THEORETICAL AND EXPERIMENTAL INVESTIGATIONS ON A SIX-COMPONENT ROTOR BALANCE

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September 10-13, 1985
London, England.

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#### Abstract

In addition to an intensive theoretical research it is necessary to verify and expand theoretical results with proper and sufficient testing in order to obtain improvements for the design of new helicopter systems. Therefore, in the DFVLR Institute for Flight Mechanics a rotor test stand for testing in large wind tunnels is operated. One of the measurement tools at this rotor test stand is the six component balance, that measures the six rotor forces and moments. As highly dynamic forces and moments have to be determined, the response of the balance is strongly dependent on the eigenfrequencies and damping in the system.

This paper concentrates on the estimation of the dynamic characteristics of the balance. The system rotor/balance is treated as two subsystems so that the balance can be considered as an isolated system. Three different methods are used for the determination of the dynamic characteristics: - A mathematical model with six degrees of freedom. - A transfer function measurement technique at the real system. - Two system identification techniques (Least Squares and Maximum Likelihood method, both working in the frequency domain).


## 1. INTRODUCTION

Wind tunnel testing with Mach scaled model rotors becomes more and more important for basic research and design of advanced helicopters. Often not only the steady state behaviour of the rotor system are of interest but also the dynamic characteristics in the case of various improving means i.e. new blade planforms and profiles, blade modal shape turning, higher harmonic control concepts and bearingless rotor systems. One possible way to get information about the dynamic response is to measure the blade bending and torsion moments using strain gauge bridges on the blades. These sensors in the rotating system are not able to measure the exact shear forces in the blades. So the calculation of the net rotor forces and moments from these signals becomes incorrect.

It is a common practice to measure the steady state rotor forces and moments by a balance which is nomally located in the fixed system [1]. All forces transfered through the balance are measurable. The calibration procedure of such balance can be very time consuming if highly couplings and nonlinearities occure in the system [2]. Nevertheless the desired set of equations in matrix notation is

$$
f_{o}=T_{0} \cdot s_{o}
$$

In this equations $f_{0}$ is the vector of the rotor forces and moments i.e.

$$
f_{o}^{T}=\left\{F_{x}, F_{y}, F_{z}, M_{x}, M_{y}, M_{z}\right\} ; \quad 6 \times 1
$$

and $s_{0}$ the vector of the sensors

$$
\mathrm{s}_{0}^{\mathrm{T}}=\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \ldots \mathrm{~s}_{\mathrm{n}}\right\} ; \quad \mathrm{n} \times 1
$$

The matrix $T$ describes the kinematic and electric relations between the sensor output (i.e. Volts) and the rotor forces and moments.

It is not necessary that the sensor vector contains only as many elements as the force vector. So the matrix $T$ has the dimension $6 \times n$. Normally the calibration procedure cannot give directly the elements of the matrix. For calibration, the balance will be loaded by increasing and de...creasing the six components of the vector $f_{o}$ individually. So one get each element in the $i-t h$ column of the $C_{0}$ matrix by a linear regression analysis of the calibration data set.

After six steps the matrix $C_{0}$ is determined and one can write for the linear case

$$
\begin{array}{lr}
s_{0}=C_{0} \cdot f_{0} & \begin{aligned}
\operatorname{dim} s_{0} & =n \\
\operatorname{dim} f_{o} & =6 \\
\operatorname{dim} C_{0} & =n x
\end{aligned} \\
T_{0}=C_{0}^{-1} & \text { if } n=6 \\
T_{0}=\left[C_{o}^{T} C_{0}\right]^{-1} \cdot C_{o}^{T} & \text { if } n>6 .
\end{array}
$$

If the balance system contains nonlinearities it is necessary to calibrate around the trim conditions i.e. a desired lift and torque. The calibration becomes very time consuming and results in a set of $T$ matrices which are valid only for the particular trim cases.

A more difficult case arises if it is necessary to measure the dynamic loads of a rotor system. Now the balance must be considered as a dynamic system with eigenfrequencies and eigenmodes. The measured loads are dependent on the dynamic system response. Several methods are existing to handle such problems e.g.

- mathematical models with the most important degrees of freedom,
- transfer function measurement technique at the real system,
- system identification techniques for determining the system matrices from measurements.

The application of these three techniques to the balance are described and the results were discussed.

