A "USER'S GUIDE" TO SYSTEM IDENTIFICATION METHODS FOR HELICOPTER AND VTOL APPLICATIONS

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Abstract
As models play an increasingly vital role throughout the life cycle of flight vehicles, the importance of system identification in aircraft development and operations continues to grow. Navigating the wide array of system identification techniques and determining the most appropriate one can be overwhelming, particularly for new users. The unique characteristics of rotorcraft and next-generation VTOL vehicles further complicate the decision-making process, introducing specific challenges and limitations on the applicable methods.This paper provides a review of available methods and model classes for system identification,with a specific focus on rotorcraft and VTOL applications.The goal is to guide users toward making the most effective choices by clarifying how these choices are influenced by various practical considerations and constraints commonly encountered during the modeling and identification process.

1 INTRODUCTION

With the increasing reliance on models throughout the life cycle of flight vehicles, the role of system identification for aircraft development and operations is growing in importance. Its many applications include, among others, comparisons to theoretical predictions, simulation development and updating, stability and control assessment, control law development, flying qualities assessments, and stability margin extraction [34]. Progress in system identification is being made not only in aeronautics, as highlighted in publishing experiences developed over the last 20 years with aircraft system identification proven for specific problems [33], but also in other fields of industry, with regular publications in conference proceedings and engineering journals (as reviewed for example in [47] and [75]).

Navigating the array of system identification approaches and selecting the most suitable ones can be overwhelming, especially for new users. To begin with, given the diversity of areas in which system identification is used, the sheer number of available techniques is huge. Despite the myriad of methods and names, a shared core actually exists, consisting of relatively few fundamental concepts derived from statistical theory [65]. Understanding the underlying principles and assumptions can help users orient themselves and make more informed choices.

Furthermore, the peculiarities of aeronautical problems impose restrictions on the range of applicable methods [16]. This is especially true when dealing with rotorcraft, as significant challenges arise from the low signal-to-noise ratio and high vibrations, the presence of often difficult to observe internal states, the typical open-loop instability, as well as the wide range of conditions encompassed by the flight envelope. Again, knowledge of the foundations of identification methods, besides a thorough understanding of the system one wants to model, can be extremely helpful.

Last but not least, the final intended use of the identified model plays a critical role: requirements will be significantly different whether the model is used to support design and certification, or as a basis for model-based flight control system development, or for implementation on a flight simulation training device – and so will be the validation process. When selecting the system identification technique, the user should be aware of the level of uncertainty this will introduce into the model.

This paper provides a review of available methods and model classes for system identification, with a specific focus on rotorcraft and next-generation VTOL applications. The goal is to help users navigate toward the most effective choice, illustrating the relationships between that choice and the various practical issues typically involved in the modeling and identification the process. Prior work along these lines was conducted by the NATO STO AVT-296 Research Task Group [112], which surveyed the state-of-the-art practices for rotorcraft simulation model fidelity assessment and improvement; this paper aims to build on this, by also including promising techniques from other industrial domains.

The text is organized as follows: Section 2 provides a general definition of the system identification problem as well as its main components. Section 3 outlines the premises for choosing suitable methods, given by the model intended use and the vehicle at hand. Section 4 gives an overview of the most common classes of rotorcraft and VTOL models. Section 5 examines available system identification methods and their characteristics, with special focus on their practical use. Section 6 outlines other practical issues re-

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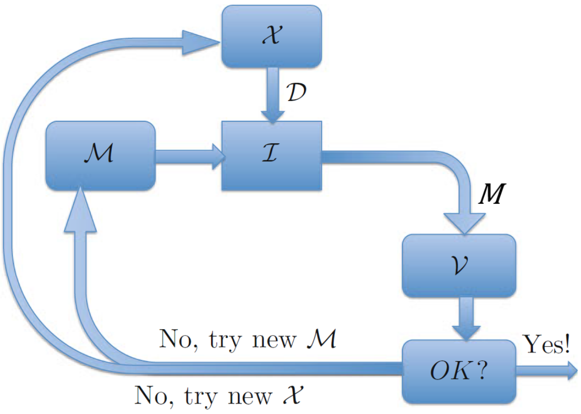

Figure 1: The identification work loop [66, adapted]

lated to rotorcraft system identification, which were not the main focus of this paper, and provides useful literature references. Finally, Section 7 provides a concluding summary and explores some possible future developments.

2 THE SYSTEM IDENTIFICATION PROBLEM

A widely accepted definition of system identification was given by Zadeh:

> System identification [is the] determination, on the basis of observation of input and output, of a system within a specified class of systems to which the system under test is equivalent. ([124])

Developed originally in the control community, the discipline expanded to other fields of science and engineering, now also including subjects like finance and network science [65, 119].

Regardless of the application and method selected, any system identification problem has five main components (Figure 1) - namely [66]:

- $\mathcal{X}$: The experimental conditions under which the data is generated
- $\mathcal{D}$: The data
- $\mathcal{M}$: The model class
- $\mathcal{I}$: The identification method, by which the "best" model $M \in \mathcal{M}$ is determined, based on the data $\mathcal{D}$
- $\mathcal{V}$: The validation process that determines whether the identified model M is fit for the intended purpose

In many cases, the model structure is known or assumed *a priori* (for example, based on physical principles), and only some parameters in the model are unknown; the model class is then parametrized as:

$$\mathcal{M} = \{M(\theta)|\theta \in D_{\mathcal{M}}\} \quad (1)$$

where the vector θ denotes the unknown parameters. In such cases, the system identification problem reduces to a *parameter identification* one - i.e., to determining the numerical values of θ. If a statistical framework is employed, the process is often referred to as *parameter estimation* [5].

As each of the five main components of system identification is a science in itself, treating all of them in detail would be impossible. This work focuses only on two components - namely, the model $\mathcal{M}$ and the identification method $\mathcal{I}$; some practical aspects related to experiment design, data analysis, and model validation are discussed briefly in section 6.

3 THE PREMISES FOR CHOOSING THE METHOD

Rotorcraft modeling presents several challenges due to the nonlinear, high-order and highly coupled dynamics and the complex physical phenomena involved. System identification is a powerful tool, as prior knowledge is typically not sufficient to build a usable model, so the model (or part of it) has to be identified from experimental data. Here again, rotorcraft pose special challenges, due not only to the complex dynamics, but also to the typical low signal-to-noise ratio and open-loop instability.

Given all these difficulties, there is no such thing as a "perfect" model or identification method: trade-offs have to be made, and the choice of the best solution depends on the final use of the model, as well as the vehicle at hand.

3.1 Modeled vehicle

The type of vehicle being modeled influences not only what constitutes an appropriate level of model complexity for a given intended use, but also how much *a priori* knowledge is available about the system. In fact, while the physics of conventional helicopters has been studied extensively and first-principles modeling techniques are well established, this may not be the case for other VTOL: for unconventional configurations, the underlying physical phenomena may not be fully understood, so physical modeling may be limited and data-driven solutions may be preferable.

For UAV and MAV, scaling effects may have to be taken into account before applying methods meant originally for full-scale vehicles (see for example [49]). Also, flight testing and model development budget are often be limited, and configuration changes are frequent - so simpler models and identification methods may be preferable.

3.2 Model intended use

Considering only models for rotorcraft flight mechanics applications, two main families of intended uses can be identified - namely, training and engineering simulation. The peculiarities and requirements of each are summarized below.

