Phi = \omega^2 M\Phi \tag{6}$$

Here, the stiffness K includes the structural stiffness and centrifugal stiffening effect when the blade is rotating, M is the structural mass matrix, ω are the natural frequencies and the vector of degrees of freedom, Φ contains the mode shapes.

For the rotorcraft aeroelastic analysis, three normal modes for flap motion, two normal modes for lag motion, and one normal mode for torsional motion are used to capture the essential dynamics of the system. The effect of uncertainty on the natural frequencies of rotating blades is studied using 5000 MCS. The baseline natural frequencies and c.o.v. are given in Table 4 and MCS results are shown in Fig. 5.

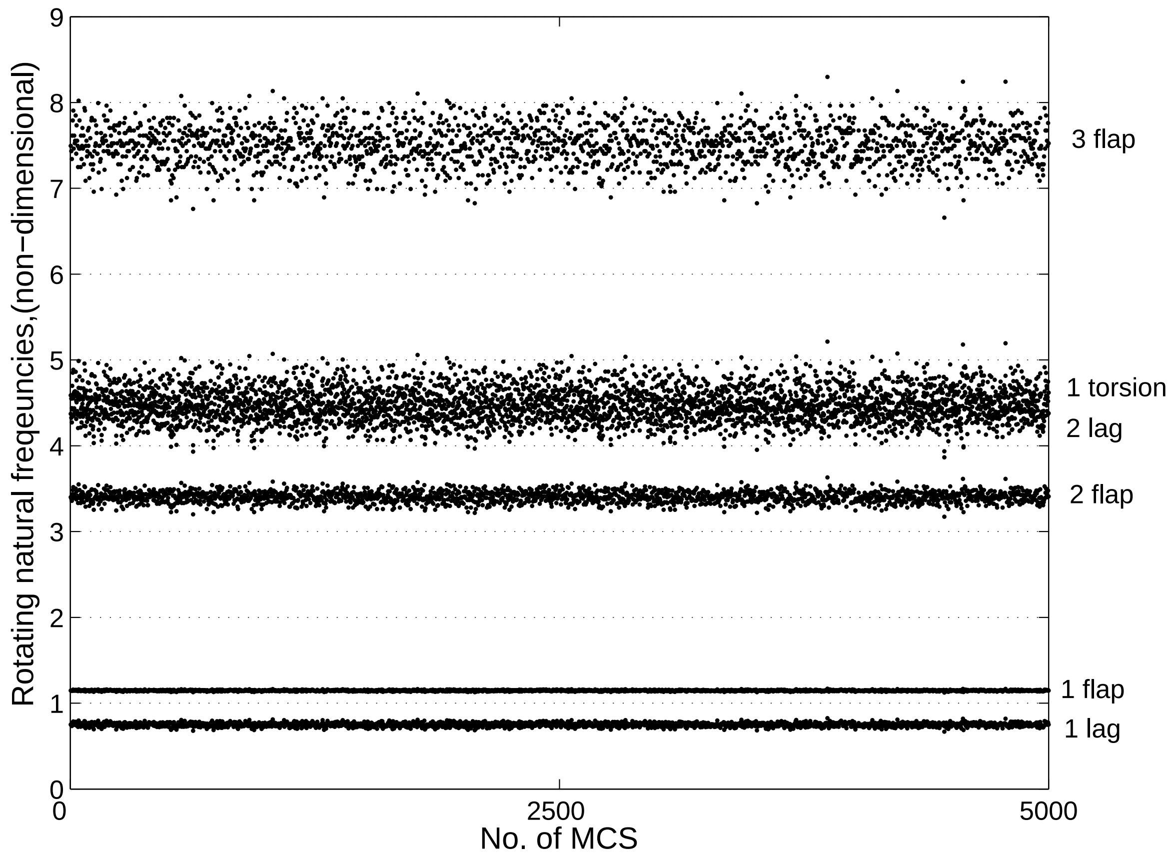

Figure 5. MCS of rotating natural frequencies

The c.o.v. of the rotating blade frequencies vary with respect to the modes. Among the flap, lag and torsional fundamental frequencies of the rotating blade, the torsion frequency exhibits the highest c.o.v. The histograms for flap, lag and torsional fundamental frequencies of rotating blade are shown in Fig. 6. The fundamental lag and torsional frequencies vary ±10% around their baseline values, while the fundamental flap frequencies vary just ±1%, as explained below.