## 2. THE SIX-COMPONENT BALANCE

Figure 1 shows the six component balance which is implemented in the rotor test stand (RTS) at the DFVLR Institute for Flight Mechanics. The balance is designed to measure the static and dynamic loads. This extended capability is achieved by a specific design. The principle of this balance is shown in Fig. 2. The construction consists of two steel plates, one lower plate and an upper plate which are connected via the force transducer systems. Four sensor equipments operate in z-direction, two in $y$-direction and one in $x$-direction. Additional a torque indicator is installed in the rotor shaft below the upper plate.

Each force transducer assembly is fitted with two strain gauge load cells and one piezo-electric force tranducer as shown in Fig. 3. The two strain gauge load cells are mechanically biased to operate at the middle of the characteristic curves to eliminate the hysteresis error which occurs around zero force. The electrical outputs of the cells are combined to form one signal whose voltage level is proportional to the loads. Calibration tests have shown that the linearity error in case of static laod is better than $0,1 \%$ of full range of 5000 N .

Generally a strain gauge load cell is able to measure static and dynamic loads. But in this case of application, the range of the dynamic forces can be very small in contrast to the static forces. To achieve a good sensitivity to dynamic loads a piezo-electric load cell is installed. In Fig. 4 typical frequency response curves of a strain gauge and a piezoelectric load cell are recorded. It shows the very good accuracy of the piezo-electric load cell while the strain gauge signal is distorted over the tested frequency range.

One problem in the design of multi component balances is the decoupling of the particular force and moment components. For the dedicated balance, flexible beams are fitted between the strain gauge and the piezoelectric cells. In this way the influences of shear forces are eliminated because they are smaller than the resolution of the load cells. In addition
the design principle allows a fully decoupling of the rotor torque and the other rotor forces and moments. This is achieved by a torque meter which is arranged between the plates. Shear forces and bending moments are eliminated by means of flexible elements between the shaft and the torque meter.

## 3. ANALYTICAL DESCRIPTION

In the early design phase a mathematical model of the balance was derived [3]. The major objective was, to see the influence of the load cell stiffness to the eigenfrequencies and eigenmodes. A six degree of freedom model was chosen for the description of the upper plate dynamics (moving part of the balance). The assumptions for the model are

- all deflections are small
- the damping in the system is neglectable
- the stiffness of the load cell systems are considerable smaller than the rest of the balance
- the eigenfrequencies of the support doesn't influence the balance dynamics.

Because the rotor support hardware and the control system are very complex a lumped mass model was built, Fig. 5. It was used to calculate the ..c.g. and the moments of inertia. The equations of motion were derived with the Langrangian formulas

$$
\frac{d}{d t}\left(\frac{\partial T}{\partial \dot{q}_{k}}\right)-\frac{\partial T}{\partial q_{k}}=Q_{k} \quad ; k=1 \ldots 6
$$

Therefore a set of generalized coordinates $q^{T}=(x, y, z, \psi, \theta, \phi)$ was chosen, Fig. 6. The origin coincidences with the c.g. of the balance which simplifies the equation of motion. The linear displacement at any location $i$ in the system is then given by

$$
d_{i}=R_{i} \cdot r+d_{0}
$$

with

$$
\begin{aligned}
& \mathrm{d}_{\dot{i}}^{\mathrm{T}}=(\mathrm{x}, \mathrm{y}, \mathrm{z}) \quad \text { net linear displacements } \\
& \mathrm{d}_{\mathrm{o}}^{\mathrm{T}}=\left(\mathrm{x}_{\mathrm{o}}, \mathrm{y}_{\mathrm{o}}, \mathrm{z}_{\mathrm{o}}\right) \text { vector of linear displacements of } \mathrm{c} \cdot \mathrm{~g} . \\
& \mathrm{r}^{\mathrm{T}}=(\psi, \theta, \phi) \quad \text { vector of Euler Angles }
\end{aligned}
$$

or

$$
\mathrm{d}_{i}=\left[I, \mathrm{R}_{\mathrm{i}}\right] \mathrm{q}=\mathrm{B}_{\mathrm{i}} \mathrm{q}
$$

with the matrix $R_{i}$ for the $i-t h$ location

$$
R_{i}=\left[\begin{array}{rrr}
-\eta & \zeta & 0 \\
\xi & 0 & -\zeta \\
0 & -\xi & \eta
\end{array}\right]_{\mathbf{i}}
$$