3.2.1 Pilot training

Flight simulation training devices (FSTD) allow pilots to practice manoeuvres and emergency procedures in a controlled environment, without the risks and costs associated with actual flight. Certification specifications such as EASA CS-FSTD(H) [26] define various classes of simulation training devices, with different levels of required fidelity; the highest qualification, "Level D", allows simulator hours to replace most of the flight hours required for a pilot's type rating or recurrent training.

As certification standards demand precise match between the model and flight test data, development and validation of flight dynamics models for Level D flight simulators require typically hundreds of test points and Qualification Test Guide (QTG) maneuvers to be collected [112]. Complications arise from the requirement for real-time simulation execution, which limits model complexity, and the fact that access to complete aircraft data may be limited. For these reasons, developers often rely on data-driven modeling or non-physical adjustments to tune models for accuracy.

3.2.2 Engineering simulation

Engineering simulation can be used to support and optimize a wide range of activities spanning the entire life cycle of an aircraft. In the context of today's efforts to make aviation more sustainable, its role is becoming ever more important: by providing an alternative to and limiting the need for flight tests, it contributes to significantly reduce risk, costs, and emissions; by allowing early problem detection as well as a fast and inexpensive way to test alternative concepts, it streamlines the design process and minimizes the number of iterations; finally, by partly replacing flight tests as a source of evidence, it can decrease the duration and costs of certification (see e.g. the recent EU-funded project RoCS, Rotorcraft Certification by Simulation, `https://www.rocs-project.org/`).

Model development can begin well ahead of first flight, using design data and theoretical knowledge, as well as results from numerical analyses (e.g. FEM, CFD, free-wake), wind tunnel and component tests as they become progressively available. Once flight tests are performed, system identification is usually employed to correct discrepancies between simulation model and experimental data.

Besides being able to represent as closely as possible the flight characteristics of the aircraft at hand, an engineering simulation should also guarantee high predictive capability, in particular when it comes to simulate untested flight conditions; this is critical for applications such as, e.g., analysis of flight mechanics and handling qualities, advanced AFCS design, envelope expansion, failure conditions analysis. Typically, physics-based mathematical models are used to represent the main vehicle components (rotors, fuselage, aerodynamic surfaces, etc.), including inertia characteristics and aerodynamic interference; simplifications such as reduced-order or semi-empirical models may be used to improve computational efficiency and allow real-time simulation. These models are usually high-order and highly nonlinear, and require high development effort. In recent years, an increasing impulse has also been towards data-driven and machine learning-based techniques; however, their use for engineering simulation is still limited, as the uncertainty when extrapolating outside the training domain is often very high, especially if highly nonlinear systems are involved [96].

One of the predominant uses of engineering simulation is to support the design of flight control systems (FCS). The selection of the most suitable mathematical models for this application depends greatly on the chosen control design method. If classic linear control theory is employed, linear flight dynamics models are often sufficient; such models are either extracted from a nonlinear engineering simulation model or identified from flight-test data when a prototype flight vehicle is available. On the other hand, specific tasks to be performed by the FCS may require that the nonlinearities of the aircraft are known, since they significantly impact closed loop dynamics and performances; a classic example are Limit Cycles Oscillations (LCO). Furthermore, model accuracy becomes crucial as the performance requirements for FCS become more and more stringent. The current tendency is therefore towards higher order models (typically also including rotor dynamics) as well as nonlinear models. Finally, specific requirements exist for applications such as adaptive control, where a mathematical model of the vehicle must be updated in real-time, or robust control, which imposes strict constraints on the acceptable model error.

4 MODEL CLASSES

The following sections provide a brief description of the most common model classes used in rotorcraft system identification.

As it is customary in system identification to work with discrete systems, dynamics equations will be written in discrete-time form; however, extension to continuous-time is possible. The treatment is valid for both single-input single-output (SISO) and multiple-input multiple-output (MIMO) systems - depending on whether the notations $u(t)$, $y(t)$, etc. are interpreted as scalars or vectors.

4.1 Linear models

One of the simplest ways to model rotorcraft dynamics is to approximate it by a linear model. As rotorcraft are inherently nonlinear systems, this operation must be done carefully: in fact, the best linear approximation is heavily dependent on both the applied inputs and the working point [96]. Typically, a linear model will be only accurate over a limited range in the neighborhood of the selected working point.

Nonetheless, linear models of reduced order are relatively easy to obtain and may be preferable for some applications, most notably flight control design [1].

The most common expressions for linear flight dynamics models are as transfer function or state space equations.

4.1.1 Linear state-space models

Linear state-space models are among the most widespread in aerospace applications (and in general in dynamic systems modeling). The classic expression (for discrete-time system) is [67]:

$$x(t_{k+1}) = A(\theta)x(t_k) + B(\theta)u(t_k) + w(t_k) \quad (2a)$$
$$y(t_k) = C(\theta)x(t_k) + D(\theta)u(t_k) + v(t_k) \quad (2b)$$

where u, x, and y are, respectively, the inputs, states, and outputs of the system, while $w(t)$ and $v(t)$ represent process and measurement noise.

In rotorcraft models, the unknown parameters θ are often representative of physical quantities - for example, stability and control derivatives, as in [58], or rotor dynamics parameters in [10].

4.1.2 Transfer function models

Transfer function models (also known as "input-output models") represent a dynamic system purely in terms of the relationship between its inputs y and outputs u. This model structure is less "transparent" than the state-space one; however, it is useful when the main focus is not on reconstructing the states, but rather on the end-to-end effect of the system

The class of transfer function models includes structures such as *Box-Jenkins, ARX, ARMAX*, and *Finite Impulse Response* (FIR) models, which are very common in the control community; some parameter identification techniques have been developed specifically for them, and can be found in classic textbooks such as [67], [105], and [123].

4.2 Nonlinear models

The variety of nonlinear models is huge. In fact, as Deistler [22] put it, "'identification of nonlinear systems' is like a statement about 'non-elephant zoology'".

A customary classification is based on how much prior knowledge (typically from physical principles) is included in the model: we thus have a "gray-scale" going from white-box (i.e., built entirely on physical principles), through gray-box, to black-box models (i.e., entirely based on experimental data) [65].

4.2.1 "Off-white": parametrized physical models

In physics-based rotorcraft models, the vehicle aerodynamic, propulsion, control, and inertial characteristics are modeled entirely from first principles. Those employed in engineering simulation are usually high-order, multi-body models, including detailed descriptions of the rotor(s) blades and inflow dynamics, fuselage and control surface airloads, and aerodynamic interference effects. A comprehensive treatment of rotorcraft physical modeling can be found in textbooks such as [25, 56, 82]; a review of state-of-the-art practices for flight mechanics applications, including both helicopters and unconventional configurations, is presented in the survey by Tischler et al. [112].

Often, the models can be reduced to a state-space form [67]:

$$x(t_{k+1}) = f(x(t_k), u(t_k), w(t_k), \theta) \quad (3a)$$
$$y(t_k) = h(x(t_k), u(t_k), v(t_K), \theta) \quad (3b)$$

where w and v account for process and measurement noise and $f(\cdot)$ and $h(\cdot)$ are nonlinear functions.