Table 4. Stochastic rotating natural frequencies

Mode	Baseline frequencies (non-dimensional)	C.O.V (%)
Flap 1	1.14	0.54
Flap 2	3.40	1.91
Flap 3	7.49	3.09
Lag1	0.75	2.97
Lag2	4.37	3.20
Torsion 1	4.58	4.02

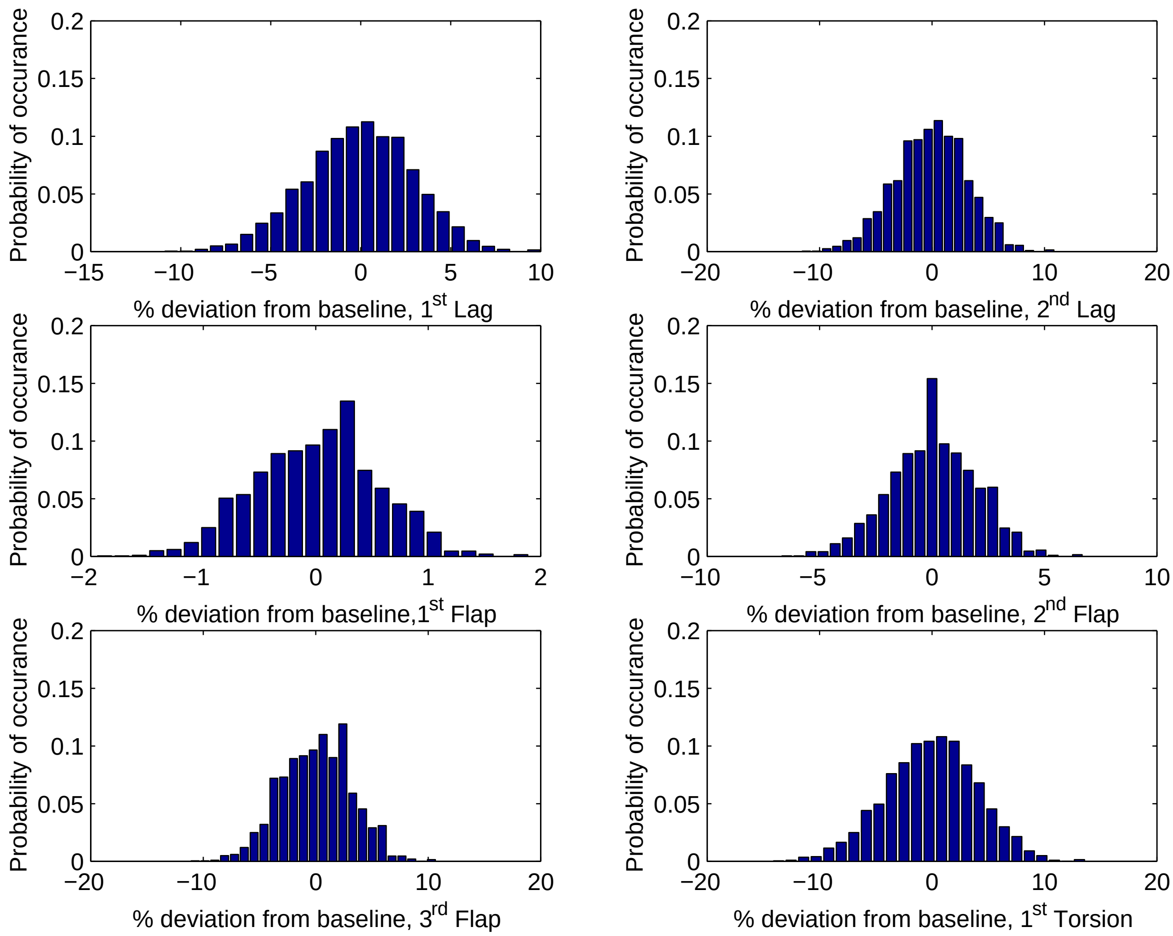

Figure 6. Histograms of rotating natural frequencies

From Table 4, it is observed that the impact of material uncertainties on natural frequencies of varies with the modes. The c.o.v. of rotating blade frequencies varies with respect to the modes. For the rotating blade, the natural frequencies depend on the structural stiffness and centrifugal stiffness. The centrifugal stiffening has different impacts on the flap, lag and torsional motions of the rotating blade [17]. For a hingeless rotor blade with uniform mass, the strain energy U has contributions from the structural and centrifugal stiffness. The centrifugal stiffness dominates the flap motion of the rotating blade compared to its structural stiffness. For the lag motion, the structural stiffness dominates the motion compared to the centrifugal stiffness. Therefore, the effect of uncertainty in the structural stiffness has a greater influence on the lag frequencies compared to the flap frequencies as shown in Table 4. For the torsional motion, the structural stiffness is comparatively higher than the centrifugal stiffness and therefore, the scattering of the torsional stiffness has a higher impact on its frequency.

3.3 Stochastic Aeroelastic Analysis in Hover

The material uncertainty impact on the aeroelastic response of helicopter is studied at hover condition. The aeroelastic response is evaluated with the stochastic beam stiffness values calculated in the earlier section. The MCS of aeroelastic blade tip response is shown in Fig. 7.

The flap, lag and torsion response show a scattering of ±3%, ±15% and ±40% from their baseline values, respectively. However, the MCS results show a discontinuity in the blade tip response. The fundamental torsion frequency and the corresponding torsion response of MCS are shown in Fig. 8. The discontinuity in the MCS of blade response needs further investigation. The uncertainty impact on the aeroelastic response in forward flight is the focus of future work.