The derivation of the equations of motion is a straightforward procedure, so here only some remarks are given and then the results will be shown. One way to calculate the stiffness matrix is that the potential energy of each load cell system will be calculated individually. The derivatives are then given by

$$
Q_{i k}=-\frac{1}{2} \sum_{i=1}^{\mathrm{m}} \frac{\partial\left(\mathrm{~d}_{i}^{\mathrm{T}} \mathrm{C}_{\mathrm{i}} \mathrm{~d}_{\mathrm{i}}\right)}{\partial \mathrm{q}_{k}} ; \mathrm{m}=\begin{aligned}
& \text { number of load cell systems } \\
& \mathrm{C}_{\mathrm{i}}
\end{aligned}=\begin{aligned}
& \text { stiffness matrix of } i-t h \text { load } \\
& \text { cell system }
\end{aligned}
$$

The $C_{i}$ is here a diagonal matrix with one element $\neq$ zero because the load cell system works only in one orientation with the stiffness $c_{i}$.

For the exact determination of the load cell system stiffness, the complete arangement as shown in Fig. 3 was tested under tension forces. It shows that the flexible beam has a not negletable effect on the overall stiffness. Figure 7 shows the results of two different load cells. The flexible beam lowers the system stiffnes. The conclusions from this tests are that it is possible to manipulate the stiffness and the sensitivity by different combinations of load cell type and flexible beams.

As all effects were considered the following stiffness matrix was derived where the $c_{i k}$ are the stiffness of the i-th load cell system in the $k-t h$ direction and $\xi_{i}, \eta_{i}$, $\zeta_{i}$ the vector components from the c.g. to the load cell system connection to the upper plate.

$$
\begin{aligned}
& \mathrm{K}= \\
& \sum c_{i z} \quad 0 \quad-\sum c_{i z} \xi_{i} \quad \sum c_{i z}{ }_{i} \\
& \text { symm. } \\
& \sum\left(c_{i x} \zeta_{i}^{2}+c_{i z} \xi_{i}^{2}\right) \quad-\sum c_{i z} \eta_{i} \xi_{i} \\
& \sum\left(c_{i y} \zeta_{i}^{2}+c_{i z} n_{i}^{2}\right)
\end{aligned}
$$

The derivation of the kinetic energy leads to the mass matrix of a rigid body with moments of deviation.


Solving the homogenous differential equation

$$
M \ddot{q}+K q=0
$$

gives the eigenfrequencies and eigenmodes.
Figure 8 shows a typical result from calculations with one earlier model configuration. As mentioned before the stiffness in the load cell system is smaller than of the load cell itself. So these stiffness were varied to show the influence to the eigenfrequencies.

One can see that the four sensor systems which are working in $z$-direction have the major influence to the eigenvalues. The conclusions are that these stiffness must be determined exactly to get a good estimation of the balance response.

## 4. EXPERTMENTAL TRANSFER FUNCTION EVALUATION

The analytical investigations are good for principal considerations but not exact enough for determining the exact transfer function. Therefore a test setup as shown in Fig. 9 was used to measure the transfer functions between the hub and each load cell system. The balance was excited by an electro dynamic shaker in each rotor force and moment component individually. A frequency response analyser was used to measure the force transmitted to the hub by the shaker and the force in the dedicated load cell. After a lot of averages the analyser computes the transfer function. In Fig. 10 one can see a typical plot for the $z_{1}$ load cell in case of $M_{x}$ excitation. The two curves indicate the eigenfrequencies and the phase lag but the gains and the phase at the rotor harmonics are most of interest. One can write now the following relation

$$
s_{v}=C_{v} \cdot f_{v}
$$

where $f_{\nu}$ is the complex vector of the excitation, $s_{\nu}$ the corresponding load cell vector and $C_{\nu}$ the transfer function. All terms are only valid at the
dedicated frequency e.g. the rotor harmonic $\nu$. For the case of a single input - single output system this equation has the form

$$
\binom{s_{c}}{s_{s}}_{\nu}=g \cdot\left[\begin{array}{rrr}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{array}\right]_{\nu} \cdot\binom{f_{c}}{f_{s}}_{\nu}
$$

where

| $s_{c}, f_{c}$ | are the cos part of the signals |
| :--- | :--- |
| $s_{s}, f_{s}$ | are the sin part of the signals |
| $\phi$ | is the phase lag between input-output |
| $g$ | is the gain of the system |
| $v$ | is the dedicated frequency |

Now one can use this relation for each force - load cell combinaton so that the resulting vectors and the matrix have the dimension $[s]=2 m$, $[\mathrm{f}]=2 \mathrm{n},[\mathrm{C}]=2 \mathrm{~m} \cdot 2 \mathrm{n}$.