The unknown parameters to be identified can be either physical quantities for which little or no knowledge is available (e.g., mass and balance data, rotor dynamics constants, aerodynamic interference factors, etc.), or non-physical corrections (e.g., the force and moment increments proposed in [69]). Clearly, adding non-physical corrections reduces the predictive capability of the model. Examples of identification of physical rotorcraft parameters can be found in [51, 13, 46, 45] (time-domain identification) as well as [112, 10, 27] (frequency-domain identification).

4.2.2 "Light-gray": composite local models

The concept behind composite local models [55, 80] is to approximate nonlinear systems by different local (often linear) models, which are only valid in the neighborhood of specific working points, and then switch among the local models based on the actual working point. Typically, the working point is expressed in terms of some observed variable (or vector of variables), referred to as the *regime variable* or *scheduling variable* and denoted by ρ.

A classic example from the control community is that of Linear Parameter Varying (LPV) models, derived from gain scheduling methods, which consist of collections of linear models whose parameters depend on the regime variable ρ. The use of local linear models is particularly attractive for control applications, as linear control theory can be used for controller design. Two approaches are available to build LPV models [114]:

- A *global approach* consists in expressing the ρ-dependent parameters as parametric functions of ρ, then fitting all the parameters at once via nonlinear optimization. This approach allows for better fit to the data but is numerically rather expensive; it has been used in [116] to identify a LPV model of helicopter rotor blade dynamics, using the blade advance ratio as a regime variable.
- A *local approach* consists in first defining local linear models for a discrete set of working points, then interpolating them with respect to ρ to build the complete

model [126]. This approach may yield less accurate results but has the advantage of being computationally efficient and easy to update. A related technique, which interprets the local linear models as Takagi-Sugeno fuzzy models [108] has been applied in [88] to the study of a miniature helicopter, with translation and rotation velocities as regime variables.

The so-called *stitched models*, developed specifically for aircraft and rotorcraft applications, can also be classified as composite local models. A model stitching simulation architecture [111, 113] is made up of:

- a series of linear state-space perturbation models, each representing the aircraft dynamic response for a specific flight condition and configuration;
- a set of trim point data, recording the variation of trim states and controls over the full envelope;
- nonlinear equations describing the aircraft 6-DoF dynamics as well as the gravitational forces acting on the CG.

The linear perturbation models and trim data are stored in lookup tables to be interpolated as functions of airspeed and altitude; they are then combined with the nonlinear equations, resulting in a a continuous, quasi-nonlinear, full-envelope model. The nonlinear equations of motion and inertia model allow extrapolation to off-nominal loading configurations and flight conditions.

The stitched model approach has become popular in models for training and control design applications, and is also well suited to the simulation of unconventional rotorcraft and UAVs, as a means to obtain nonlinear full-envelope models even without full knowledge of the underlying physics. Examples can be found, among others, in [112], [10], [21], and [38]; in a recent work [100], the linearization is applied only to rotor dynamics, and all the other components (equations of motion, inertia, airframe and control surfaces aerodynamics) are modeled by nonlinear equations.

4.2.3 "Dark-gray": block-oriented models

In system identification, the term *block-oriented model* refers to a model made up of interconnected components having different properties (e.g., linear and nonlinear) [125].

A frequent assumption is that each model component must be either a linear dynamic model or a static nonlinearity (i.e., a simple nonlinear algebraic equation). The most common models belonging to this family are the so-called *Wiener models* and *Hammerstein models*, together with their combinations (Figure 2). This models can be identified via dedicated methods (see, e.g., [6, 36, 122, 35, 7]), and may sometimes be interpreted in a physical sense: for example, a Hammerstein-Wiener model can well approximate a system stabilized around a working point (hence having a nearly linear behavior), but whose actuators and sensors show significant nonlinear effects [65].

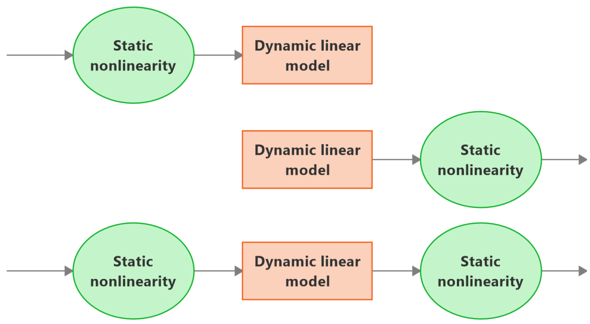

Figure 2: Classic block-oriented models: top: Hammerstein model; center: Wiener model; bottom: Hammerstein-Wiener model.

Another block-oriented approach has been described in [112] and [92] and consists in combining low-order, linear black-box models with a baseline (typically nonlinear) rotorcraft model, in order to ensure a better fit to flight test data. This method can be employed as a quick and straightforward means to update dynamic models for training and control design, without any modifications to the structure of the baseline model.

4.2.4 "Black": basis function expansion

Nonlinear black-box models do not rely on any prior knowledge and use instead a general parametrization, capable (ideally) of approximating any possible system behavior. The most common parametrizations include [125, 81]:

- polynomial and rational functions
- splines and B-splines
- wavelets
- multi-layer perceptron networks (e.g., using sigmoid functions)
- radial basis networks (e.g., using Gaussian or other bell-shaped functions)
- fuzzy-neural networks

Typically, these models can be reduced to the form [101]:

$$y(t) = g(\phi(t), \theta) + v(t) \tag{4}$$

where $v(t)$ is an additive error term and $\phi(t)$ is a function of observed inputs and outputs up to the time $t-1$. The nonlinear function $g(\cdot)$ is usually parametrized as a basis function expansion, e.g.:

$$g(\phi(t), \theta) = \sum_{l=1}^{m} \gamma_l \kappa(\alpha_l(\phi(t) - \beta_l)) \tag{5}$$

where $\kappa(\cdot)$ is a "mother" basis function which is first dilated and translated by α and β, then weighted by γ in the sum forming $g(\cdot)$. The set of all coefficients $\{\alpha, \beta, \gamma\}$ constitutes the model parameters to be identified.

Nonlinear black-box models are becoming more and more popular thanks to the increasing computational power of modern computers. Examples of neural and radial basis network models for rotorcraft system identification can be found, for instance, in [40, 16, 24]. B-splines are a common choice to approximate stall regions and nonlinear aerodynamics functions [19, 15]; however, they have been also applied to UAV dynamics modeling: for instance, in [118] they were employed to model the thrust/torque/rotor inflow dynamics of a quadrotor.

5 IDENTIFICATION METHODS

The following sections provide an overview of available methods for rotorcraft system identification. The various techniques have been grouped by their use, starting from prediction error family - which is by far the most widely used and is applicable to most model structures - then moving on to more specific methods for identifying linear or nonlinear systems or dealing with uncertainty quantification problems. Section titles try to reflect practical questions, such as "how can one deal with unstable systems in time domain?".

5.1 Prediction error methods

Most techniques used in aircraft applications belong to the family of *prediction error methods* [61, 50, 111]. These are *parameter identification* methods, i.e., assume that a model structure (1) has already been established, and the only unknowns left to determine are the numerical values of the parameters θ.

At the basis of prediction error methods is the idea that a model of a system - for example, the nonlinear one from (4) - can be seen as a means to predict future outputs based on past data [67]:

$$\hat{y}(t|\theta) = g(t, \theta, Z^N) \tag{6}$$

where Z^N is a set of observed input and output data:

$$Z^N = \{u(t_1), y(t_1), u(t_2), y(t_2), ..., u(t_N), y(t_N)\} \tag{7}$$

The error between the model's prediction and the actual system output is then:

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t|\theta) \tag{8}$$

A natural criterion to choose the model parameters θ is to select the ones yielding the smallest possible error ε. This is the core of prediction error methods.