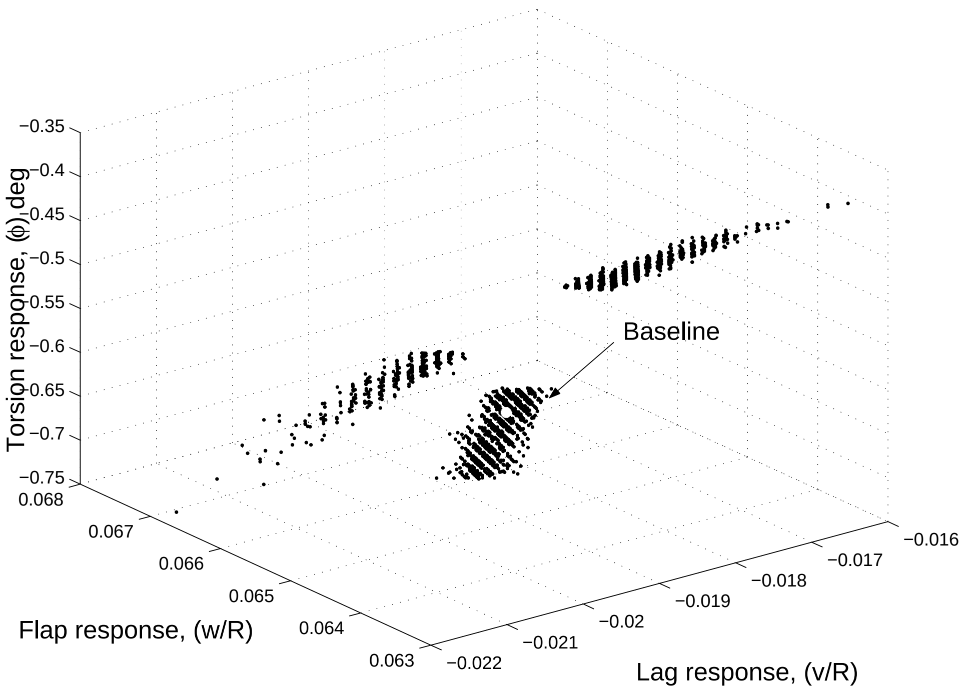

Figure 7. MCS of rotor blade response in hover

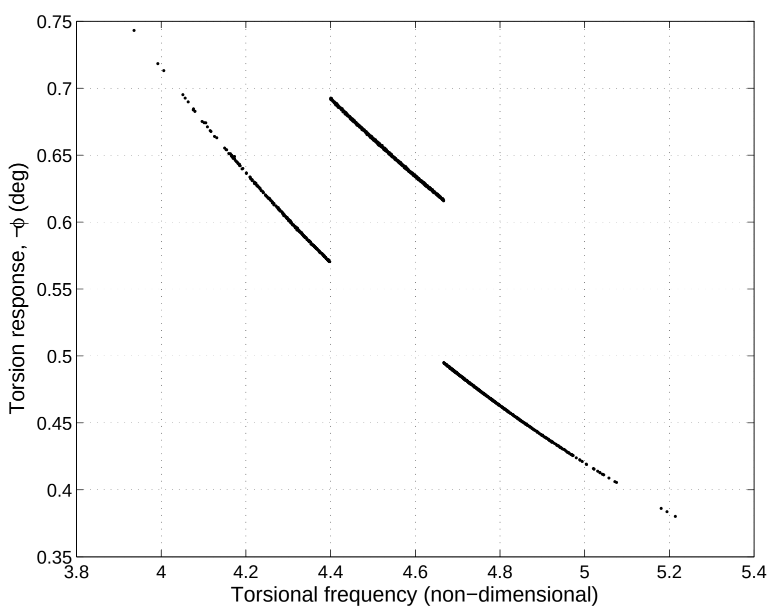

Figure 8. Effect of uncertainty on blade torsion response

5. CONCLUSION

The aeroelastic response of a composite helicopter rotor with material uncertainty is studied. The composite material properties E_1, E_2, G_{12} and ν_{12} are modeled as independent normally distributed random variables. The effect of material uncertainties on the cross-sectional stiffness, rotating natural frequencies, and aeroelastic response of the composite rotor blade in hover are evaluated using Monte Carlo simulations. The following conclusions are drawn from this study.

1) The flap, lag and torsional stiffnesses of the composite rotor blade show c.o.v. of 9.33, 9.33 and 8.39%, respectively when uncertainty is considered in the material properties. The uncertainty impact on the blade stiffness varies with respect to the ply angle sequence of the laminates.

2) The flap, lag and torsional fundamental natural frequencies of the rotating composite blade show c.o.v. of 0.54, 2.97 and 4.02%, respectively. The flap natural frequencies are less sensitive to material uncertainty compared to the lag and torsional frequencies. The c.o.v. of the rotating blade natural frequencies varies with modes because of the relative importance of structural stiffness vis-à-vis the centrifugal stiffness.

3) The aeroelastic response of the composite helicopter rotor in hover shows a considerable scattering from the baseline value response due to material uncertainty. The deviation in the aeroelastic response affects the accurate prediction of aeroelastic performance parameters. Therefore, aeroelastic design of rotorcraft should consider the randomness in composite material properties.

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