During the frequency response test it is not necessary to apply both parts of $f v$ because one can set $t_{o}$ for the excitation in the manner that one part of $f_{v}$ ( $f_{c}$ or $f_{s}$ ) becomes zero. After all force - load cell combinations where measured over the frequency range of interest (e.g. at the desired rotor harmonics) the transfer matrices for the dedicated frequency can be calculated

$$
T_{v}=\left[c_{v}^{T} C_{v}\right]^{-1} c_{v}^{T}
$$

In the case of dynamic measurement, the $f_{\nu}$ vectors are constructed by means of harmonic analysis of the $s_{i}$ time signals and then solving the matrix equation for each frequency $v$.

$$
f_{v}=T_{v} \cdot s_{v}
$$

Now it becomes obvious that the static case is only a special condition with $\phi=0$. The data reduction program at the RTS use this general equation for the cases $\nu=0,1, \ldots n$ (rotor harmonics) for the straightforward computation of the rotor forces and moments.

However, this way of solution can contain a lot of errors. One of the major sources are the differences in the frequencies $v$ and the actual frequency during the wind tunnel runs. If the derivatives $\frac{\mathrm{dT}}{\mathrm{d}}$ are not equal zero at $\omega=\nu$ the measured $T_{\nu}$ matrix is not valid at $\omega \neq \nu$ which leads to considerable errors in the $f_{v}$ vector. For small deviations in $\omega$ one can expand the equation to

$$
f_{\omega}=\left(T_{\nu}+\frac{d T(\omega)}{d \omega} \quad \omega=v \cdot d \omega\right) \cdot s_{\omega} \cdot
$$

but this is not satisfactory for a general formulation of the problem. A further error source can be the data path and the analysis if there unidentified or unexpected phase shifts and gains occour. Nevertheless this analyse technique is only practicable in steady state conditions because one must perform a lot of averages to get proper results from the harmonic analysing. Therefore further analytical and experimental investigations were made to come closer to a satisfactory solution of this measurement problem. The first step was a simulation study with different data reduction techniques at a 2 DOF model [4]. Besides the presented procedure three different Kalmanfilter were formulated for this problem. Special emphasis was taken to the cases with errors in the system matrix respectively in the T matrix. The simulation results have shown that the Kalmanfilter analysis leads to better results than the frequency domain technique, Fig. 11. So in the next chapter two systems indentifications procedures are discussed which were used to estimate the balance parameters.

## 5. SYSTEM IDENTIFICATION IN FREQUENCY DOMAIN

The general problem of system identification (Fig. 12) is to determine certain characteristics of the physical systems from experimental test data. Measurements are made of external inputs $u$ and resulting output responses $y$ which depend on the system characteristics to be determined. A mathematical model must be selected that adequately represents the physical system to be measured. The parameters in the model can be evaluated from the test data by means of statistical methods.

The rotor balance is a system which has 6 degrees of freedom and many parameters to be evaluated. The eigenfrequencies of the balance are high, so that a high sampling rate should be taken in the data acquisition. The eigenmodes of the balance are spread over a wide frequency range. It is not possible to use only one input to excite all these modes at one test. A good estimate of the system parameters can only be obtained if adequate information about the eigenmodes of the system is provided in the measurement data. For technical reasons it is not possible to apply several inputs simultaneously within one test. Therefore independent tests (multiple-run) are made with different inputs at the same test conditions. By applying a multiple-run-technique in system identification the system parameters were evaluated by using all information from separate tests.

The application of the multiple-run-technique for parameter identification in time domain not only enlarges the data to be evaluated but also increases the number of parameters (initial values of states and bias of measurements for each run) to be identified. This leads to high requirements not only for computing time but also for storage.