Usually, the "size" of the prediction error is measured through a scalar norm $\ell(\varepsilon(t, \theta))$, and the performance of the model over the whole data set is quantified by a cost function [67]:

$$J(\theta) = \sum_{k=1}^{N} \ell(\varepsilon(t_k, \theta)) \tag{9}$$

The "best" parameters $\hat{\theta}$ are then found by minimizing (9):

$$\hat{\theta} = \hat{\theta}(Z^N) = \underset{\theta \in D_{\mathcal{M}}}{\text{minimize}}\, J(\theta) \tag{10}$$

which essentially becomes a numerical optimization problem.

The form (10) is common to all prediction error methods [67]. The various techniques differ in the choice of the norm $\ell(\cdot)$ and cost function V_N, and in the assumptions about system disturbances and properties of the error ε.

5.1.1 The simplest approach: Least squares

The simplest prediction error criterion is built by choosing a quadratic norm:

$$\ell(\varepsilon) = \frac{1}{2}\varepsilon^2 \tag{11}$$

Which results in a cost function:

$$\begin{aligned} J_{\mathsf{LS}}(\theta) &= \frac{1}{2}\sum_{k=1}^{N} \varepsilon^2(t_k) \\ &= \frac{1}{2}[y(t_k) - \hat{y}(t_k|\theta)]^T [y(t_k) - \hat{y}(t_k|\theta)] \end{aligned} \tag{12}$$

This approach is known as the *Least Squares (LS)* or *Equation Error* method [60, 67, 50]. It can be applied in either time- or frequency-domain, as will be discussed in detail in section 5.1.3.

An attractive characteristic of this technique is that, if the model at hand is *affine in the parameters* - i.e., can be reduced to a linear parametrization of the type:

$$\hat{y}(t|\theta) = \phi^T(t)\theta + \mu(t) \tag{13}$$

then the cost function (12) is quadratic in θ and can be minimized analytically. This means that the parameters can be calculated in a single step, as:

$$\hat{\theta} = \left[\frac{1}{N}\sum_{t=1}^{N} \phi(t_k)\phi(t_k)^T\right]^{-1} \frac{1}{N}\sum_{t=1}^{N} \phi(t_k) y(t_k) \tag{14}$$

It should be noted that the concept of affinity in the parameters is not connected to the model being linear or not, so the condition in (13) is also satisfied by many nonlinear models; for example, the following simple model for an airfoil pitching moment coefficient [61]:

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_M} M + C_{m_\alpha M}\alpha M \tag{15}$$

is a nonlinear function of angle of attack α and Mach M, but is affine in the parameters $C_{m_{(\cdot)}}$. For models not satisfying (13), the cost function minimization problem must be solved iteratively.

A slight refinement of the LS approach (the so-called *Weighted Least Squares*, WLS) involves assigning different weights to the individual data points - usually, based on their relative precision or on the degree of confidence [50]; the cost function then becomes:

$$J_{\mathsf{WLS}}(\theta) = \frac{1}{2}[y(t_k) - \hat{y}(t_k|\theta)]^T W [y(t_k) - \hat{y}(t_k|\theta)] \tag{16}$$

where W is a weighting matrix.

The main advantage of LS methods lies in their simplicity and in the existence of a closed-form analytical solution for many model structures, which can be computed quickly. Thanks to this feature, the LS approach is well suited for on-line applications where model parameters must be calculated/updated in real time, as well as for dealing with unstable aircraft, since the solution is calculated without integrating the dynamics equations [18]. Examples of rotorcraft applications can be found, among others, in [2, 88, 97] (time-domain) and [76, 120] (frequency-domain). Another classic use of these techniques is as a preliminary step in the parameter identification process, to evaluate different model structures (see section 6.2.1) or to compute initial parameter guesses and initialize more sophisticated identification algorithms [32, 107].

A major limitation of LS methods is that they often perform poorly in the presence of noise and measurement errors, resulting in asymptotically biased and inconsistent parameter estimates (i.e., not converging to the true values) [50]. Unless very high-quality measurements are available, one way to obtain more accurate results is to pre-process the data in a *flight path reconstruction* step prior to the actual parameter identification; more details on this approach are given in section 6.2.

5.1.2 Using a statistical framework: Maximum Likelihood methods

It is often convenient to embed the prediction error approach in a statistical framework, so that results from statistical estimation theory can be exploited [67].

One of the first families of parameter estimation methods to be developed, and one of the most widely used for aircraft, is that of Maximum Likelihood methods [4]. These are based on the assumption that minimizing the prediction error is equivalent to maximizing the joint probability:

$$p(y|\theta) = \mathbb{L}(\theta) \tag{17}$$

i.e., the probability that the outputs y (seen as realizations of stochastic variables) are observed, given a certain value of the parameter vector θ; this is called the *likelihood function* [67].

Formulating the likelihood function requires some kind of assumption about the statistical properties of the prediction error. A classic hypothesis is that the error is a Gaussian process with zero mean and covariance matrix R, leading to the well known *Output Error* method (OE) [48]:

$$\hat{\theta} = \underset{\theta \in D_{\mathcal{M}}}{\text{minimize}}\, J_{\text{ML}}(\theta) \tag{18}$$

with:

$$\begin{aligned} J_{\text{ML}}(\theta) &= -\ln\left(\mathbb{L}(\theta)\right) = \\ &= \frac{1}{2}\sum_{k=1}^{N}[y(t_k) - \hat{y}(t_k|\theta)]^T R^{-1}[y(t_k) - \hat{y}(t_k|\theta)] + \\ &\quad + \frac{N}{2}\ln\left(\det\left(R\right)\right) + \frac{N n_y}{2}\ln\left(2\pi\right) \end{aligned} \tag{19}$$

where N is the total number of samples and n_y is the number of output variables.

The minimization of $J_{\text{ML}}(\theta)$ is a nonlinear optimization problem and is usually solved employing an iterative numerical optimization method (e.g., Gauss-Newton, Levenberg-Marquardt, etc. [103]). Typically, since both the error covariance R and the parameter vector θ are unknown, a relaxation technique is used, where at each iteration first an estimate of R is computed, then θ is optimized for that R; the process is repeated until both R and θ reach convergence. The initial guess for θ can be derived from prior knowledge or computed using Least Squares.

Compared to LS, the Output Error method allows a better match to measured outputs and produces parameter estimates with more favorable asymptotic properties [76]. Also, it can be applied indiscriminately to any model structure and in both time- and frequency-domain. On the other hand, OE is more complex and computationally more expensive, requiring an iterative, nonlinear optimization process in which the model equations of motion are solved at each iteration to compute the predicted outputs. Difficulties and numerical instabilities in the optimization procedure may arise due to poor choices of the initial parameter values, or when dealing with unstable aircraft [18].

In literature, there are countless examples of the use of Maximum Likelihood methods for rotorcraft parameter estimation; recent surveys, including both time- and frequency-domain approaches, can be found in [10], [18], and [21].