For the system identification in the frequency domain the data can be reduced by choosing the frequency range of interest. The system identification in the frequency domain is non-biased if the information of the zero-frequency of Fourier variables is not taken into account $[5,6,7]$.

## 6. METHODS OF ANALYSIS

The basic approaches of parameter identification in the frequency donain presented in this paper are 1) Method of Least Squares and 2) MaximumLikelihood Method, both with the application of multiple-run techniques.

Method of Least Squares (LS)

The mathematical model used for this approach is

$$
j \omega x(\theta)=A(\theta) x(\omega)+B(\theta) u(\omega)
$$

where

$$
j=\sqrt{-1}
$$

$\omega$ the independent variable of frequency,
$x(\omega)$ state vector
$u(\omega)$ control vector

A( $\theta$ ) stability matrix
$B(\theta)$ control matrix and
$\theta$ unknown system parameter vector

The cost function is
(1) $J(\theta)=\sum_{i k} \sum_{k} e^{*}\left(\omega_{k}, i\right) e\left(\omega_{k}, i\right)$
where $i$ number of run
$k \quad$ dedicated frequency
$e\left(\omega_{k}, i\right)$ equation error and
$e^{*}\left(\omega_{k}, i\right)$ complex conjugate of $e$.

Maximum-Likelihood Method (ML)

The mathematical model used in this approach is composed of equations of state and equations of observation. It can be written in the frequency domain as following:

Equations of state:

$$
j \omega x(\theta)=A(\theta) x(\omega)+B(\theta) u(\omega) .
$$

Equations of observation:

$$
y(\omega)=C(\theta) x(\omega)+D(\theta) u(\omega)
$$

where

$$
\begin{aligned}
& y(\omega) \text { observation vector } \\
& C(\theta) \text { and } D(\theta) \text { are transformation matrices. }
\end{aligned}
$$

The cost function of the Maximum-Likelihood estimate is given by

$$
\begin{equation*}
J(\theta)=\sum_{i} \sum_{k} \varepsilon^{*}\left(\omega_{k}, i\right) S^{-1} \varepsilon\left(\omega_{k}, i\right)+\ln |S| \tag{2}
\end{equation*}
$$

where

$$
\begin{aligned}
& \varepsilon\left(\omega_{k}, i\right) \text { measurement error vector } \\
& \varepsilon^{*}\left(\omega_{k}, i\right) \text { transpose complex conjugate of } \varepsilon \\
& S
\end{aligned}
$$

The LS estimate of the unknown parameters is obtained by minimizing the cost function (1) with respect to $\theta$. This leads to a linear system of equations in the parameter $\theta$ which can be solved for $\theta$ exactly. The LSestimate is simple and effective but the estimated parameters are not free from biases.

In the ML parameter identification the cost function (2) will be minimized w.r.t. $\theta$. This leads to a system of non-linear equations which can be solved iteratively by the principle of Quasi-linearization. In contrast to LS-estimates the ML-estimates are unbiased, but a-priori-values of the parameter must be given for starting the estimation.

To combine the both approaches within one estimation procedure one can first quickly get the primary values of the parameters by using LS-estimates and then start the $M L-t e c h n i q u e$ to improve these results and to obtain unbiased estimates.

## 7. MATHEMATICAL MODEL FOR PARAMETER IDENTIFICATION

The balance was excited through the controls $u$. Under consideration of damping due to frictions the equations of motion can be written:

$$
\begin{equation*}
\mathrm{M} \ddot{\mathrm{q}}+\mathrm{D} \dot{\mathrm{q}}+\mathrm{Kq}=\mathrm{F} \mathrm{u} \tag{3}
\end{equation*}
$$

where

```
q state vector q ( }\textrm{x},\textrm{y},\textrm{z},\psi,0,\phi
    u control vector u(Fx, Fy, Fz, Mx, My)
    M inertia matrix
    D damping matrix
    K stiffness matrix
    F control matrix
```