5.1.3 Time domain vs. Frequency domain

Although the expressions reported so far assume that time-domain data are used, both LS and Maximum Likelihood methods can be applied just as easily to frequency-domain data such as Fourier transform data or frequency response data. For example, a typical formulation of the Weighted Least Squares cost function is [111]:

$$\begin{aligned} J_{\text{WLS}}(\theta) = \frac{20}{n_\omega}\sum_{k=1}^{n_\omega} W_\gamma \Big[& W_g\big(|T(\omega_k)| - |\hat{T}(\omega_k,\theta)|\big)^2 + \\ & + W_p\big(\angle T(\omega_k) - \angle\hat{T}(\omega_k,\theta)\big)^2 \Big] \end{aligned} \tag{20}$$

where $T(\omega)$ indicates a frequency response function, and different weighting factors are applied to gains and phases to make their errors comparable.

The choice between time- and frequency-domain is critical and problem-specific, as both domains have advantages and limitations that make them more or less suitable for a given model structure or final use.

Frequency-domain techniques are particularly suitable for rotorcraft identification, thanks to their robustness to high noise and vibration levels, and straightforward applicability to unstable dynamic systems; their computational cost is also usually lower compared to time-domain, as the number of data points is limited and the solution to differential equations involves only algebraic operations. Finally, working in the frequency domain is usually more convenient for applications such as control system design and handling qualities analysis [111]. Due to these practical advantages, frequency-domain methods are by far the most commonly used by rotorcraft manufacturers and research institutions, as shown in a recent survey by the NATO STO AVT-296 Research Task Group [112]. The frequency response-based software suite CIFER® [111, 10] is also widely used.

An important limitation of frequency-domain techniques is that they are essentially limited to linear models. While there are many examples of frequency-domain identification applied to nonlinear models, including the update of physics-based engineering simulations [112, 10, 27], all methods ultimately rely on a local linear approximation of the nonlinear model, which is typically valid only within a limited range of the working point and may vary depending on the inputs applied [96]. In addition, any losses of information in the transformation from time to frequency domain (for example, the loss of biases and linear trends) result in missing or erroneous representation of the corresponding dynamic effects. Unless all these factors are properly accounted for, the parameters identified in the frequency domain risk being far from optimal in a nonlinear framework.

Unlike frequency-domain methods, time-domain techniques do not pose any theoretical limitations regarding the identification of nonlinear systems. However, practical issues arise, due mainly to two aspects:

- When applied to nonlinear systems, the optimization problem connected to parameter identification is also nonlinear and often ill-posed [53, 61]: in addition to requiring higher computational effort, the optimization algorithm may reach a local instead of a global minimum, or not converge altogether.
- In time domain, classic methods such as OE involve numerical integration of the system dynamics equations; for inherently unstable and highly sensitive systems, this can lead to numerical divergence in both the simulation and the optimization [50]. On the other hand, more general methods such as filter-error (see section 5.1.4) may be impractical due to excessive computational cost, as well as convergence and identifiability problems [61].

For these reasons, traditional time-domain parameter identification techniques are widely applied to fixed-wing aircraft (see for example [50, 61]), but remain uncommon for rotorcraft and are mostly limited to the identification of simple, lower-order models as in [58]. Two rare examples of the use of classic time-domain OE to identify a nonlinear, physics-based helicopter model can be found in [51] and [46]. Possible approaches to overcome the limitations of time-domain identification of unstable systems are discussed in 5.1.6.

5.1.4 Dealing with unknown states

Most dynamic system models can be expressed in a state-space form ((2) or (3)), where outputs are calculated explicitly as a function of the state variables. The Output Error method described in section 5.1.2 assumes that the states can be computed without error, by integrating the model dynamics equations. However, this is not always feasible, as some states may be difficult to observe (this is a common issue with rotor states, for instance) or corrupted by noise (e.g., if flight data are collected in turbulence).

Sometimes it may be possible to reduce state errors in a data pre-processing step (see 6.2): for example, in the case of turbulence, wind components may be derived from other measurements and used to correct the data [50, 115]. If pre-processing the data is not enough, a parameter identification method is needed that can account not only for measurement errors (i.e., errors in the outputs) but also for process errors (i.e., errors in the states): the most common one is the so-called *Filter Error* method (FE) [73, 40].

FE is a maximum likelihood parameter estimation method in which model states are assumed to be stochastic quantities and must be estimated using a state estimation technique. The resulting iterative process similar to the OE process, but now the states are also estimated at each step and used to compute the output predictions [50].

The additional state estimation task requires assumptions about the statistical properties of the process errors and the selection of an appropriate estimator. The classic choice is to assume Gaussian process errors and use a Kalman filter to estimate the states. For nonlinear models, an extended Kalman filter (essentially based on a local linear approximation of the model equations) is typically used [40, 50]; however, a recent trend is toward sequential Monte Carlo methods such as particle filters and particle smoothers, which are computationally more expensive but better suited for nonlinear/non-Gaussian systems [93, 64].

Besides the capability to deal with unknown/noise-corrupted states, another advantage of the FE method is that it is less sensitive to stochastic disturbances and dynamically unstable systems, often resulting in a more nearly-linear optimization problem with fewer local minima and faster convergence rate [50]. The combination of FE with (extended) Kalman filter can also be adapted into a recursive algorithm and used for real-time parameter estimation [61].

A major drawback of FE is its increased complexity, particularly the risk of convergence and identifiability issues arising from low information content in the data relative to the large number of unknowns to be estimated [61, 76]. In addition, the method is effective for estimating the process noise distribution matrix for a single flight maneuver or for tests carried out in similar conditions, but can become cumbersome when applied across multiple experiments. Finally, while FE always ensures a good match of the dynamic responses, it does not guarantee the accuracy of the model or parameters: in fact, the method may potentially mask discrepancies due to neglected effects and conceal the need for model improvements [50].

Similar to FE, but more general, is the *Expectation Maximization* approach proposed by Dempster, Laird, and Rubin [23]. This method assumes that the observed outputs are only "incomplete data", and that the maximum likelihood function would be easier to compute if certain unobserved "latent variables" were known [66]; these latent variables may be states, as in FE, but can also be defined in a more general way [93]. The iterative algorithm alternates between an "expectation" step estimating the likelihood function (based on assumptions about the latent variables), and a "maximization" step optimizing the parameters.

5.1.5 Dealing with input errors

The parameter identification techniques described so far assume that errors or measurement noise affect only the system outputs, while the inputs are error-free. This is not always the case - for example, control input deflection measurements may be noisy due to poor sensor quality and/or high vibration levels - and has a significant impact on physical modeling, making it difficult to gain physical insight into the system [104].

Identification techniques that consider both input and output errors are usually called *Errors-in-Variables* (EIV) methods. They often derive from extensions of classic methods such as LS, OE, and FE [50, 86]. As the identification problem becomes more difficult, they also typically require restrictive assumptions about the properties of the measurement errors [95]. A comprehensive summary of the different EIV methods and their properties can be found in [104].

5.1.6 Time-domain identification of unstable systems

As discussed in 5.1.3, time-domain methods cannot be easily applied to unstable system (such as most rotorcraft), due to diverging solutions in both the simulation and the optimization process. The only exception is the Least Squares method, which derives the input-output behavior purely from the measured data, without requiring the integration of the state equations; however, this method is not always a viable option due to the limitations seen in 5.1.1.

Therefore, the classic approach to dealing with unstable systems has been to use either the Filter Error method or some *ad hoc* modifications of the Output Error. These modifications include:

- The *regression startup* procedure [50], which reformulates the OE problem as a LS problem by incorporating the measured states as control inputs.
- The *equation decoupling* method [87], which uses state measurements to reformulate the model in such a way that each differential equation can be integrated independently.
- Artificial stabilization of the OE algorithm by applying an arbitrary correction to the states, typically proportional to the fit error [70].

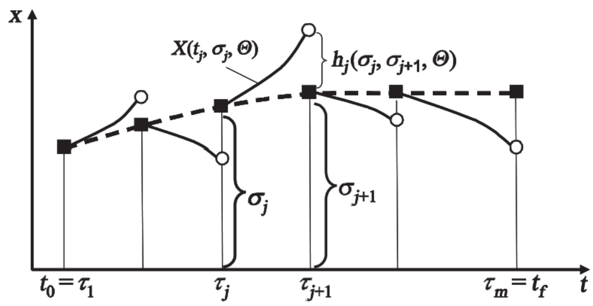

Figure 3: Basic principle of the multiple shooting method citejategaonkar2015

As pointed out in [52], while these techniques can be effective in certain situations, they are not universally applicable: in particular, the first two rely on the availability of high-quality state measurements, and the third requires substantial engineering judgment.

More recently, a promising approach has been derived from optimal control techniques. The method is based on the concept of multiple shooting [12] (Figure 3): rather than integrating the system equations over a whole maneuver, the time domain is subdivided into a series of intervals and the system equations are integrated only within each interval; this reduces the chances of simulation divergence caused by the dramatic amplification of small perturbations. To still ensure continuity of the states over the whole time domain, "gluing" conditions are added which constrain the states at the interface between intervals to be equal; this turns parameter identification into a constrained optimization problem, with the unknowns including not only the parameters, but also the state values at the boundaries between shooting segments.

In a recent work by Göttlicher [31], the multiple shooting approach is driven to the extreme of a full discretization, where as many shooting segments are taken as there are sample points. This method has been successfully applied in [45] to the identification of a nonlinear physics-based model of an ultralight helicopter. Although the full discretization technique shows good convergence properties and can seamlessly handle unstable dynamics, it also requires considerable computational effort due to the large size of the optimization problem; as also pointed out in [31], this can be a major drawback when dealing with large amounts of flight data. In such cases, hybrid approaches may be a better solution: for instance Bottasso, Luraghi, and Maisano [13] proposed a single-multiple shooting method in which multiple shooting is applied only to the flight mechanics states, while the rotor states are left unconstrained.

5.1.7 Exploiting auxiliary information

Sometimes, in addition to dynamic response data, the user may have access to other prior information about the system being identified. A common scenario is that the user already has some knowledge about the parameter values: for example, in the case of aerodynamic derivatives, approximate values may already be known from wind tunnel tests. Incorporating this prior knowledge, typically referred to as *auxiliary information*, into the parameter identification process is often beneficial and can both result in a better defined nonlinear optimization problem and reduce the variance of the identified model due to uncertain data [54]. The theoretical basis for this comes from regularization theory [110, 81].

When an *a priori* value θ^* of the parameter vector is available, a classic approach [66] is to modify the cost function of the chosen identification method by adding a quadratic norm:

$$J^*(\theta) = J(\theta) + \lambda(\theta - \theta^*)^T \Lambda(\theta - \theta^*) \tag{21}$$

where λ is a scaling factor and Λ is a weighting matrix. Basically, the modified criterion adds a penalty if the estimated parameter vector is too far from the nominal value; the amount of penalty is tuned by λ and Λ.

If, in addition to θ^*, also the probability distribution of the θ is known/assumed, the Bayesian approach allows to account for auxiliary information in a probabilistic way. This approach extends the Maximum Likelihood framework by also treating the parameters random variables. The parameters are then estimated by maximizing:

$$P(\theta|Y) = \frac{P(\theta \cap Y)}{P(Y)} = \frac{P(Y|\theta)P(\theta)}{P(Y)} \tag{22}$$

i.e., the *a posteriori* probability of θ given the observations y; for this reason, the method is also known as *maximum a posteriori estimation* (MAP) [67, 84, 61]. The resulting cost function is similar to (21), where $J(\theta) = J_{\mathsf{ML}}(\theta)$ is the log likelihood function from (19) and λ and Λ derive from the statistical properties of θ.

Other types of auxiliary information can also be incorporated into the prediction error framework, usually either as additional objectives or as constraints on the optimization; a detailed discussion can be found in [53] and [54].

5.2 Methods for linear systems

5.2.1 Nonparametric frequency response identification

A simple frequency response characterization of the aircraft is often sufficient for many tasks in handling quality analysis or classical flight control system design [111]; for these applications, a simple nonparametric model may be enough. Such a model can be constructed in various ways, as discussed (among others) in [111, 86, 85]; one often used method is based on spectral analysis and is summarized below.

The first step in the spectral analysis method is to calculate the auto- and cross-spectra of the inputs u and outputs y (G_{xx}, G_{yy}, G_{xy}). This is typically done by applying the fast Fourier transform or the Chirp-Z transform to the measured time-domain data; criteria for selecting window sizes for spectral averaging can be found in [111, 61]. At each frequency point f, the frequency response function $H_{xy}(f)$ can then be estimated directly from these spectra; for example, in the Single-Input/Single-Output (SISO) case:

$$H_{xy}(f) = \frac{G_{xy}(f)}{G_{xx}(f)} \tag{23}$$

The concept can be extended to MIMO (Multiple-Input/Multiple-Output) systems as well [111, 10]; if highly correlated inputs are present, the Joint Input-Output technique proposed in [10] can be used in a post-processing step to improve the quality of the estimated frequency response.

Despite having an increased variability compared to parametric models, nonparametric frequency response estimates provide insight into the aircraft (linear) input-output behavior for a large number of discrete frequencies without requiring any information about the model structure or parameters. Therefore, they are often used to get an initial idea of the system complexity as well as for model validation purposes (e.g., to detect unmodeled dynamics) [85]. A useful tool in this sense is the coherence function (here in the SISO formulation):

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{|G_{xx}(f)||G_{yy}(f)|}, \qquad 0 \le \gamma_{xy}^2(f) \le 1 \tag{24}$$

which measures the linearity between inputs and outputs and can be used as a metric to evaluate whether a linear model is adequate to characterize the dynamic system. $\gamma_{xy}^2(f) = 1$ means that process is perfectly linear, while $\gamma_{xy}^2(f) < 1$ implies that either nonlinearities are present or the outputs are contaminated by measurement error, process error, and/or unmeasured or secondary inputs [111, 61]; Tischler and Remple [111] suggest $\gamma_{xy}^2(f) \ge 0.6$ as an indication of an acceptable linear model.

Another common practice in aircraft frequency-domain parameter identification is to start the procedure with a nonparametric frequency response model, then identify the parameters by fitting a parametric frequency response curve to the nonparametric one [9]. This is, for instance, the approach used in the CIFER software [111, 10].

5.2.2 Identifying black-box state-space models: Subspace methods

Subspace methods [43] address the problem of identifying a linear state-space model (2) of a MIMO dynamic system when no insight into the internal structure is available. The goal is to determine a *minimal state-space representation* of the system - i.e., to find a set of matrices A, B, C, and D (together with the covariance matrices of the noise sequences $w(t)$ and $v(t)$) that give a good description of its input-output behavior, and where the matrix A has the minimal dimension [30, 67].