The spring force $s$ which was measured from the force transducer $i$ according to Hooke's law can be written as

$$
s_{i}=c_{i} d_{i}
$$

or, in matrix form:
(4) $\quad s=C d$
where

$$
\begin{aligned}
& \mathrm{s}^{\mathrm{T}}=\left(\mathrm{s}_{\mathrm{x}}, \mathrm{~s}_{\mathrm{yl}}, \mathrm{~s}_{\mathrm{y} 2}, \mathrm{~s}_{\mathrm{z} 1}, \mathrm{~s}_{\mathrm{z} 2}, \mathrm{~s}_{\mathrm{z} 3}, \mathrm{~s}_{\mathrm{z} 4}\right) \\
& \mathrm{d}^{\mathrm{T}}=(\mathrm{x}, \mathrm{y} 1, \mathrm{y} 2, \mathrm{z} 1, \mathrm{z} 2, \mathrm{z} 3, \mathrm{z} 4) \\
& \mathrm{C} \text { is a diagonal matrix. }
\end{aligned}
$$

It can be differentiated with respect to time, giving

$$
\dot{s}=c \dot{d}
$$

and

$$
\ddot{s}=c \ddot{d}
$$

The local displacement of the load cell $i$ was given in section 3 .

$$
d_{i}=B_{i} q
$$

The matrix form is given by
(5) $\quad d=B q$
where

$$
B=\left[\begin{array}{cccccc}
1 & 0 & 0 & -\eta_{x} & \zeta_{\mathrm{x}} & 0 \\
0 & 1 & 0 & \xi_{\mathrm{y} 1} & 0 & -\zeta_{\mathrm{y} 1} \\
0 & 1 & 0 & \xi_{\mathrm{y} 2} & 0 & -\zeta_{\mathrm{y} 2} \\
0 & 0 & 1 & 0 & -\xi_{z 1} & \eta_{\mathrm{z} 1} \\
0 & 0 & 1 & 0 & -\xi_{z 2} & \eta_{\mathrm{z} 2} \\
0 & 0 & 1 & 0 & -\xi_{z 3} & \eta_{z 3} \\
0 & 0 & 1 & 0 & -\xi_{z 4} & \eta_{z 4}
\end{array}\right]
$$

The time derivatives of $d$ are
and

$$
\begin{aligned}
& \dot{\mathrm{d}}=\mathrm{B} \dot{\mathrm{q}} \\
& \ddot{\mathrm{~d}}=\mathrm{B} \ddot{\mathrm{q}} .
\end{aligned}
$$

The relation between $q$ and $s$ can be obtained from equation (4) and (5) using the matrix operation

$$
q=\left(B^{T} B\right)^{-1} B^{T} K^{-1} s
$$

The variable transformation from $q$ to $s$ leads to indefinite transformation if the dimension of $n$ is not equal to 6 . The dimension of $n$ can be reduced to 6 by introducing the resultant of the two vectors $s_{z 3}$ and $s_{z 4}$ • The vector $s$ is then

$$
s^{T}=\left(s_{x}, s_{y 1}, s_{y 2}, s_{z 1}, s_{z 2}, s_{z 34}\right)
$$

The vector $d$ and the matrix $B$ become

$$
\begin{aligned}
\mathrm{d}^{\mathrm{T}} & =(\mathrm{x}, \mathrm{yl}, \mathrm{y} 2, \\
\mathrm{z} 1, \mathrm{z} 2, & \mathrm{z} 34) \\
\mathrm{B} & =\left[\begin{array}{cccccc}
1 & 0 & 0 & -\eta_{\mathrm{x}} & \zeta_{\mathrm{x}} & 0 \\
0 & 1 & 0 & \xi_{\mathrm{y} 1} & 0 & -\zeta_{\mathrm{y} 1} \\
0 & 1 & 0 & \xi_{y 2} & 0 & -\zeta_{y 2} \\
0 & 0 & 1 & 0 & -\xi_{z 1} & \eta_{z 1} \\
0 & 0 & 1 & 0 & -\xi_{z 2} & \eta_{z 2} \\
0 & 0 & 1 & 0 & -\xi_{z 34} & \eta_{z 34}
\end{array}\right]
\end{aligned}
$$