The basic subspace algorithm consists of two steps [59]:

1. To build a coordinate basis for the state-space model, an estimated state vector is constructed from the input-output data. This step exploits results from realization theory [43] and typically uses a state estimator (e.g., Kalman filter) to compute the states.
2. The matrices of the state-space model are calculated by solving a least-squares problem.

These steps can be implemented in a number of different ways; a survey of available techniques can be found for instance in [117] and [59].

The key advantage of the subspace procedure is that it eliminates the need to solve a nonlinear optimization problem and requires the user to choose only a few design parameters, resulting in a non-iterative process with higher numerical robustness and efficiency compared to most prediction-error methods, especially when dealing with high-order MIMO systems. The main drawbacks derive from the "black-box" nature of the identified models and their often sub-optimal statistical properties, typically resulting in higher variance compared to models identified by other methods [117]. For this reason, subspace techniques are often used as an initialization step for prediction-error or other parameter identification methods; examples of such applications to helicopter models are given in [9] and [63].

5.2.3 Correlation approaches

Correlation approaches to parameter identification operate on a principle similar to prediction error methods; however, rather than minimizing a specific error norm, these techniques define a "good" model as one whose prediction error is uncorrelated with the available information [67].

The most common methods in this class are called *Instrumental Variable* (IV) methods and calculate the model parameters as:

$$\hat{\theta} = \operatorname*{sol}_{\theta \in D_{\mathcal{M}}} \left[\sum_{k=1}^{N} \varepsilon(t_k, \theta) \zeta(t_k, \theta) = 0 \right] \tag{25}$$

where $\varepsilon(t,\theta)$ is the prediction error, while the sequence $\zeta(t,\theta)$ is a set of *instrumental variables* constructed as function of the observed data and possibly of the parameters [66]. Possible methods to define $\zeta(t,\theta)$ are discussed in [67, 106, 123]; in general, these variables should be strongly correlated with the model independent variables but uncorrelated with the noise. Most procedures are based either on *a priori* values of the parameters θ (incorporated with a filter or estimated via LS) or on time-delayed values of the inputs.

Similarly to Least Squares methods, IV techniques allow the parameters to be calculated in a single step using simple simple matrix algebra operations. However, the use of IV overcomes some limitations of LS and yields asymptotically unbiased estimates even in the presence of noise. The main disadvantage of these methods, which has so far prevented their widespread use in aircraft parameter identification, is due to the difficulty of defining appropriate sets of instrumental variables [50].

5.3 Identifying nonlinear black-box models: Machine learning techniques

Thanks to the availability of larger computing resources and their ability to process and identify patterns in vast amounts of data, machine learning techniques are becoming increasingly popular. The core of machine learning is essentially function estimation - i.e., inferring input-output mappings from empirical data, where the inputs could be signals, images, texts, etc. and the outputs could be numerical values or classifications [89, 94]. The similarities with system identification problems are evident - in fact, the problem of identifying a dynamic system could be seen as a special case of machine learning called *sequence learning* [94].

A general survey of machine learning methods for system identification is given in [68] and [94], among others; for aircraft applications, the main techniques have been reviewed in [40] and [24]. Most methods are based on *Artificial Neural Networks* (ANN or NN), which provide a nonlinear mapping of input-output data through a set of flexible basis functions (called *nodes* or *neurons*) defined over an adaptive number of grid points in the input space [40, 94].

The unknown parameters in the nodes' basis functions are *learned* from the data in the model *training* process, typically using a stochastic optimization algorithm. The scheme is similar to prediction-error methods, where the parameters are determined by minimizing a cost function, usually defined in terms of prediction error or entropy loss.

The nodes can be arranged in a number of different architectures; the most promising are:

Feedforward NN (FNN) This widely used architecture is characterized by the unidirectional flow of information from the inputs to the outputs; the nodes are typically defined as in (5) - where classic choices of the basis function κ (often called *activation function*) include the sigmoid ($\kappa(x) = 1/(1+e^{-x})$) and the rectified linear unit (ReLU) ($\kappa(x) = \max(x, 0)$). The popularity of FNNs is due their flexibility and to the fact that they are relatively fast and easy to train, although not always capable of extracting complex features from the data; applications to aircraft dynamics prediction are reported in [24].

Recurrent NN (RNN) In these NNs, outputs are expressed as function of internal hidden states, which are evolved in time through equations describing the dynamics of the network. Modifications such as *Long Short-Term Memory (LSTM) models* [44] and *Gated Recurrent Unit (GRU) models* [17] also allow efficient handling of long-term dependencies. The RNN approach is conceptually similar to state-space models and promises to be particularly suitable for modeling dynamic systems. The training process, however, is challenging and significantly affects the final model performance [40]. As a result, applications to aircraft parameter identification have been limited; some examples are mentioned in [24].

Convolutional NN (CNN) This architecture is based on hierarchical convolution and is frequently used for applications such as image processing, object detection, and natural language processing [24, 8]. Recent work on a modified version of CNNs called *temporal convolutional networks* has shown that these may be suitable for sequence learning tasks and may even outperform RNNs [8]. A major limitation is that these NNs may need pre-training and require in general a relatively slow and computationally heavy training process [24].

Lagrangian NN This approach combines machine learning with classic physical principles by embedding Lagrangian mechanics into the neural network architecture. By doing so, the network is constrained to generate only physically plausible solutions, while still retaining its capacity to fit complex nonlinear systems. Compared to other NNs, the Lagrangian approach is expected to provide better network interpretability, faster convergence, and better performance for inputs outside the training domain. A recent application to modeling and control of a quadrotor UAV is presented in [14].

The NN concept can be further extended by concatenating several *layers*, each consisting of one of the architectures described above, in a hierarchical model to form a *deep neural network* (DNN). The learning process is then called *deep learning* and has proven to be considerably more efficient than using a single layer; furthermore, although DNNs have a slightly less flexible structure than their single-layer counterparts, they are much less sensitive to variability in the training data, resulting in models with a better bias-variance trade-off [94].

An alternative to NNs for identifying nonlinear dynamic systems is Support Vector Regression (SVR), a variant of the Support Vector Machine (SVM) method. This machine learning technique uses structural risk minimization to minimize an upper bound of the error on the training data [102]. The advantage over NNs is that, while the latter can suffer from local minima and overfitting, SVR provides a unique solution and is thus easier to apply in practical scenarios. The method has been used with promising results in [72] for the identification of a helicopter training simulation model.

5.4 Uncertainty-based methods

Usually, parameter estimation methods also provide metrics for assessing the quality of the estimated model, based on the assumed statistical properties of the measurement error [71]. A classic example from maximum likelihood estimation is the *Fisher information matrix* [50]:

$$\mathcal{F} \approx \sum_{k=1}^{N} \left[\frac{\partial y(t_k)}{\partial \theta}\right]^T R^{-1} \left[\frac{\partial y(t_k)}{\partial \theta}\right] \qquad (26)$$

which provides a lower bound for the covariance of the estimated parameter with respect to their "true" values [67, 50]. However, these metrics have two important limitations:

- They rely on specific hypotheses that may not always hold true in reality. For instance, using a Gaussian framework to describe process and measurement noise becomes unrealistic when structural model errors dominate over random ones, as is common with nonlinear models. Although some methods have been proposed [42, 77, 31], determining reliable uncertainty bounds for these models remains an open research challenge [96].
- They are often difficult to test, especially since it is generally impossible to have complete knowledge of the "true system" [67, 71].