Then the variable transformation from $q$ to $s$ is

$$
\mathrm{q}=\mathrm{B}^{-1} \mathrm{~K}^{-1} \mathrm{~s}
$$

By introducing the variables s the equation (3) becomes

$$
\begin{equation*}
\ddot{s}+M^{-1} D \dot{s}+M^{-1} K s=B_{F} u \tag{6}
\end{equation*}
$$

where

$$
B_{F}=M^{-1} B \quad K \quad F
$$

The equation (6) can be written as a first-order system of dimension 12 by introducing the vector $x$

$$
\mathrm{x}^{\mathrm{T}}=\left(\dot{s}_{\mathrm{x}}, \dot{s}_{\mathrm{y} 1}, \dot{s}_{\mathrm{y} 2}, \dot{s}_{z 1}, \dot{s}_{z 2}, \dot{s}_{z 34}, \mathrm{~s}_{\mathrm{x}}, \mathrm{~s}_{\mathrm{y} 1}, \mathrm{~s}_{\mathrm{y} 2}, \mathrm{~s}_{\mathrm{zl}}, \mathrm{~s}_{z 2}, \mathrm{~s}_{\mathrm{z} 34}\right)
$$

which leads to

$$
\dot{x}=\left[\begin{array}{cc}
M^{-1} D & M^{-1}  \tag{7}\\
I & 0
\end{array}\right] x+\left[\begin{array}{c}
B_{F} \\
0
\end{array}\right] u
$$

Equation(7) can be represented in the frequency domain by using Fourier transformation. Non periodic signals in the state variables were considered as described in $[5,6]$.

The equations of observation used here are

$$
y=I x
$$

where $I$ is an identity matrix.
For the parameter identification the diagonal elements of matrix $M^{-1} D$ and all elements of matrixes $M^{-1} \mathrm{~K}$ and $\mathrm{B}_{\mathrm{F}}$ are evaluated.

## 8. IDENTIFICATION RESULTS AND DISCUSSION

The sampling rate of the measuements was 0.0025 sec . The Nyquist frequency is 400 Hz . The upper boundary of the frequency range of interest is about 140 Hz which corresponds to $8 / \mathrm{rev}$ of the rotor. It is evident that high noise level could occur in the measurements at high frequencies. Therefore a band limit at 200 Hz was taken by using a 6-order low pass filter during the data acquisation.

For the evaluation a frequency range from 10 to 125 Hz was considered. The eigenvalues of the balance have been calculated from the simulated model whose parameters were estimated by using system identification techniques. For comparison, the elgenfrequencies which were calculated from the analytical method with those calculated from the simulated model are shown in Fig. 13. One can see that only four of the six eigenfrequencies are in the same range. Two modes of the analytical model have considerable higher eigenfrequencies than the true system. One explanation for these discrepancies is that these modes are not considered in the analytical 6-DOF model, but a valid answer to this problem can only be given by an experimental mode analysis.

Plots of transfer functions of the simulated model and measurements are shown in Fig. 14. Figures $14(a)$ and (b) illustrate the transfer functions $\dot{s}_{x} / F_{x}$ and $\dot{s}_{x} / M_{y}$ respectively. A good agreement is obtained in the medium frequency range. Differences at high frequencies are due to the high noise level from the $x$-force transducer measurement for $\omega>80 \mathrm{~Hz}$. Figure 14 (c) shows the curve fit of the transfer function $\dot{s}_{y 1} / F_{y}$. It is not very good in quantity. The energy level of $\dot{S}_{y}$ - measurements are relatively lower than other force measurements, so that the parameters in the $s$-equation cannot be estimated exactly. Figure 14 (d) shows the transfer function $\dot{s}_{z l} / F_{z}$. The curves of transfer functions of $z$-force transducers provide a good fitness for other input cases, too. One dominant system response is caused by the $\mathrm{F}_{z}$ force whereas the other inputs excite different eigenmodes.

## 9. CONCLUSIONS

The application of three different methods for estimating the dynamics of the rotor balance leads to the following results.

- The dedicated balance cannot be described with a 6-DOF rigid body model.
- The transfer function measurement technique gives good estimates in the calibration case but in the measurement case a lot of unidentifyable errors leads to inexact results.
- System identification can give a satisfactory estimation of the inputoutput relations at the balance. Especially in combination with a data reduction using Kalman filter techniques, this method seems to be a practicable way to get valid dynamic force measurements.


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Figure 1. Six Component Rotor Balance


Figure 2. Principle of the Rotor Balance


Figure 3. Force Transducer Assembly


Figure 12. Principle of System Identification Using Output-Error-Method


Figure 13. Comparison of Computed Eigenfrequencies


Figure 14. Fit of Transfer Functions

- Measurement
+     + Parameter Identification