The above limitations are undesirable for applications such as robust control, where keeping model error within acceptable limits is essential [30]. As a result, specialized identification techniques such as *set membership* and *worst-case* methods have been developed which make no assumptions about the nature of the error, but only impose an upper hard bound [20, 71]. In a recent study [121], this deterministic approach was successfully applied to helicopter flight dynamics modeling, also demonstrating good convergence properties. The main drawback is that, by considering the worst case of identification uncertainty, the obtained results are typically more conservative than those produced by traditional methods [30].

6 SOME PRACTICAL ISSUES

The practical application of rotorcraft system identification requires a multidisciplinary approach that integrates modeling and simulation, flight testing, measurement, data analysis and optimization techniques (Figure 4). Although the focus of this paper is mainly on the choice of models and identification methods, practical aspects from all steps in the process must be taken into consideration; the following sections provide a brief overview of some of these aspects, along with references for further reading.

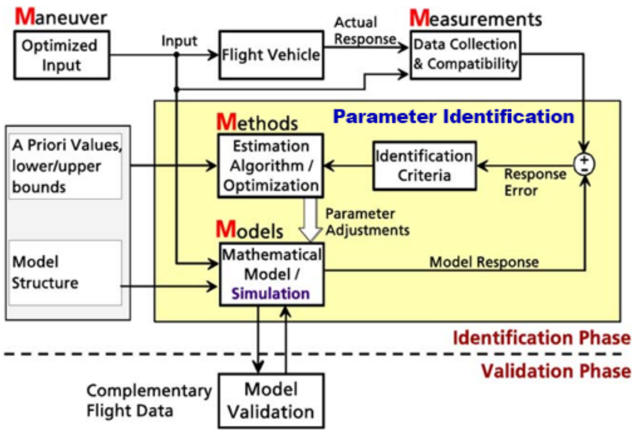

Figure 4: Basic scheme of a generic flight vehicle parameter identification process [50, adapted]

6.1 Experiment design

The success of any system identification activity depends ultimately on how much information is contained in the data. The task of experiment design is therefore critically important.

Flight test design for rotorcraft system identification has been covered extensively in the literature [57, 79, 98], with dedicated maneuvers and optimal input design methods developed to maximize the information content in the data [62, 78, 91, 109]. The topic has also been treated in a few AGARD lecture series [28, 37, 41], and useful guidelines are provided in classic textbooks such as [50, 61, 111].

The following general considerations should always be kept in mind [1, 96]:

- The experiment should be as close as possible to the conditions of interest for the intended model application.
- All the relevant dynamics should be contained in the data; unless physical modeling is used, missing dynamics in the data will not appear in the model.
- The design of experiments depends on the type of model to be identified. For example, when identifying a linear model, inputs should be kept small to avoid exciting a nonlinear response; whereas for a nonlinear model, since the goal is precisely to observe nonlinearities, larger amplitudes and/or different input shapes are used.

Finally, sometimes a model may have to be identified from pre-existing data, collected from tests not specifically designed for system identification. In this case, rather than applying proper design of experiments, it is necessary to analyze the data and extract meaningful windows for the identification; some techniques are presented, e.g., in [11] and [90].

6.2 Data consistency and flight path reconstruction

As has been pointed out in section 5, some problems in the data, such as errors in the state or input variables which require the use of more advanced system identification methods, can potentially be removed by pre-processing the data in a *flight path reconstruction* (FPR) step.

The purpose of FPR is to detect instrument errors and ensure the consistency of the measured variables. This is achieved by comparing signals from different sensors using the well-defined equations of aircraft motion: for example, measurements of airflow angles (angle of attack, sideslip) can be compared to those reconstructed from linear accelerations and angular rates, typically obtained from higher-quality sensors. The practical implementation of this approach involves either the estimation of parametric sensor error models (using classic methods such as LS or OE [50, 61, 111]), or a more elaborate state estimation based on filtering-smoothing algorithms (typically, an extended Kalman filter combined with a Rauch smoother [50, 16]).

6.2.1 Model structure determination

Even after selecting a class of models for identification, determining the exact model structure, e.g. which and how many parameters to include, remains a complex challenge. An important factor to consider, especially when using black-box models, is the so-called *bias/variance tradeoff* [81]: while adding more parameters reduces model bias, i.e., its ability to exactly represent the system, it simultaneously increases model variance, which affects its sensitivity to new data and its ability to extrapolate.

Simpler system identification techniques, like least squares or nonparametric frequency response estimation, can support model structure determination, as shown for instance in [50, 61, 111, 67]; in addition, a specific approach for structure selection of aerodynamic models has been proposed in [74]. In addition, general methods can be found, e.g., in [67], [1] and [81].

6.3 Closed-loop system identification

The identification of closed-loop systems is particularly challenging, as the controller not only tends to suppress suppresses and distort pilot inputs, but also creates correlations between the inputs and with unmeasurable noise - violating the basic assumptions of several identification methods and leading to their failure. A comprehensive study of available approaches to deal with this problem is provided in [29]; their application to rotorcraft identification is discussed in [13, 111].

6.4 Model validation

The definition of metrics for model validation and uncertainty quantification is still the subject of ongoing research. In rotorcraft modeling, the ultimate validation is performed by testing the model's predictive capability on data that were not used in the identification [76]; this process is known as *cross-validation* [67]. Model fidelity can then be measured using quantitative metrics, which are typically defined as a function of the *residuals* - i.e., of the errors between simulated model outputs and experimental data, for a set of quantities of interest.

An in-depth discussion of these metrics is provided in [83]; in general, they are built either as a weighted sum of the (root) mean squared residuals (where the weights are defined by engineering judgement), or by assuming that the residuals are Gaussian distributed and using results from Chi-square statistics. Both approaches have limitations: the first relies heavily on experience and thus cannot be considered as an absolute statement on model validity; the second, for the reasons seen in section 5.4, is only valid in specific cases and does not account for account for structural model errors. Therefore, the need for new metrics is still urgent; some examples of recent developments can be found, e.g., in [3] and [39].

7 SUMMARY AND FUTURE WORK

The problem of system identification applied to rotorcraft and next-generation VTOL was explored by reviewing the characteristics of available methods and model classes, with a special focus on the practical choices that the user has to make. Despite the wide range of options and numerous factors to take into account, this article shows that it is indeed possible to navigate toward the most effective choice, as long as the following points are clear:

1. The premises for the choice:
   (a) The intended use of the model
   (b) The extent of knowledge available about the modeled vehicle

   These points will help understand which model shall and can be identified.
2. The characteristics of the chosen model; these in turn influence the selection of data for the identification or the design of experiments.
3. The available data - in particular, how much useful information can be extracted from them, and whether any limitations are present.

Once the constraints imposed by these elements are clear, and obviously having the properties and limitations of the methods well known, the choice of the most suitable identification method becomes almost automatic.

The future eVTOL aircraft configurations developed in the Advanced Air Mobility (AAM) will present new system identification challenges and a combination of fixed-wing and rotary-wing identification methods (see for example [99]). The next step following this article will be to represent this decision-making process graphically, providing the user with a clear roadmap to guide their choices. To further streamline and refine this process, a potential advancement could involve creating a computerized database, such as a knowledge graph, that brings together all available options and can flexibly accommodate future developments.

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